RAJEEV GANDHI MEMORIAL COLLEGE OF ENGINEERING AND TECHNOLOGY

AUTONOMOUS

NANDYAL-518501, KURNOOL DIST., A.P., INDIA

DEPARTMENT OF MECHANICAL ENGINEERING

B.Tech –IV Year – I Sem

DEPARTMENT OF MECHANICAL ENGINEERING

(ESTD-1995)

Course Material

Mechanics of Composite Materials

Prepared By

M. ASHOK KUMAR

Assoc. Professor,

Department of Mechanical Engineering, RGMCET.

Dr K. THIRUPATHI REDDY
B.E(Mech), M.Tech,Ph.D,MISTE,ASME
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NANDYAL-518501, KURNOOL DIST. A.P
Approved by AICTE, New Delhi, Affiliated to J.N.T.University, Anantapur Accredited by NBA, New Delhi, participated in World Bank aided TEQIP
Ph: 08514-275201, 275203,275204, Fax: 08514-275123
DEPARTMENT OF MECHANICAL ENGINEERING

CONTENTS

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Autonomous

MECHANICAL ENGINEERING

IV B.Tech, I-Sem (ME)

T	C
3+1*	3

[A0338158] MECHANICS OF COMPOSITE MATERIALS

(Department Elective-III)

OBJECTIVE:

- This course provides students a background in modern lightweight composite materials which are being used in an ever-increasing range of applications and industries. Basic knowledge of composites will allow engineers to understand the issues associated with using these materials, as well as gain insight into how their usage differs from metals, and ultimately be able to use composites to their fullest potential.

OUTCOMES: At the end of the course, the student will be able to:

- Know the fundamental concepts of composite materials.
- Understand various manufacturing methods of composites.
- Learn macro and micro-mechanical analysis of a lamina.
- Understand failure theories, and to determine the strength of a lamina.

UNIT-I

Introduction to Composite Materials: Introduction, Classification: Polymer Matrix Composites. Metal Matrix Composites, Ceramic Matrix Composites, Carbon–Carbon Composites, Fiber. Reinforced Composites and nature-made composites, and applications.

UNIT-II

Reinforcements: Fibres- Glass, Silica, Kevlar, carbon, boron, silicon carbide, and boron carbide. fibres. Particulate composites, Polymer composites, Thermoplastics, Thermosets, Metal matrix and ceramic composites.

UNIT-III

Manufacturing Processes: Hand lay-up, Spray lay-up, Vacuum bagging, Pultrusion, Resin Transfer Molding (RTM), Filament winding.

UNIT-IV

Macro-Mechanical Analysis of a Lamina: Introduction, Definitions: Stress, Strain, Elastic Moduli, Strain Energy. Hooke's Law for Different Types of Materials – Anisotropic material, monoclinic material and orthotropic material, Hooke's Law for a Two Dimensional Unidirectional Lamina - Plane Stress Assumption, Reduction of Hooke's Law in Three Dimensions to Two Dimensions, Relationship of Compliance and Stiffness Matrix to Engineering Elastic Constants of a Lamina, Angle Lamina.

UNIT-V

Hooke's Law for a Two-Dimensional Angle Lamina, Engineering Constants of an Angle Lamina, Invariant Form of Stiffness and Compliance Matrices for an Angle Lamina, Strength Failure theories of an angle lamina- Maximum stress Failure Theory, Tsai–Hill Failure Theory, Tsai–Wu Failure Theory.

UNIT-VI

Micro-Mechanical Analysis of a Lamina: Introduction, Volume and Mass Fractions, Density, and Void Content, Evaluation of the Four Elastic Moduli – Longitudinal young's modulus, Transverse young's modulus, Major Poisson's ratio and In-plane shear modulus by Strength of Materials Approach, Semi Empirical Models, Ultimate Strengths of a Unidirectional Lamina- Longitudinal tensile strength, Transverse tensile strength, Longitudinal compressive strength, Transverse compressive strength. Ir

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strength.

TEXT BOOKS:

1. Mechanics of Composite Materials- Autar K. Kaw, 2/e, CRC Pubi.
2. Analysis and performance of fibre Composites, B. D. Agarwal and L.J. Broutman Wiley- Inter science,

REFERENCE BOOKS:

1. Engineering Mechanics of Composite Materials- Isaac and M Daniel, Oxford Univ. Press.
2. Mechanics of Composite Materials, R. M. Jones, Mc Graw Hill Company, New York.
3. Composite Materials Science and Engineering, Kishan K. Chawla, Springer.
4. Analysis of Laminated Composite Structures, L.R. Calcote, Van Nostrand Rainfold, New York,
5. Machanics of Composite Materials and Structures, madhujit Mukhpadhyay, New York.

DEPARTMENT OF MECHANICAL ENGINEERING(Autonomous) :: NANDYAL - 518 501

FIRST SEMESTER CENTRAL TIME TABLE FOR 2020 -2021 ACADEMIC YEARW.e.f:12.02.2021

DAY	PERIOD → / CLASS	9.30-10.20	10.20-11.10	11.30-12.20	12.20-01.10	02.10-03.00	03.00-03.50	03.50-04.50
MON	II-A	MOS	PYTH	T.D		MSM	NM	AARC
	II-B	MSM	AARC	MOS	NM	T.D		PYTH
	II-C	NM	TD	AARC	PYTH	MOS & MSM LAB		
	III-A	EEA	DEM-I	TE	DOM	TE,D&I & CAD LAB		
	III-B	MT	DOM	CAD	MM	DME-I		EEA
	III-C	TE	TE,D&I & CAD LAB			DOM	MT	EEA
	IV-A	OR	C/C	FEM	MCM	PM-II / CAM /MP LAB		
	IV-B	NCES	MCM	PM-II	NCES	OR	FEM	C/C
	IV-C	FEM	PM-II / CAM / MP LAB			NCES	C/C	
TUE	II-A	PYTH	MOS & MSM LAB			BE	MSM	MOS
	II-B	NM	PYTH LAB			TD	PYTH	MSM
	II-C	AARC	MSM	T.D		MOS	NM	TD
	III-A	MM	MT	CAD	EEA	MT	TE	DOM
	III-B	DME-I	TE	EEA	DOM	TE,D&I & CAD LAB		
	III-C	MT	DOM	MM	DME-I	CAD	EEA	TE
	IV-A	C/C	MCM	OR	MCM	NCES	PM-II	RAC
	IV-B	OR	FEM	C/C	RAC	PM-II / CAM / MP LAB		
	IV-C	NCES	FEM	RAC	C/C	RAC	MCM	OR
WED	II-A	NM	AARC	BE	PYTH	T.D		MOS
	II-B	MSM	BE	T.D		MOS	PYTH	NM
	II-C	BE	PYTH	MOS	MSM	NM	BE	AARC
	III-A	DOM	TE	DME-I	CAD	MT	MM	EEA
	III-B	MM	TE,D&I & CAD LAB			DOM	TE	MT
	III-C	EEA	TE	DME-I	DOM	TE,D&I & CAD LAB		
	IV-A	RAC	PM-II	NCES	OR	MCM	FEM	C/C
	IV-B	FEM	PM-II / CAM / MP LAB			C/C	RAC	OR
	IV-C	RAC	MCM	PM-II	NCES	PM-II / CAM / MP LAB		
THU	II-A	BE	MSM	NM	MOS	MOS & MSM LAB		
	II-B	NM	MOS & MSM LAB			BE	MSM	MOS
	II-C	MSM	PYTH LAB			MOS	BE	PYTH
	III-A	TE	EEA	MM	DME-I	TE,D&I & CAD LAB		
	III-B	CAD	MT	DOM	TE	EEA	DME-I	MM
	III-C	MM	MT	CAD	DME-I	TE	MT	MM
	IV-A	FEM	PM-II	RAC	OR	PM-II / CAM / MP LAB		
	IV-B	RAC	C/C	MCM	NCES	OR	RAC	FEM
	IV-C	MCM	OR	NCES	FEM	RAC	MCM	OR
FRI	II-A	PYTH	PYTH LAB			MOS	MSM	PYTH
	II-B	Placement & Training		NM	MSM	MOS & MSM LAB		
	II-C	NM	MOS & MSM LAB			PYTH	MSM	MOS
	III-A	CAD	MM	DOM	DME-I	EEA	MT	TE
	III-B	TE	EEA	MM	MT	TE,D&I & CAD LAB		
	III-C	DOM	MM	CAD	TE	EEA	DME-I	
	IV-A	MCM	NCES	FEM	C/C	RAC	NCES	OR
	IV-B	PM-II	FEM	MCM	NCES	PM-II / CAM / MP LAB		
	IV-C	C/C	OR	NCES	MCM	FEM	PM-II	FEM
SAT	II-A	AARC	TD	MSM	NM	NM	Placement & Training	
	II-B	MOS	BE	PYTH	AARC	PYTH	AARC	MOS
	II-C	Placement & Training		MOS	MSM	TD	PYTH	NM
	III-A	MT	TE,D&I & CAD LAB			DOM	DME-I	MM
	III-B	DOM	MM	DME-I	EEA	TE	MT	CAD
	III-C	EEA	MM	MT	DOM	TE,D&I & CAD LAB		
	IV-A	NCES	PM-II / CAM / MP LAB			C/C	RAC	FEM
	IV-B	MCM	C/C	RAC	MCM	PM-II	OR	NCES
	IV-C	PM-II	C/C	RAC	OR	PM-II, CAM & PM LAB		

II B.Tech:

NM & PT:Dr. P. Sreedevi(A,B&C)

Python : Mr. V. Ravi Kanth(A,B&C)

MOS : Dr. G. Venkatesh(A,B&C)

MSM : Dr. Syed Altaf Hussain (A), Mr.K.Viswanath (B&C)

TD : Dr. V. Siva Reddy(A,B&C)

BE:Dr.Nayab Rasool (A,B,C)

AARC :Mr.Y.Rajaobul Reddy(A,B&C)

Python Lab: Mr.V.Ravi Kanth(A,B&C)

MOS Lab: Dr.BSR &Mr.N.U(A)

Dr. BSR & Mr.Vinrendra (B)

Dr.YSK Reddy &Dr.G.V(C)

MSM Lab:Dr.SAH &Mr.Alamgir - (A)

Dr. AshifPerwez&Dr.YSKR(B)

)

III B.Tech:

EEA : Dr.G.C.Venkataiah (A & B), Mrs.K.PushpaLatha (C)

DME-I: Dr.Syed Altaf Hussain (A) Dr.K.SudhaMadhuri (B&C)

T.E : Dr.B.Rama Krishna (A&B), Mr.John Babu.T (C)

DOM : Dr.V.Nageswar Reddy (A&B), Mr.B.ChinnaAnkanna (C)

MM : Mr.Dinesh Babu.B (A&B), Mr. MD.Alamgir (C)

MT : Mr.Khaja Gulam Hussain(A&B) Dr.Uffaith Hussain Quadri(C)

CAD :Mr.Suresh.B (A,B & C)

D&I LAB: Dr.VNR&Mr.BCA - (A)

Dr.MAK &Dr.VNR - (B)

Mr.BCA/KAN[M]/Dr.MAK-(C) TE LAB:

Dr.BRK & BDB- (A)

Mr.BDB & Dr.BRK - (B)

Mr.John Babu/Dr.Razak -(C)

CAD LAB: Mr.B.Suresh& Mr.Anees-(A),

Mr.MD.Anees/Suresh – (B)

Mr.MD.Anees/Dr.YSKR –(C)

IV B.Tech:

CAD/CAM:Mr.Y.Suresh Babu (A&B), Mr.K.Viswanath (C)

OR : Mr.K.Aswarthanarayana (A,B&C)

FEM : Mr.N.Upendra (A&B), Dr.UpendraRajak

RAC : Dr.V.Chandra Sekhar (A&B), Dr. Y. Siva Kumar Reddy (C)

NCES : Mr. B.Veerendra (A&B), Mr. MD. Alamgir (C)

MCM : Dr. M. Ashok Kumar (A&B) Mr. B. Chinna Ankanna(C)

PM-II: Dr. Manoj Panchal (A,B&C)

CAM LAB:Mr.KGH& YSB - (A)

Mr.YSB&Dr.KSM- (B)

Dr.KSM&Mr.KGH- (C)

PM-II LAB:Dr.ManojPanchal&Dr.Quadri(A)

Mr.N.Upendra&Dr.Qu

Dr.K.Viswanath/Dr.Q

Dr.ManojPanchal(C)

Mini Project:Dr.Ashif

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(ESTD-1995)

Academic Diary for II-B.Tech., I-Semester (R19)
Academic Year: 2020-21 (After Mid-I exams)

	Jan-21		Feb-21		Mar-21		Apr-21	
Day	Date	Class	Date	Class	Date	Class	Date	Class
Sun								
Mon			1	5	1	29		
Tue			2	6	2	30		
Wed			3	7	3	31		
Thu			4	8	4	32	1	
Fri	1		5	9	5	33	2	Good Friday
Sat	2		6	10	6	34	3	II-I End
Sun	3		7		7		4	
Mon	4		8	11	8	35	5	BJR's B'Day
Tue	5		9	12	9	36	6	II-I End
Wed	6		10	13	10	37	7	
Thu	7		11	14	11	Sivaratri	8	II-I End
Fri	8		12	15	12	38	9	
Sat	9		13	16	13	39	10	II-I End
Sun	10		14		14		11	
Mon	11		15	17	15	40	12	Labs
Tue	12		16	18	16	Mid-II	13	Ugadi
Wed	13		17	19	17	Mid-II	14	BRA's B'Day
Thu	14		18	20	18	Mid-II	15	Labs
Fri	15		19	21	19	Mid-II	16	Labs
Sat	16		20	22	20	Mid-II	17	Labs
Sun	17		21		21		18	
Mon	18		22	23	22	Mid-II	19	**II-Sem**
Tue	19		23	24	23	Mid-II	20	
Wed	20		24	25	24	Preparation	21	
Thu	21		25	26	25	Preparation	22	
Fri	22		26	27	26	Preparation	23	
Sat	23		27	28	27	II-I End	24	
Sun	24		28		28		25	
Mon	25				29	II-I End	26	
Tue	26				30		27	
Wed	27	1			31	II-I End	28	
Thu	28	2					29	
Fri	29	3					30	
Sat	30	4						
Sun	31							

1. **Second Spell of Instructions** : **27/01/2021 – 15/03/2021**
2. **Slot for Assignment-II** : **10/03/2021 – 15/03/2021**
3. **Mid-II Examinations** : **16/03/2021 – 23/03/2021**
4. **Preparation** : **24/03/2021 – 26/03/2021**
5. **End Examinations** : **27/03/2021 – 10/04/2021**
6. **End Practical Examinations** : **12/04/2021 – 17/04/2021**
7. **Commencement of Class Work for II-Sem:** **19/04/2021 Onwards**

C.E. PRINCIPAL

Date: 24-01-2021

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Our Institution Vision

- To develop this rural based engineering college into an institute of technical education with global standards
- To become an institute of excellence which contributes to the needs of society
- To inculcate value based education with noble goal of " Education for peace and progress"

Our Institution Mission

- To build a world class undergraduate program with all required infrastructure that provides strong theoretical knowledge supplemented by the state of art skills
- To establish postgraduate programs in basic and cutting edge technologies.
- To create conductive ambiance to induce and nurture research
- To turn young graduates to success oriented entrepreneurs To develop linkage with industries to have strong industry institute interaction.
- To offer demand driven courses to meet the needs of the industry and society To inculcate human values and ethos into the education system for an all-round development of students.

Our Institution Quality Policy

- To improve the teaching and learning
- To evaluate the performance of students at regular intervals and take necessary steps for betterment
- To establish and develop centers of excellence for research and consultancy
- To prepare students to face the competition in the market globally and realize the responsibilities as true citizen to serve the nation and uplift the country's pride.

Department of Mechanical Engineering Vision

Vision:

To be a center of excellence by offering UG, PG and Research programs in cutting edge technologies of Mechanical Engineering in collaboration with industries

Department of Mechanical Engineering Mission

- To Produce Mechanical Engineers who are exceptionally competent, disciplined and have a sense of devotion to their profession by adapting modern teaching and learning process.
- To establish modern laboratory facilities to impart quality education in association with Industry- Institute interaction.
- To inculcate research orientation among the student community.

Department of Mechanical Engineering Program Specific Outcomes (PSO's)

1. The graduate will be able to design systems, components or process for broadly defined engineering technology problems appropriate to programme educational objectives
2. The graduates will be able to apply modern engineering tools viz., CAD/CAM packages for modeling, analysis and predicting simple to complex engineering activities with an understanding of the limitations
3. The graduate will be able to apply oral and graphical communication in both technical and non-technical environment
4. The graduate will be able to engage in self directed continuing professional development and have a strong commitment to address ethical and professional responsibilities.

Department of Mechanical Engineering Program Educational objectives (PEO's)

1. To apply modern computational, analytical, simulation tools and techniques to address the challenges faced in mechanical and allied engineering streams.
2. To Plan, design, construct, maintain and improve mechanical engineering systems that are technically sound, economically feasible and socially acceptable to enhance quality of life.

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3. To Exhibit professionalism, ethical attitude, team spirit and pursue lifelong learning to achieve career and organizational goals
4. To communicate effectively using innovative tools and demonstrates leadership & entrepreneurial skills.

Department of Mechanical Engineering Program Outcomes (PO's) - Engineering Graduates will be able to:

1. Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. Design/development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.
6. The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

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10. Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

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Lesson Plan

NAME OF THE FACULTY: Dr. M.ASHOK KUMAR **ACADEMIC YEAR:** 2020-2021

CLASS/SEM: IV B.TECH/ISEM **TOTAL HOURS: 50**

NAME OF THE SUBJECT: [A0338158] MECHANICS OF COMPOSITE MATERIALS

S.No	DATE	TOPIC	HOURS	REMARKS
		Introduction to Composite Materials: Classification: Polymer Matrix Composites (PMCs), matrix materials,reinforcements used in PMCs Metal Matrix Composites(MMCs) Ceramic Matrix Composites(CMCs) Carbon–Carbon Matrix Composites (CCMCs), Fiber Reinforced Composites	7	I
		nature-made composites, and applications Reinforcements: Fibres, characteristics Glass Fiber, types Silica fiber Kevlar fiber Carbon fiber Boron fiber Boron carbide fiber, silicon carbide fiber. Particulate composites	10	II
		Introduction to Manufacturing Processes:, Hand lay-up Spray lay-up, Vacuum bagging, Pultrusion,	9	

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	Resin Transfer Molding (RTM), Filament winding		**III**
27 28 29 30	Macro-Mechanical Analysis of a Lamina: Introduction, Definitions: Stress, Strain, Elastic Moduli, Strain Energy. Hooke"s Law for Different Types of Materials – Anisotropic material, monoclinic material and orthotropic material, Hooke"s Law for a Two Dimensional Unidirectional Lamina - Plane Stress Assumption, Reduction of Hooke"s Law in Three Dimensions to Two Dimensions, Relationship of Compliance and Stiffness Matrix to Engineering Elastic Constants of a Lamina, Angle Lamina Problems Problems	12	**IV**
	Hooke"s Law for a Two-Dimensional Angle Lamina Engineering Constants of an Angle Lamina, Invariant Form of Stiffness and Compliance Matrices for an Angle Lamina, Strength Failure theories of an angle lamina-Maximum stress Failure Theory, Tsai–Hill Failure Theory, Tsai–Wu Failure Theory.		**V**

	Micro-Mechanical Analysis of a Lamina: Introduction, Volume and Mass Fractions, Density, and Void Content,		
	Evaluation of the Four Elastic Moduli – Longitudinal young's modulus, Transverse young's modulus, Major Poisson's ratio and In-plane shear modulus by Strength of Materials Approach, Semi Empirical Models, Ultimate Strengths of a Unidirectional Lamina- Longitudinal tensile strength, Transverse tensile strength, Longitudinal compressive strength, Transverse compressive strength. In-Plane shear strength		**VI**

Signature of faculty **HSME**

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Lecture Plan

NAME OF THE FACULTY: Dr. M.ASHOK KUMAR **ACADEMIC YEAR:** 2020-2021

CLASS/SEM: IV B.TECH/ISEM **TOTAL HOURS: 50**

NAME OF THE SUBJECT: [A0338158] MECHANICS OF COMPOSITE MATERIALS

S.No	DATE	TOPIC	HOURS	REMARKS
1		Introduction to Composite Materials:	1	
2		Classification: Polymer Matrix Composites (PMCs), matrix materials,reinforcements used in PMCs	1	
3		Metal Matrix Composites(MMCs)	1	
4		Ceramic Matrix Composites(CMCs)	2	
5		Carbon–Carbon Matrix Composites (CCMCs), Fiber	1	I
6		Reinforced Composites	1	
7		nature-made composites, and applications	1	
8		Reinforcements: Fibres, characteristics	1	
9		Glass Fiber, types	1	
10		Silica fiber	2	
11		Kevlar fiber	2	
12		Carbon fiber	1	
13		Boron fiber	1	
14		Boron carbide fiber, silicon carbide fiber. Particulate composites	1	II
15		Introduction to Manufacturing Processes:,	2	
16		Hand lay-up	1	
17		Spray lay-up,	1	
18		Vacuum bagging,	1	

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19		Pultrusion,	1	
20		Resin Transfer Molding (RTM),	2	
21		Filament winding	1	**III**
22		Macro-Mechanical Analysis of a Lamina: Introduction,	2	
23		Definitions: Stress, Strain, Elastic Moduli, Strain Energy.	2	
24		Hooke‟s Law for Different Types of Materials – Anisotropic material, monoclinic material and orthotropic material,	1	**IV**
25		Hooke‟s Law for a Two Dimensional Unidirectional Lamina - Plane Stress Assumption,	1	
26		Reduction of Hooke‟s Law in Three Dimensions to Two Dimensions,	2	
27		Relationship of Compliance and Stiffness Matrix to Engineering Elastic Constants of a Lamina,	1	
28		Angle Lamina	1	
29		Problems	1	
30		Problems	1	
31		Hooke‟s Law for a Two-Dimensional Angle Lamina	1	
32		Engineering Constants of an Angle Lamina,	1	**V**
33		Invariant Form of Stiffness and Compliance Matrices for an Angle Lamina,	2	

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34		Strength Failure theories of an angle lamina-	2	
35		Maximum stress Failure Theory, Tsai–Hill Failure Theory, Tsai–Wu Failure Theory.	1	
36		Micro-Mechanical Analysis of a Lamina:	1	
37		Introduction, Volume and Mass Fractions, Density, and Void Content,	1	**VI**
38		Evaluation of the Four Elastic Moduli – Longitudinal young's modulus, Transverse young's modulus, Major Poisson's ratio and In-plane shear modulus by Strength of Materials Approach,	1	
39		Semi Empirical Models, Ultimate Strengths of a Unidirectional Lamina-Longitudinal tensile strength,	2	
40		Transverse tensile strength, Longitudinal compressive strength, Transverse compressive strength. In-Plane shear strength	2	

Signature of faculty **HSME**

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UNIT-I

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INTRODUCTION

- A composite material can be defined as a combination of two or more materials (having significantly different physical or chemical properties) that results in better properties than those of the individual components.
- The constituents retain their identities in the composite; that is, they do not dissolve or otherwise merge completely into each other, although they act in concert.
- Composites are one of the most widely used materials because of their adaptability to different situations and the relative ease of combination with other materials to serve specific purposes and exhibit desirable properties.
- The main advantages of composite materials are their high strength and stiffness, combined with low densitv. when compared with bulk materials.

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Why Composites?

Steel	Vs	Composites
Low material cost		High material cost
High installed cost		Low installed cost
Corrosive		Non-corrosive
Heavy		Lightweight
Fabrication required		No fabrication required
High maintenance		Low maintenance

Matrix Phase: continuous phase, surrounds other phase
(e.g.: metal (Cu, Al, Ti, Ni¼); , ceramic (SiC¼), or polymer (Thermosets, thermoplastics, Elastomers)

Reinforcement Phase: dispersed phase, discontinuous phase
(e.g.: Fibers, Particles, or Flakes)

?→ Interface between matrix and reinforcement
Interfacial properties - the interface may be regarded as a third phase.

Examples:
± Straw in mud
± Wood (cellulose fibers in hemicellulose and lignin)
± Bones (soft protein collagen and hard apatite minera
errite and cementite)

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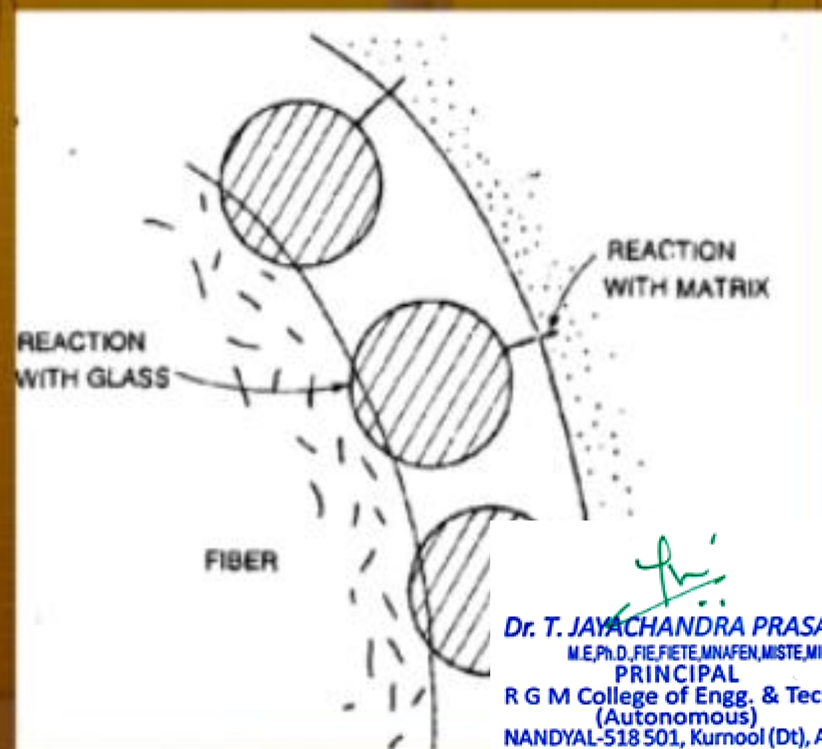
Composites Offer

- High Strength to weight ratio
- High Stiffness to weight ratio
- High Modulus to weight ratio
- Light Weight
- Directional strength
- Corrosion resistance
- Weather resistance
- Dimensional stability -low thermal conductivity
  -low coefficient of thermal expansion
- Radar transparency
- Non-magnetic
- High impact strength
- High dielectric strength (insulator)
- Low maintenance
- Long term durability
- Part consolidation
- …o large part geometry possible
- …d surface finish
- … Flexibility

Composite strength depends on the following factors:

- Inherent fiber strength, Fiber length, Number of flaws
- Fiber shape
- The bonding of the fiber (equally stress distribution)
- Voids
- Moisture (coupling ...nts)



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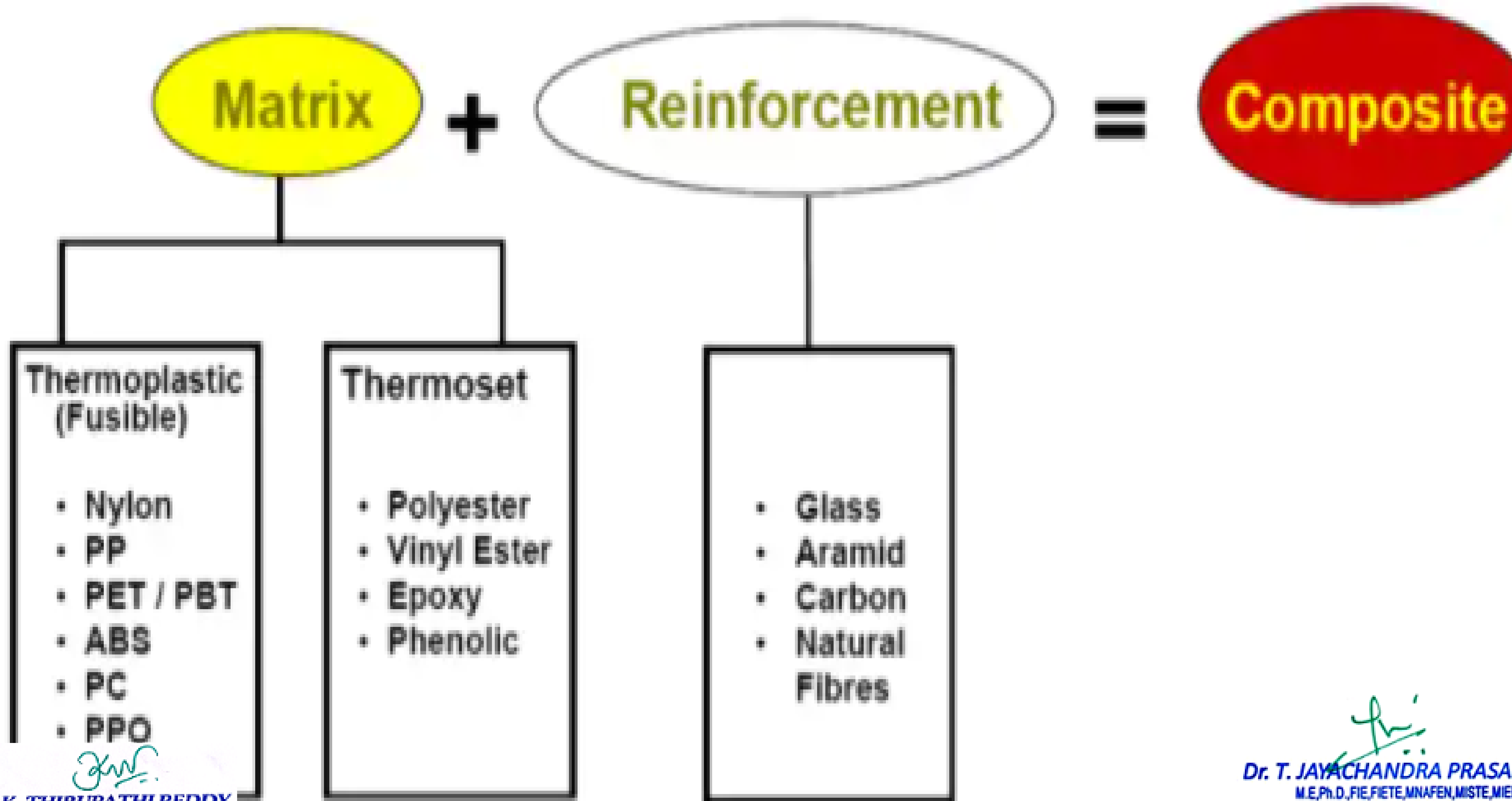
What are composites ?



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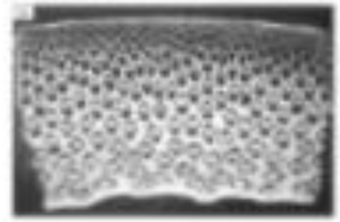
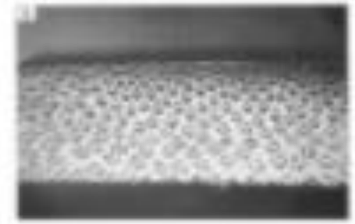
What are composites made of ?

- Human learns from 'mother nature' to develop new composite materials
- Natural Composites: wood and bamboo, shells, bones, muscles, other tissues and natural fibres (silk, wool, cotton, jute, sisal)





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Definition

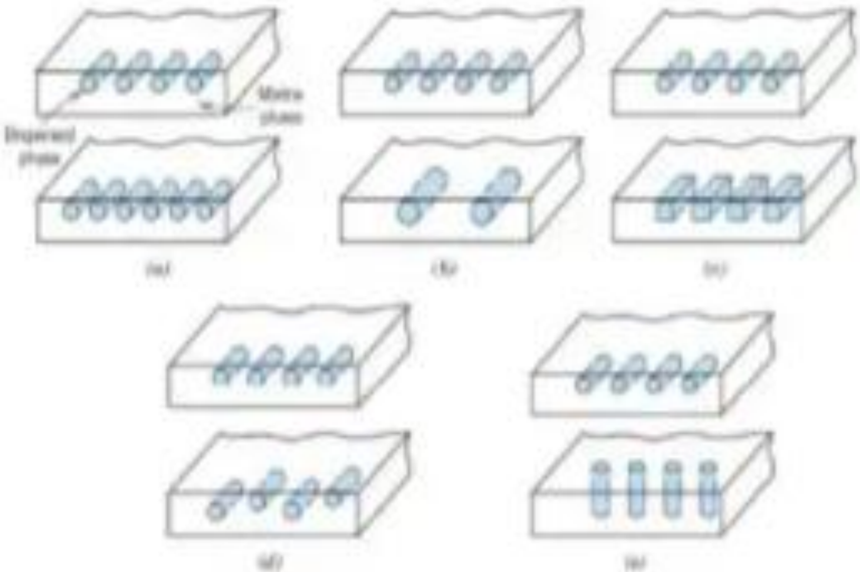

Figure 16.1 Schematic representations of the various geometrical and spatial characteristics of particles of the dispersed phase that may influence the properties of composites: (a) concentration, (b) size, (c) shape, (d) distribution, and (e) orientation. (From Richard A. Flinn and Paul K. Trojan, *Engineering Materials and Their Applications*, 4th edition. Copyright © 1990 by John Wiley & Sons, Inc. Adapted by permission of John Wiley & Sons, Inc.)

Two phase composite:

- **Matrix** is the continuous phase and surrounds the reinforcements
- **Reinforcement** is the dispersed phase, which normally bears the majority of stress

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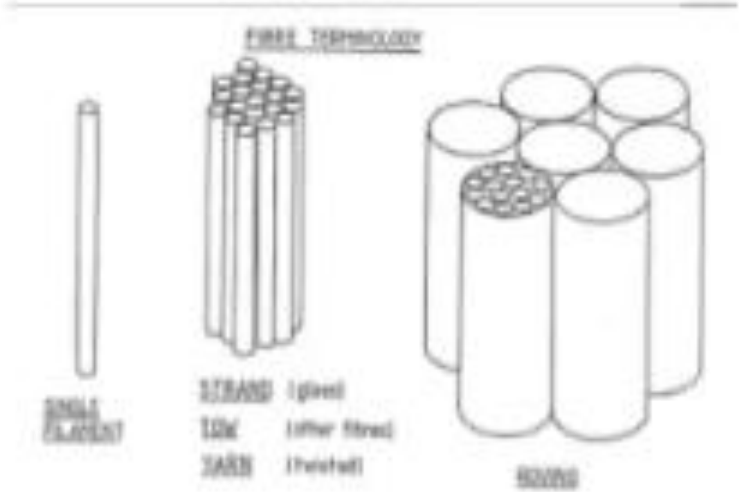
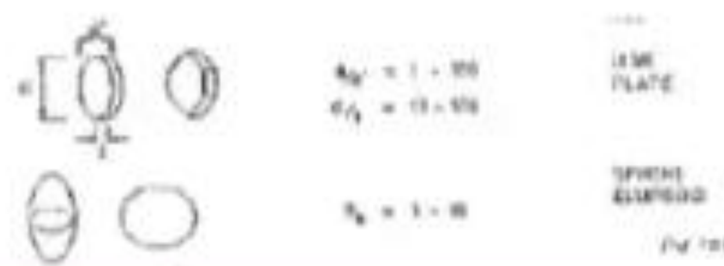
Reinforcements

- A reinforcement is the strong, stiff integral component which is incorporated into the matrix to achieve desired properties
- The term 'reinforcement' implies some property enhancement
- Different types
  - Fibres or Filaments: continuous fibres, discontinuous fibres, whiskers
    - Particulates reinforcements may be of any shape, ranging from irregular to spherical, plate-like or needle-like, nanoparticles
- They have a low ductility





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Matrix

- Made from Metal, polymer or ceramic
- Continuous phase
- Some ductility is desirable
- Functions
  - Binds the reinforcements (fibers/particulates) together
  - Mechanically supporting the reinforcements
  - Load transfer to the reinforcements
  - Protect the reinforcements from surface damage due to abrasion or chemical attacks
  - High bonding strength between fiber and matrix is import

So why use composites?

- The greatest advantage of composite materials is strength and stiffness combined with lightness. By choosing an appropriate combination of reinforcement and matrix material, manufacturers can produce properties that exactly fit the requirements for a particular structure for a particular purpose.
- Modern aviation, both military and civil, is a prime example. It would be much less efficient without composites. In fact, the demands made by that industry for materials that are both light and strong has been the main force driving the development of composites. It is common now to find wing and tail sections, propellers and rotor blades made from advanced composites, along with much of the internal structure and fittings. The airframes of some smaller aircraft are made entirely from composites, as are the wing, tail and body panels of large commercial aircraft.
- In thinking about planes, it is worth remembering that composites are less likely than metals (such as aluminium) to break up completely under stress. A small crack in a piece of metal can spread very rapidly with very serious c… …n the case of aircraft). The fibres in a composite act to b… …any small crack and to share the stress around.

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- The right composites also stand up well to heat and corrosion. This makes them ideal for use in products that are exposed to extreme environments such as boats, chemical-handling equipment and spacecraft. In general, composite materials are very durable.
- Another advantage of composite materials is that they provide design flexibility. Composites can be moulded into complex shapes – a great asset when producing something like a surfboard or a boat hull.
- The downside of composites is usually the cost. Although manufacturing processes are often more efficient when composites are used, the raw materials are expensive. Composites will never totally replace traditional materials like steel, but in many cases they are just what we need. And no doubt new uses will be found as the technology evolves. We haven't yet seen all that composites can do.

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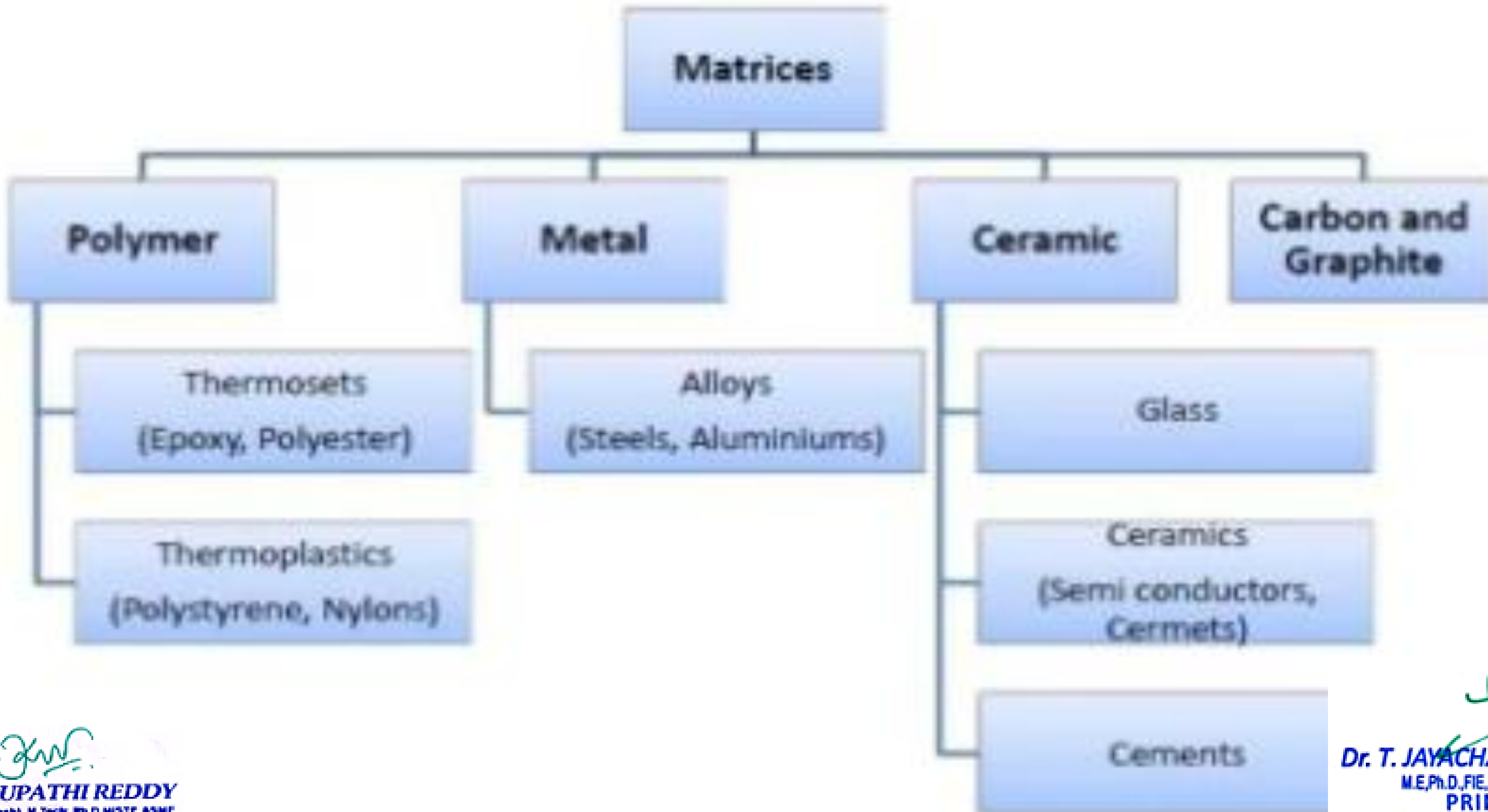
CLASSIFICATION OF COMPOSITE MATERIALS

The composites are classified as mainly two constituents are matrix and a reinforcement



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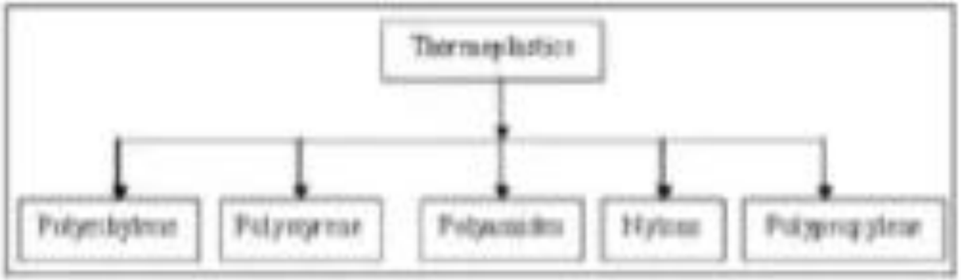
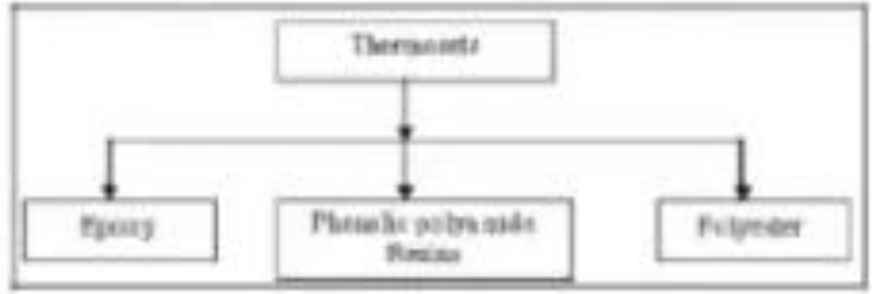
ORGANIC/POLYMER MATRIX COMPOSITE (PMCs)

Two main kinds of polymers are thermosets and thermoplastics





- Thermosets have qualities such as a well-bonded three dimensional molecular structure after curing. They decompose instead of melting on hardening.
- Thermoplastics have one or two dimensional molecular structure and they tend to at an elevated temperature and show exaggerated melting point. Another advantage is that the process of softening at elevated temperatures can reversed to regain its properties during cooling.

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Factors influence the performance processing method

Impact Resistance
Delamination
Interphase
Fiber Orientation
Properties Of Raw Materials

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Branches of composites

- Hybrid Composites
- Nanocomposites
- Blended Composites
- Blended Nanocomposites
- Hybrid Fiber Reinforced Composites
- Laminated Composites
- Particulate Composites

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Factors affecting the composites

- ❑ Properties Of Constituents
- ❑ Shape Of The Fiber
- ❑ Geometry Of The Fiber
- ❑ Cross Sectional Area Of The Fiber
- ❑ Manufacturing Method
- ❑ Time Of Mixing
- ❑ Interface Between The Constituents
- ❑ Processing Temp
- ❑ Fiber distribution and orientation

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Polymer matrix Composites (PMCs)

- It is a multi phase material.

- ‘Poly’ means many and ‘mers’ means units

- polymer is a large molecule prepared by many repeated subunits.

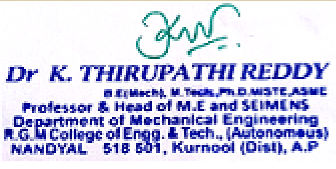
- Prepared by long and short continuous fibers bound together by polymer matrix.

- These yield superior strength and stiffness.

- Three types of polymers are used such as
- Thermoplastics, (high processing temp.)
- Thermosets, and (less processing temp.)
- Elastomers (i.e. rubber).
- Both synthetic and natural fibers can also be used as a reinforcements

- Glass fibers, Kevlar fibers, carbon fibers, aramid fibers are some of the synthetic fibers.

ement is in discontinuous phase and matrix in in continuo



- Majority of polymers are made by petroleum based products.

- Polymers are made by chemical reaction by bonding of monomers by polymerization. Some polymers are made by organism.

- Proteins have polypeptide molecules which are natural polymers made from various amino-acids monomer unit.

- Fiber length with less diameter imparts more mechanical strength rather than width.

- these PMCs do not need any furnace to produce.

- Temperature resistance of these polymers are up to $250^{o}C$.
- Continuous fibers(glass, carbon, aramid, basalt or polymer fibers), chopped fibers(chopped CFs and chopped GFs), woven fabric fibers are fibers available commercially.
- Degree off polymerization is depends on the how many no of units in the chain.
- Thermoplastics- addition polymerization, thermo-sets- condensation polymerization

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Nanofillers (also called nanocomposites)

- ❑ Carbon nanotubes
- ❑ Exfoliated clay platelets
- ❑ Carbon black nanoparticles

Length is less than 0.5 microns (i.e.500 nanometers)

Dramatic Improvements

increased modulus

Strength, dimensional stability, thermal stability, electrical conductivity, flame retardency, chemical resistance, optical clarity, decreased gas water, oil permeability, surface appearance.

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Classifications of polymers

❑ Linear Polymers

• molecules are in the form of chains.

❑ Thermoplastic Polymers

• molecules are linear or branched but not inter connected

❑ Thermoset Polymers

• polymers are heavily cross linked to produce strong 3D network structures.

❑ Elastomers

• lightly cross linked and its elastic deformation is >200%

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Advantages of PMCs

Light weight

High strength and stiffness

High impact resistance

Good Corrosion resistance

Good abrasion and wear resistance

Disadvantages

Environmental degradation

Moisture absorption causes swelling

Thermal mismatch between the fiber and matrix. Due to'α'' and causes debonding.

Low working temperature

Sensitive to radiation

Applications

Medical field

MRI scanners, X-ray couches, C-scanners, mammography plates, tables, surgical target tools, wheel chairs, prosthetics. etc

Transportation vehicles

Automotive:

belts, seats, hoses, sports cars (Bugatti uses CF to construct the body of the car)

fuel tanks, mirror and light housing, engine parts, body panels, wind s

-protective coatings for paintworks.

Aerospace Vehicles: tires, interiors,fuselages, rudders, windows,

Marine Ships: fishing boats, ships

Personal protective equipments:

fire fighters, while facing the deadly weapons

Others:

industrial equipments, foot wear, packaging, building, construction and civil Engg(impellers, blades, housing and covers), power tool housings, lawn mover hoods, mobile phones, Energy storage devices(batteries)

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Metal Matrix Composites

- Conventional materials have some limitations in achieving the good combination of strength, stiffness, toughness, and low density.
- So these shortcomings are overcome.
- MMCs posses significantly improved properties
- such as
- high specific strength,
- high specific modulus,
- high damping capacity, and
- high wear resistance.

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METAL MATRIX COMPOSITE (MMCs)

- Metal matrix composites are High strength, fracture toughness and stiffness are offered by metal matrices than those offered by their polymer counterparts. They can withstand elevated temperature in corrosive environment than polymer composites.
- MMCs are widely used in engineering applications where the operating temperature lies in between 250 °C to 750 °C.
- Matrix materials: Steel, Aluminum, Titanium, Copper,

er alloys.

CERAMIC MATRIX COMPOSITE (CMCs)

- Ceramics can be described as solid materials which exhibit very strong ionic bonding in general and in few cases covalent bonding. High melting points, good corrosion resistance, stability at elevated temperatures and high compressive strength
- CMCs are widely used in engineering applications where the operating temperature lies in between 800°C to 1650°C

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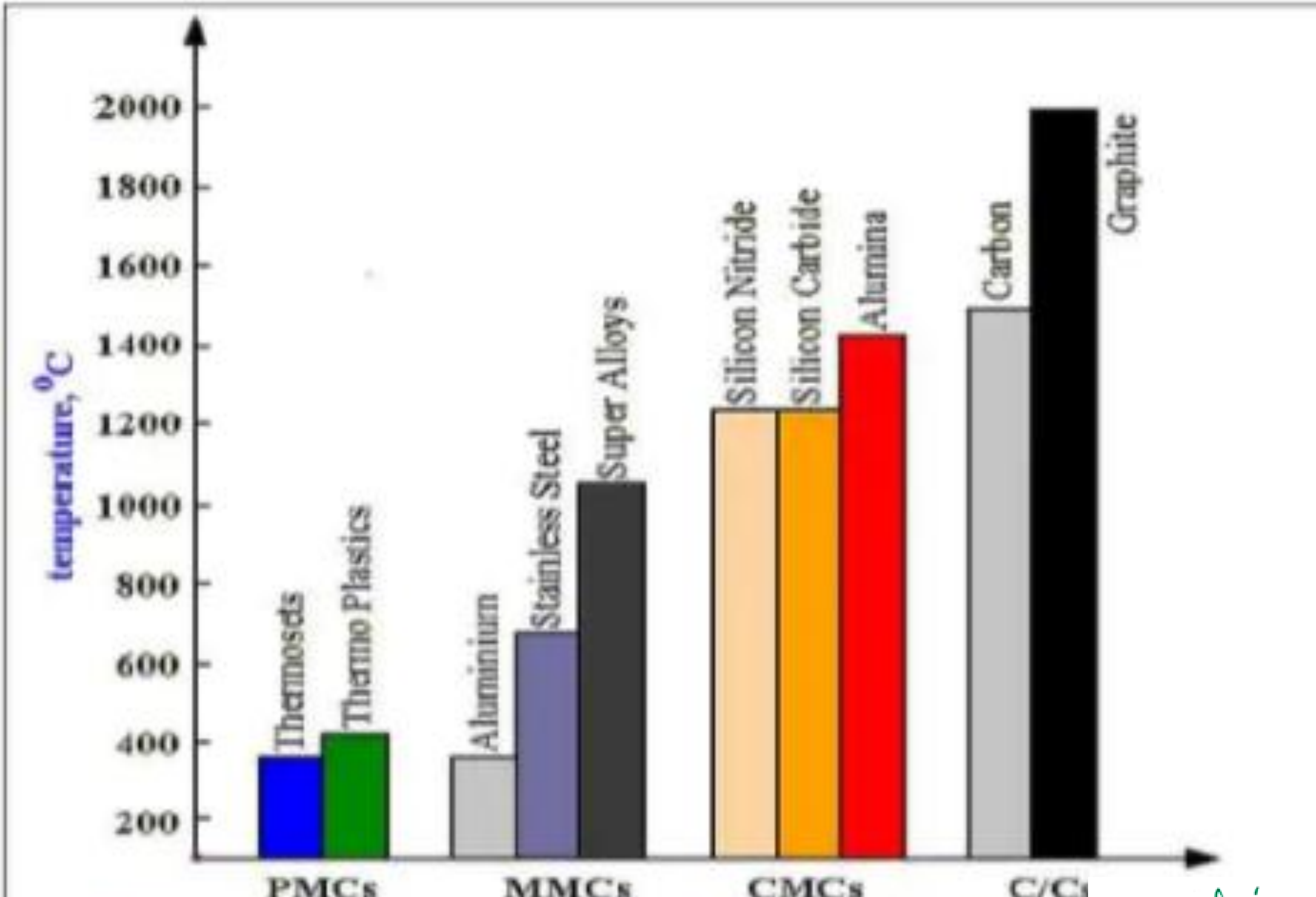

Service temperature ranges for different types of

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FUNCTIONS OF A MATRIX

- Holds the fibers together.
- Protects the fibers from environment.
- Distributes the loads evenly between fibers so that all fibers are subjected to the same amount of strain.
- Enhances transverse properties of a laminate.
- Improves impact and fracture resistance of a component.
- Carry inter laminar shear.

DESIRED PROPERTIES OF A MATRIX

- Reduced moisture absorption.
- Low shrinkage.
- Low coefficient of thermal expansion.
- Strength at elevated temperature (depending on application).
- Low temperature capability (depending on application).
- ent chemical resistance (depending on application).

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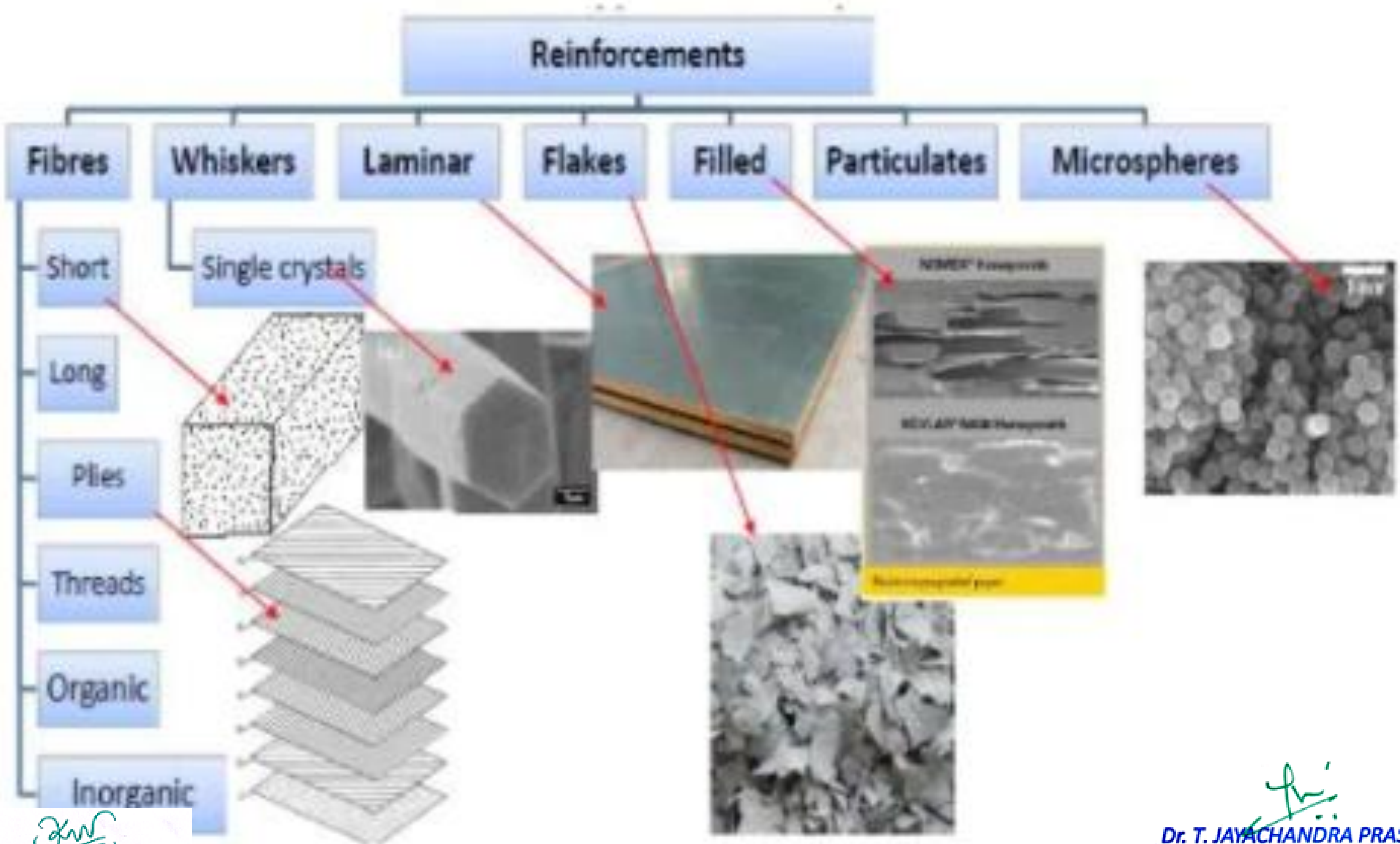
CLASSIFICATION OF COMPOSITE MATERIALS



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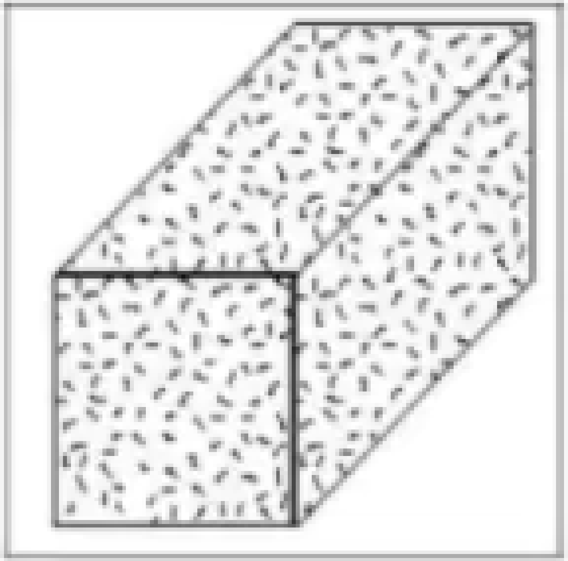
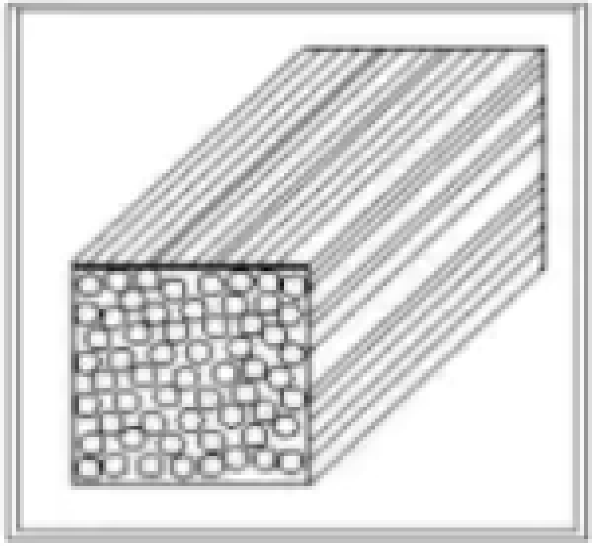
FIBER REINFORCED COMPOSITES

Fibers are the important class of reinforcements, as they satisfy the desired conditions and transfer strength to the matrix constituent influencing and enhancing their properties as desired.



Random fiber (short fiber) reinforced composites



Continuous fiber (long fiber) reinforced composites

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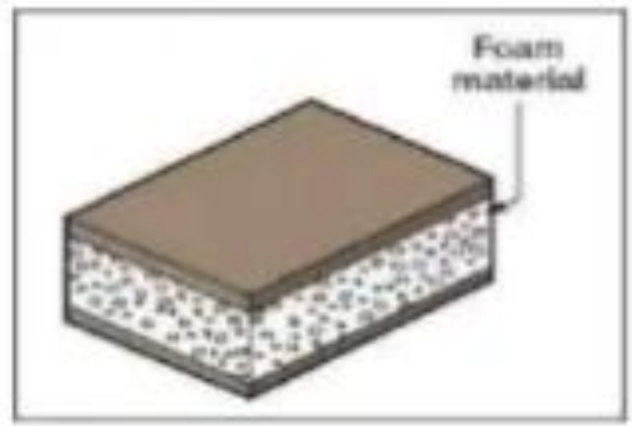
LAMINAR COMPOSITES

Laminar composites are found in as many combinations as the number of materials. They can be described as materials comprising of layers of materials bonded together. These may be of several layers of two or more metal materials occurring alternately or in a determined order more than once, and in as many numbers as required for a specific purpose.

Laminar Composite



Sandwich (

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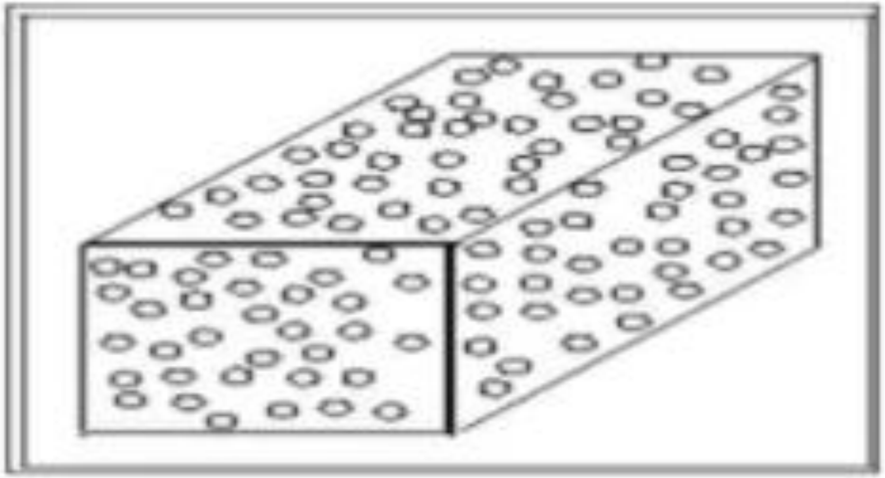
PARTICULATE REINFORCED COMPOSITES

Microstructures of metal and ceramics composites, which show particles of one phase strewn in the other, are known as particle reinforced composites. Square, triangular and round shapes of reinforcement are known, but the dimensions of all their sides are observed to be more or less equal. The size and volume concentration of the dispersed distinguishes it from dispersion hardened materials.



Particulate reinforced composites

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Particulate composites

- These provide reinforcement,
- improves conductivity, improves operating temp.,
- oxidation resistance,
- cost to the matrix
- Combination of matrix and reinforcement can provide us very special material
- Nanoparticles saves material and also improves strength.
- Usually isotropic because particles are added randomly
- Size of the particles is <0.25 microns
- Ex: chopped fibers, platelets, hollow spheres, nan-oclay, carbon nanotubes,
- Traditional manufacturing methods such as injection moulding reduces the cost.
- Al-alloys with sic particle dispersed are widely used for piston and brake applications.
- carbon or ceramic particulates used for brakes
- Applications:
- cutting tools
- automotive parts, brakes
- Computer housings
- cell phone casings
- office furniture
- helmets

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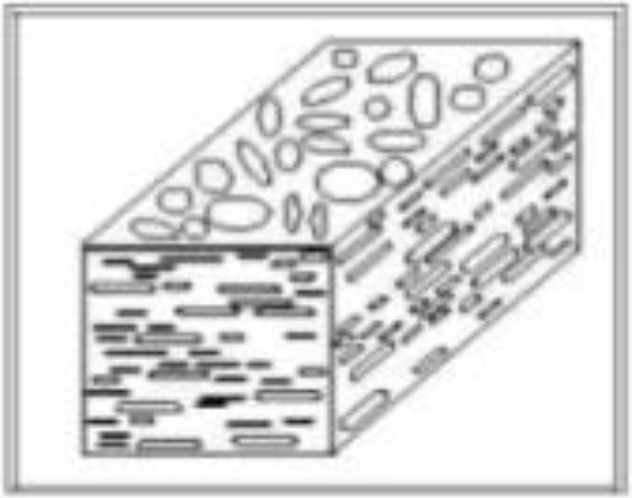
FLAKE COMPOSITES

Flakes are often used in place of fibers as can be densely packed. Metal flakes that are in close contact with each other in polymer matrices can conduct electricity or heat, while mica flakes and glass can resist both. Flakes are not expensive to produce and usually cost less than fibers.



Flake composites

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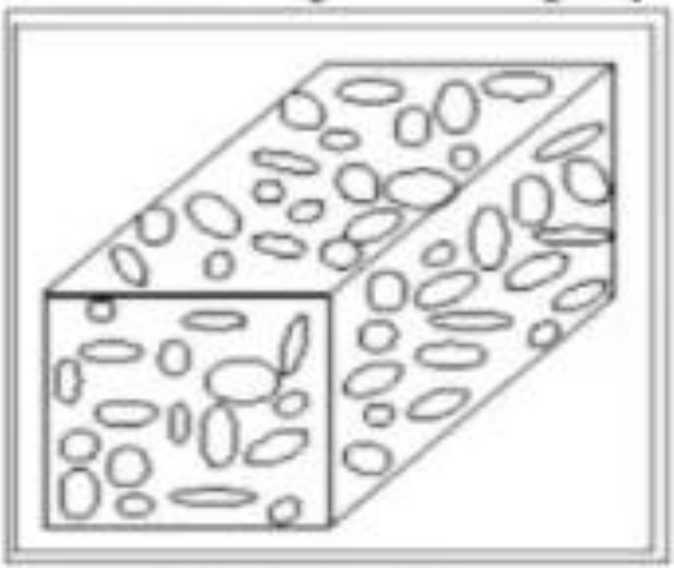
FILLED COMPOSITES

Fillers may be the main ingredient or an additional one in a composite. The filler particles may be irregular structures, or have precise geometrical shapes like polyhedrons, short fibers or spheres.



Filled composites

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MICROSPHERES

Microspheres are considered to be some of the most useful fillers. Their specific gravity, stable particle size, strength and controlled density to modify products without compromising on profitability or physical properties are it's their most-sought after assets.

Solid Microspheres have relatively low density, and therefore, influence the commercial value and weight of the finished product. Studies have indicated that their inherent strength is carried over to the finished molded part of which they form a constituent.

Hollow microspheres are essentially silicate based, made at controlled specific gravity. They are larger than solid glass spheres used in polymers and commercially supplied in a wider range of particle sizes.

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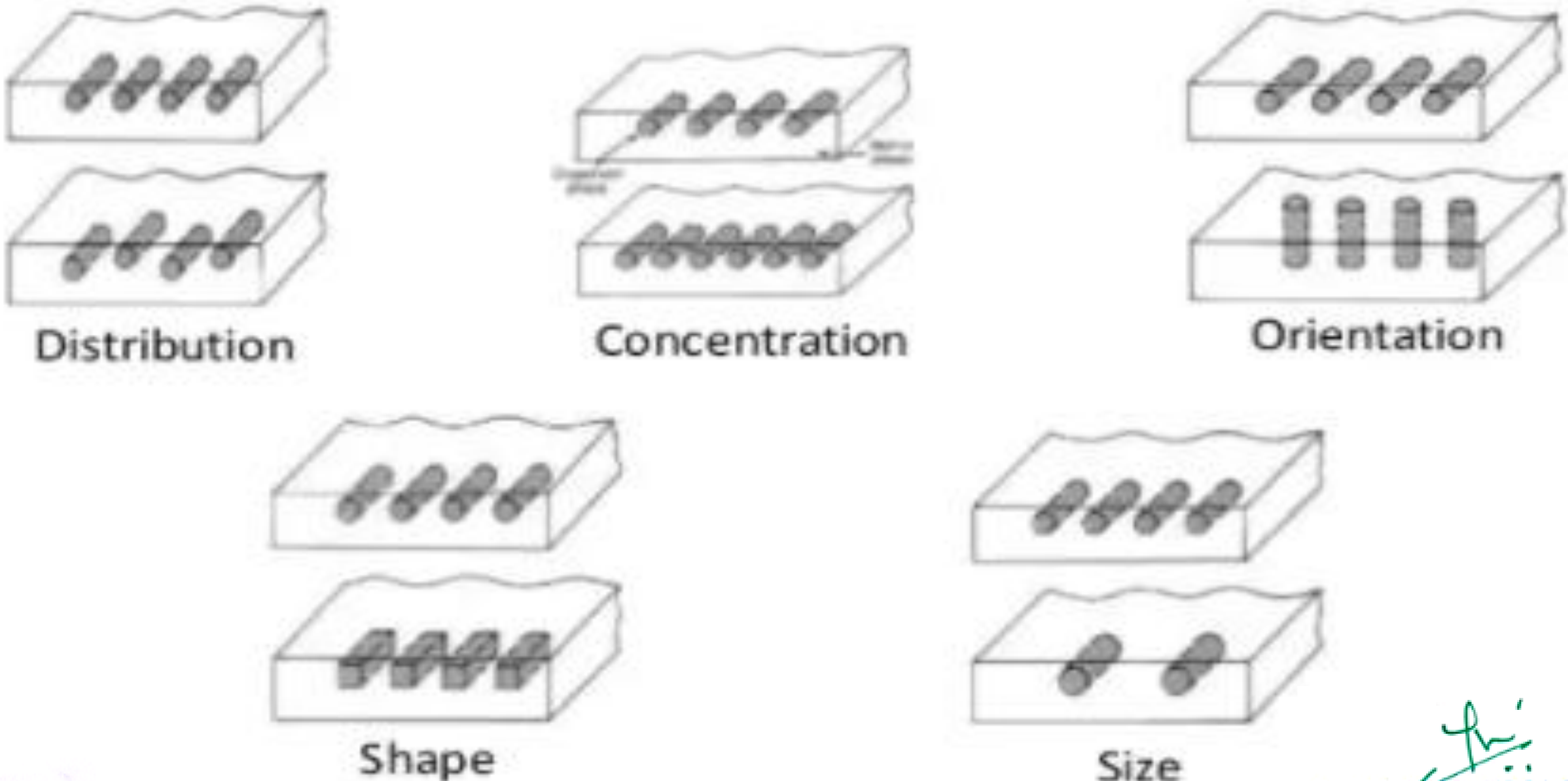
FACTORS AFFECTING PROPERTIES OF COMPOSITES

- The type, distribution, size, shape, orientation and arrangement of the reinforcement will affect the properties of the composites material and its anisotropy



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FAILURE MODES OF COMPOSITE MATERIALS

- Delamination
- Matrix tensile failure
- Matrix compression failure
- Fiber tensile failure
- Fiber compression failure

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- **INTRODUCTIONS**
- **Examples of naturally found composites.**
- Examples include wood, where the lignin matrix is reinforced with cellulose fibers and bones in which the bone-salt plates made of calcium and phosphate ions reinforce soft collagen.
- **What are advanced composites?**
- Advanced composites are composite materials that are traditionally used in the aerospace industries. These composites have high performance reinforcements of a thin diameter in a matrix material such as epoxy and aluminum. Examples are graphite/epoxy, Kevlar®†/epoxy, and boron/aluminum composites. These materials have now found applications in commercial industries as well.

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CLASSIFICATION

- **How are composites classified?**
- Composites are classified by the geometry of the reinforcement
  - Particulate
  - Flake
  - Fibers
- Composites are classified by the type of matrix
  - Polymer
  - Metal
  - Ceramic
  - Carbon.

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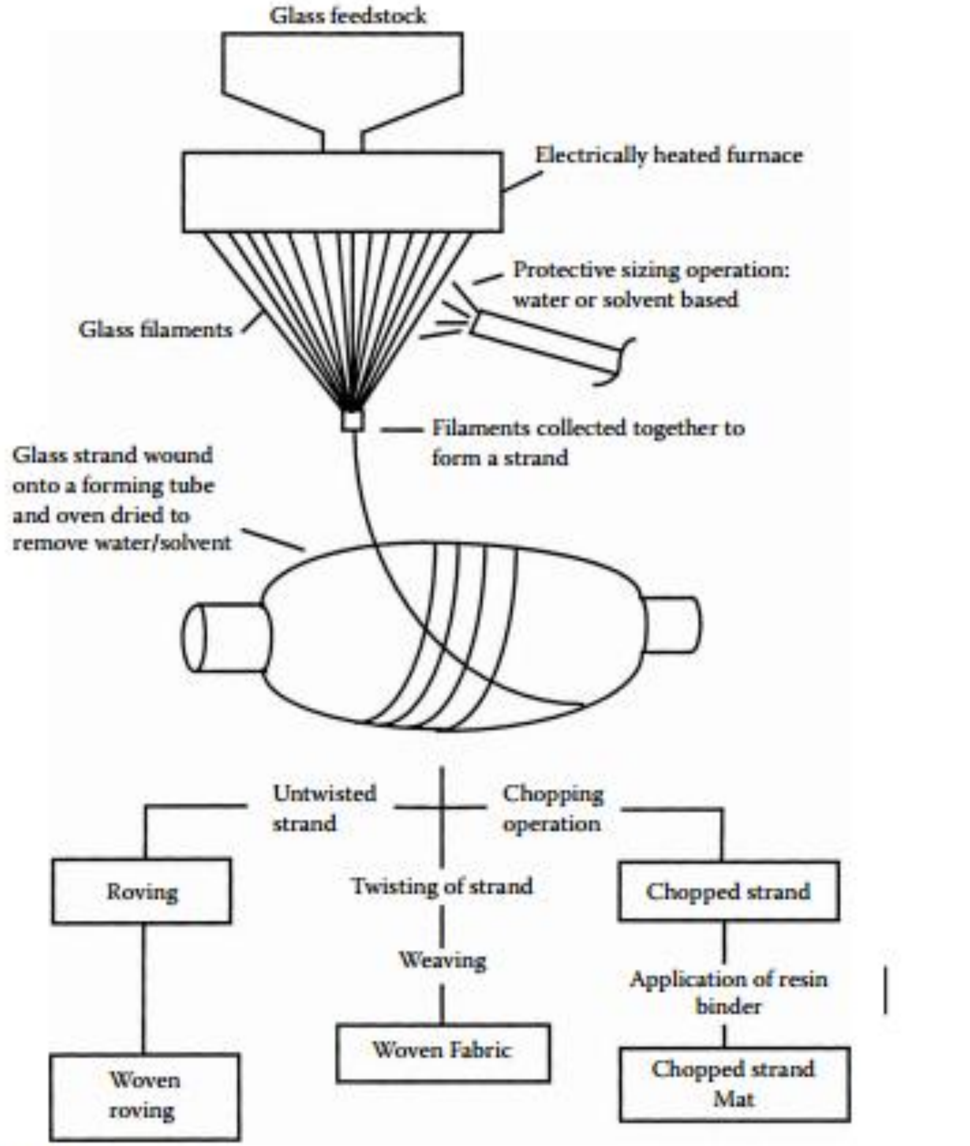

FIGURE 1.9

Schematic of manufacturing glass fibers and available glass forms. (From Bishop, W., in *Advanced Composites*, Partridge, I.K., Ed., Kluwer Academic Publishers, London, 1990, Figure 4, p. 177. Reproduced with kind permission of Springer.)

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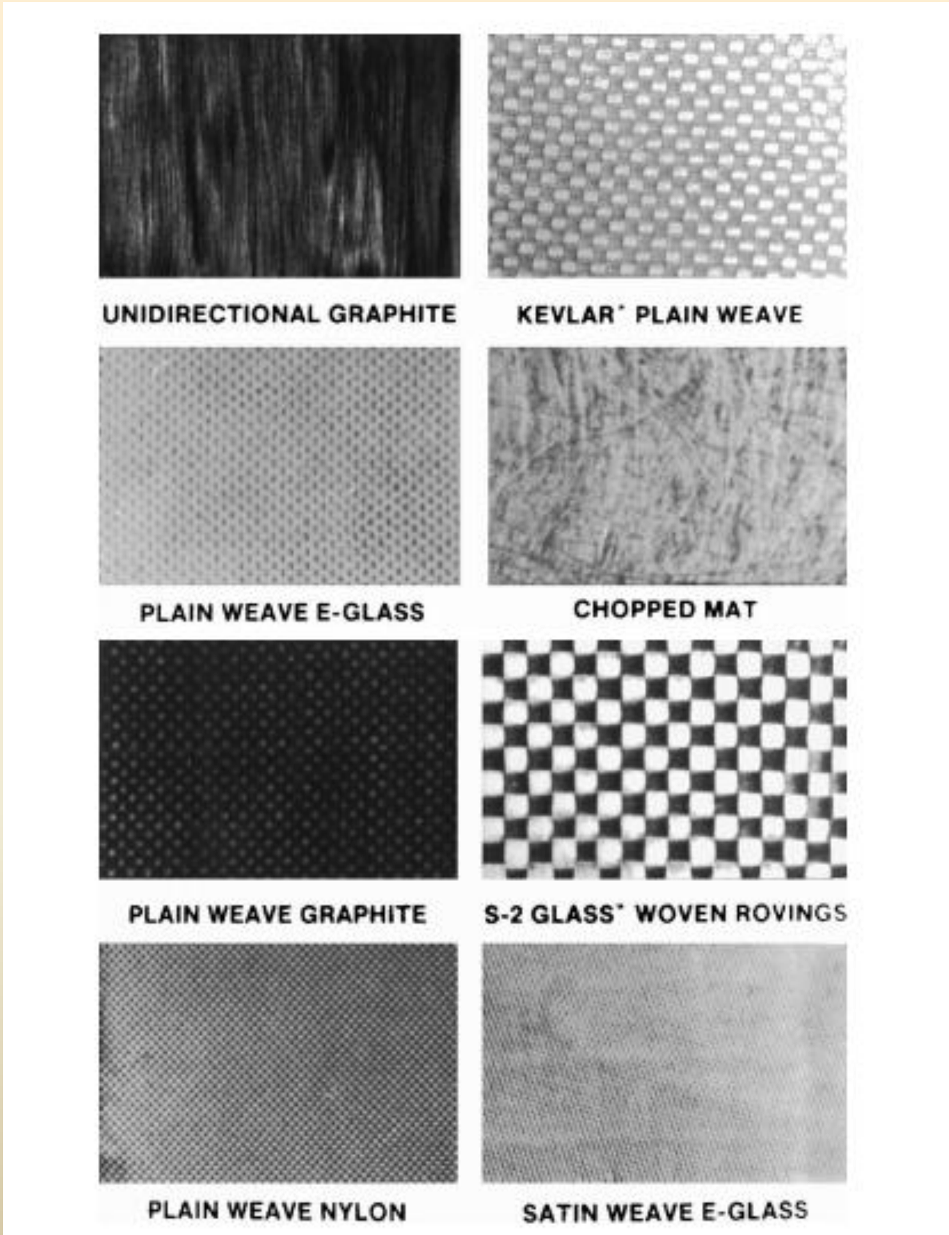
UNIDIRECTIONAL GRAPHITE
KEVLAR* PLAIN WEAVE
PLAIN WEAVE E-GLASS
CHOPPED MAT
PLAIN WEAVE GRAPHITE
S-2 GLASS* WOVEN ROVINGS
PLAIN WEAVE NYLON
SATIN WEAVE E-GLASS

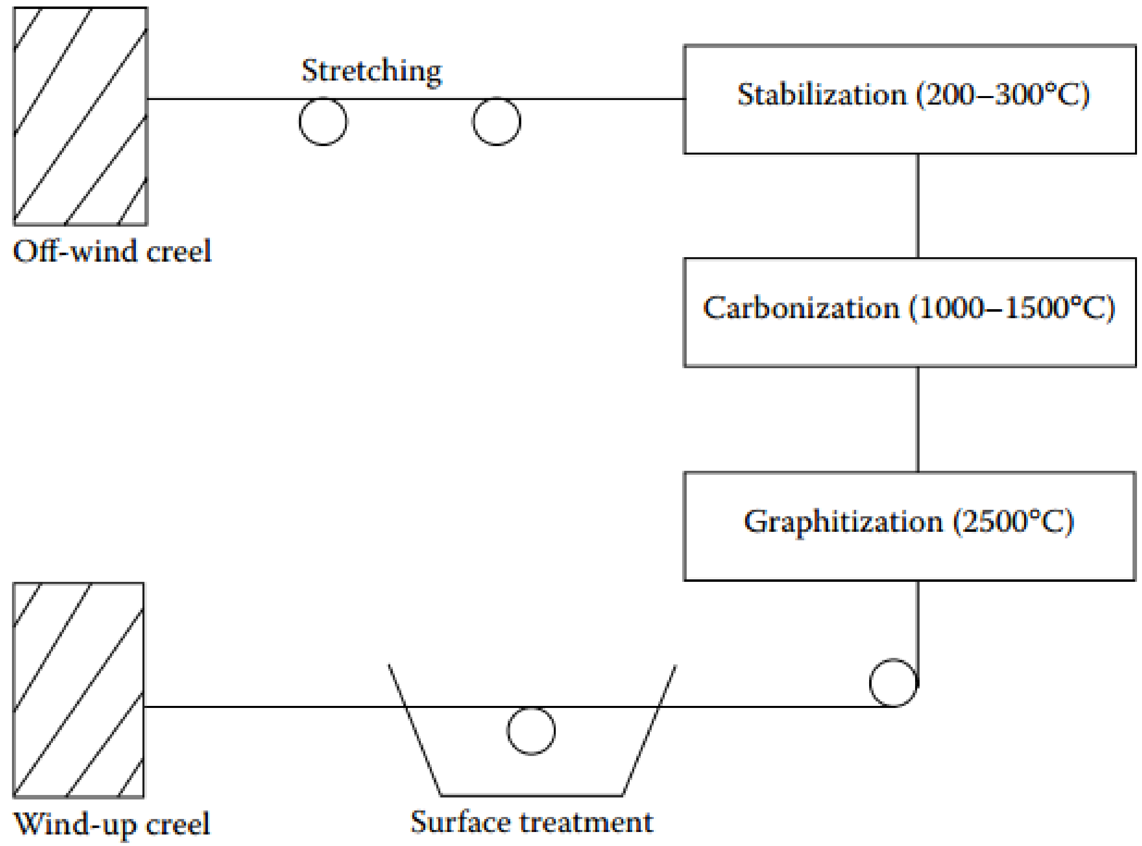

FIGURE 1.11
Stages of manufacturing a carbon fiber from PAN-based precursors.

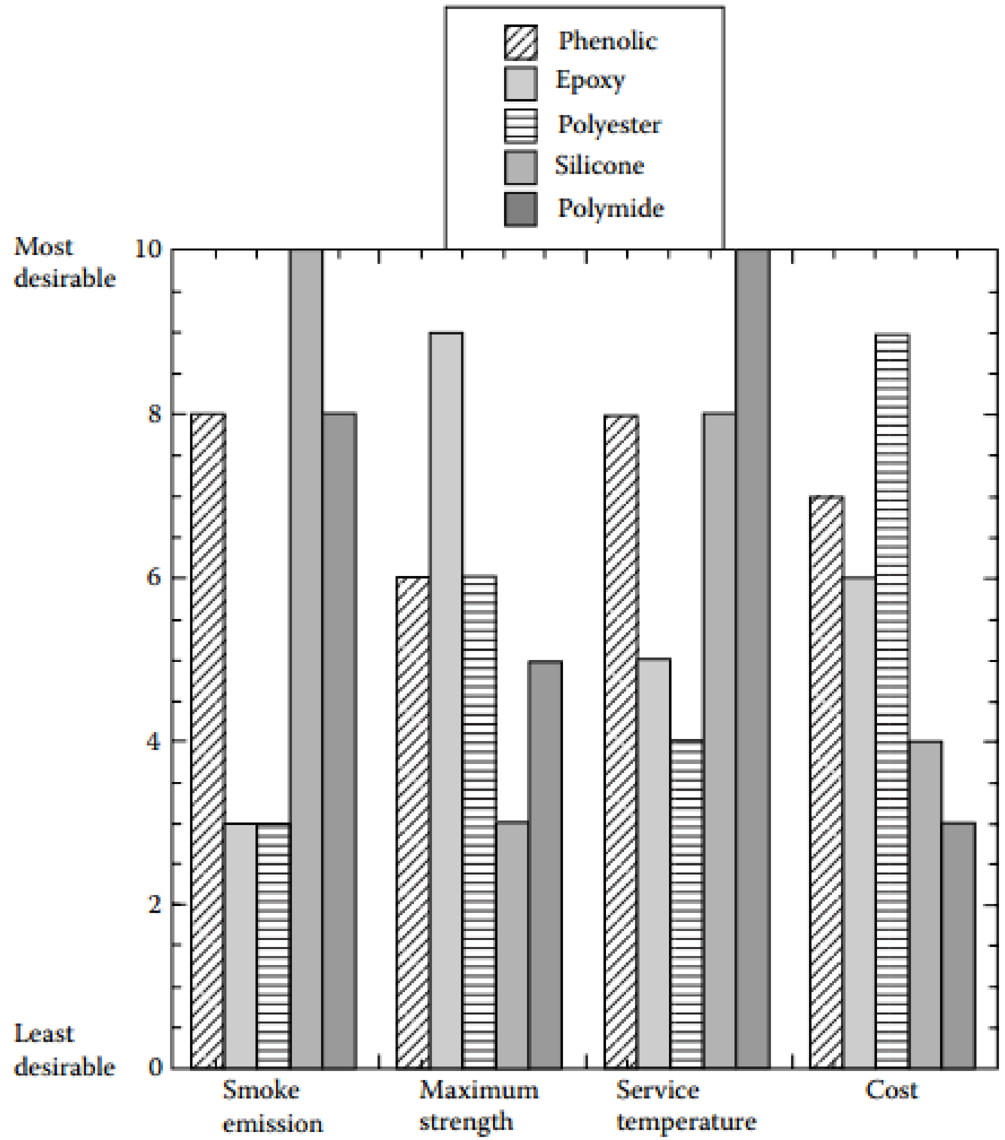

FIGURE 1.12
Comparison of performance of several common matrices used in polymer matrix composites. (Graphic courtesy of M.C. Gill Corporation, http://www.mcgillcorp.com.)

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- **Polymers are classified as thermosets and thermoplastics. What is the difference between the two? Give some examples of both.**
- Thermoset polymers are insoluble and infusible after cure because the chains are rigidly joined with strong covalent bonds; thermoplastics are formable at high temperatures and pressure because the bonds are weak and of the van der Waals type. Typical examples of thermoset include epoxies, polyesters, phenolics, and polyamide; typical examples of thermoplastics include polyethylene, polystyrene, polyether–ether–ketone (PEEK), and poly phenylene sulfide (PPS). The differences between thermosets and thermoplastics are given in the following table

Thermoplastics	Thermoset
Soften on heating and pressure, and thus easy to repair	Decompose on heating
High strains to failure	Low strains to failure
Indefinite shelf life	Definite shelf life
Can be reprocessed	Cannot be reprocessed
Not tacky and easy to handle	Tacky
Short cure cycles	Long cure cycles
Higher fabrication temperature and viscosities have made it difficult to process	Lower fabrication temperature
Excellent solvent resistance	Fair solvent resistance

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- **What are prepregs?**
- Prepregs are a ready-made tape composed of fibers in a polymer matrix (Figure 1.13). They are available in standard widths from 3 to 50 in. (76 to 1270 mm). Depending on whether the polymer matrix is thermoset or thermoplastic, the tape is stored in a refrigerator or at room temperature, respectively. One can lay these tapes manually or mechanically at various orientations to make a composite structure. Vacuum bagging and curing under high pressures and temperatures may follow

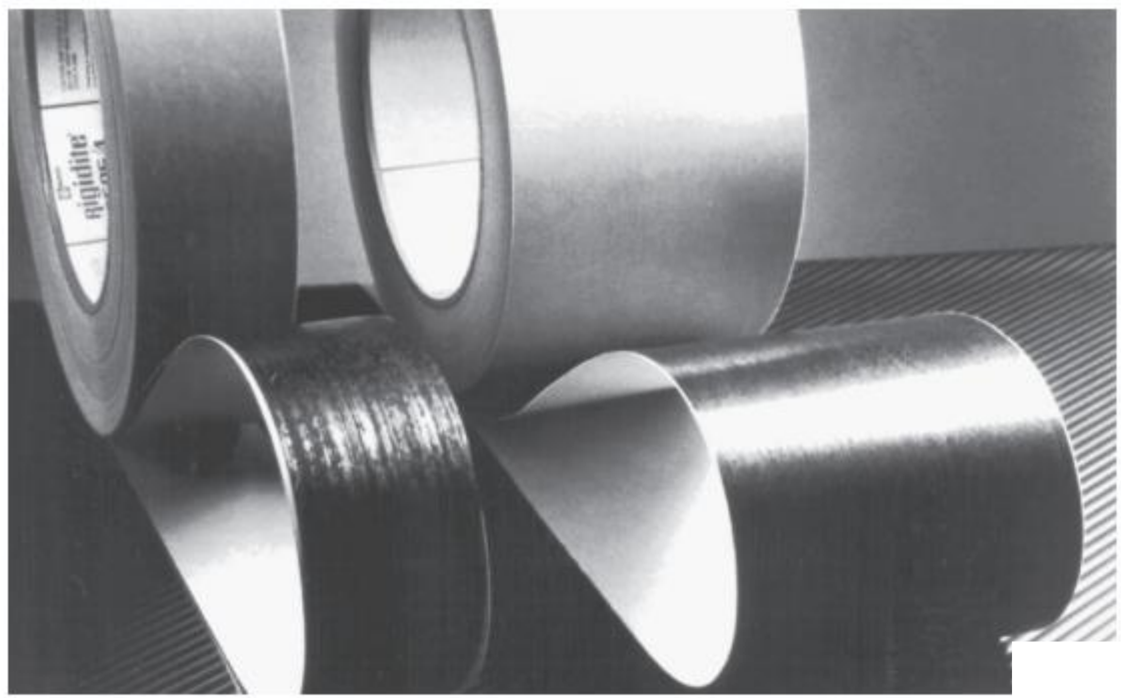

FIGURE 1.13
Boron/epoxy prepreg tape. (Photo courtesy of Specialty Materials, Inc., http://www.specmaterials.com.)

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- Figure 1.14 shows the schematic of how a prepreg is made. A row of fibers is passed through a resin bath. The resin-impregnated fibers are then heated to advance the curing reaction from A-stage to the B-stage. A release film is now wound over a take-up roll and backed with a release film. The release film keeps the prepregs from sticking to each other during storage

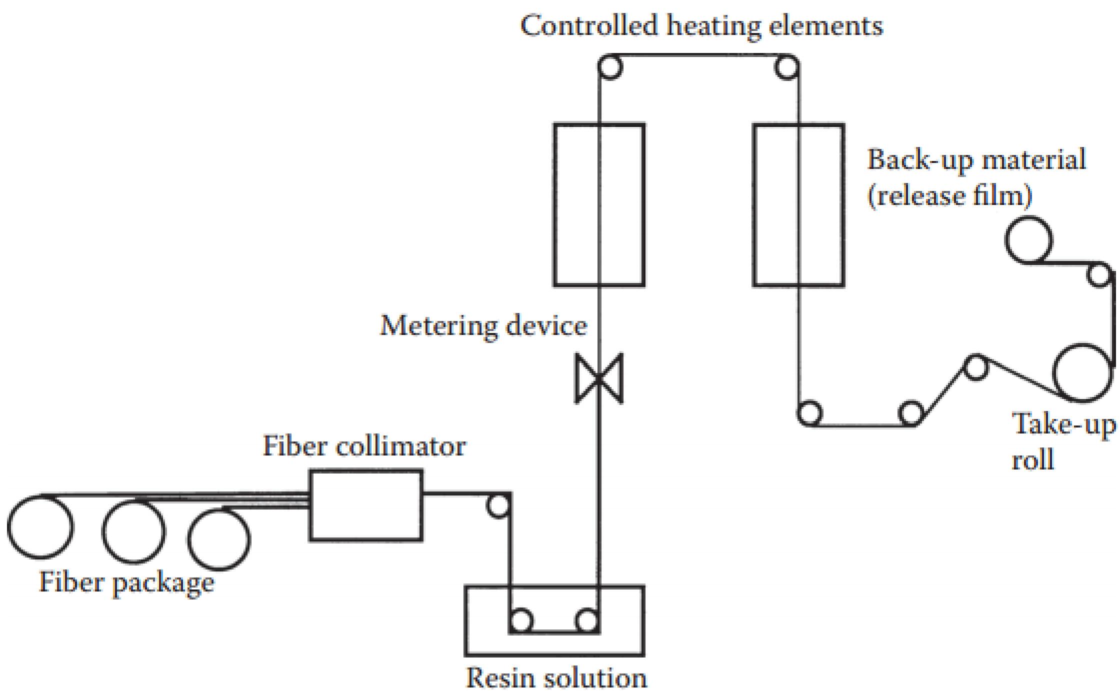

FIGURE 1.14
Schematic of prepreg manufacturing. (Reprinted from Mallick, P.K., *Fiber-Reinforced Composit*
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rtesy of CRC Press, Boca Raton, FL.)

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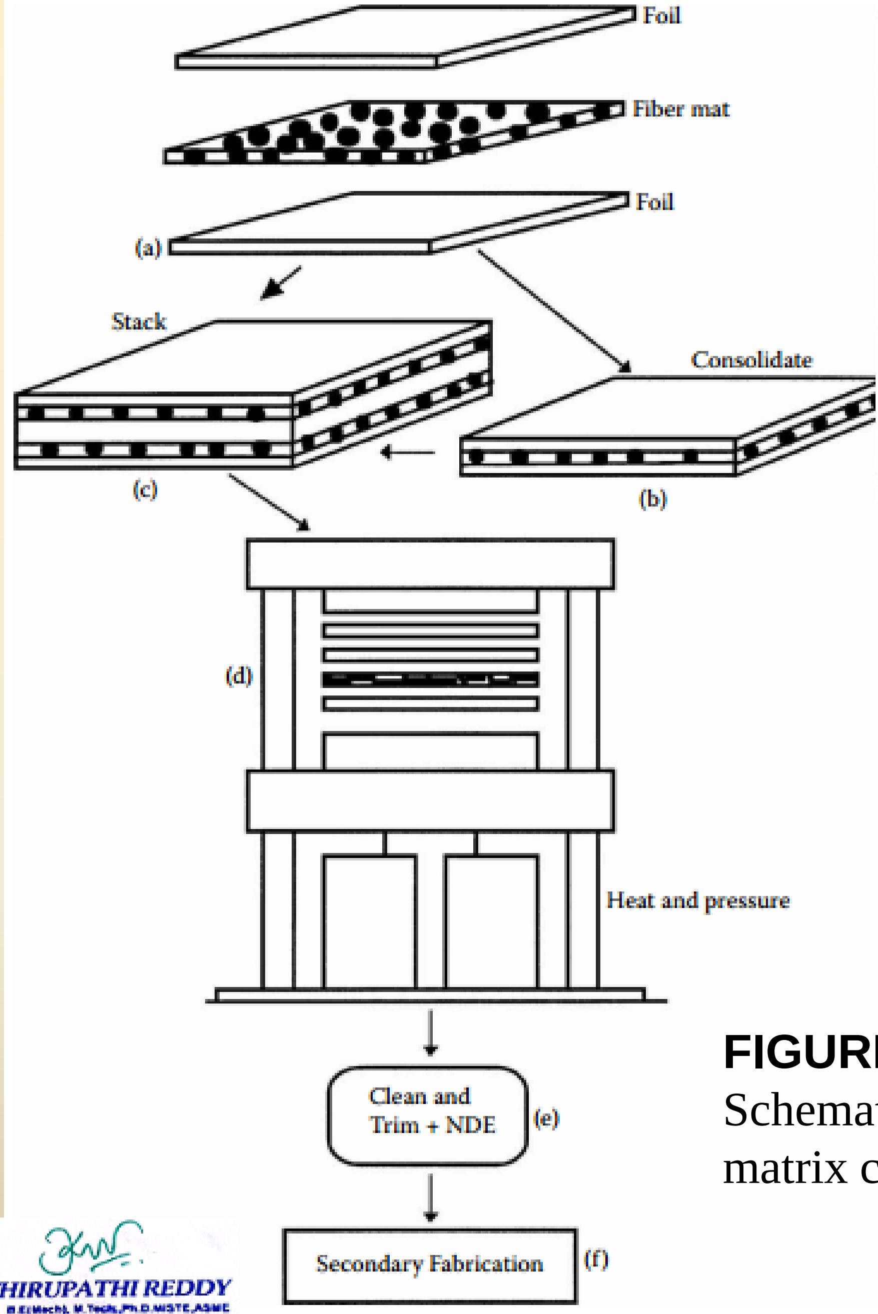

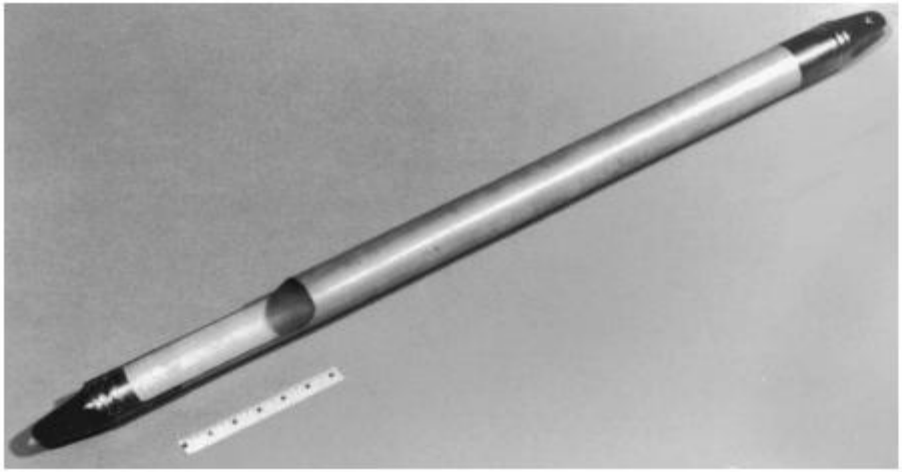

FIGURE 1.29
Boron/aluminum component made from diffusion bonding.

FIGURE 1.28
Schematic of diffusion bonding for metal matrix composites.

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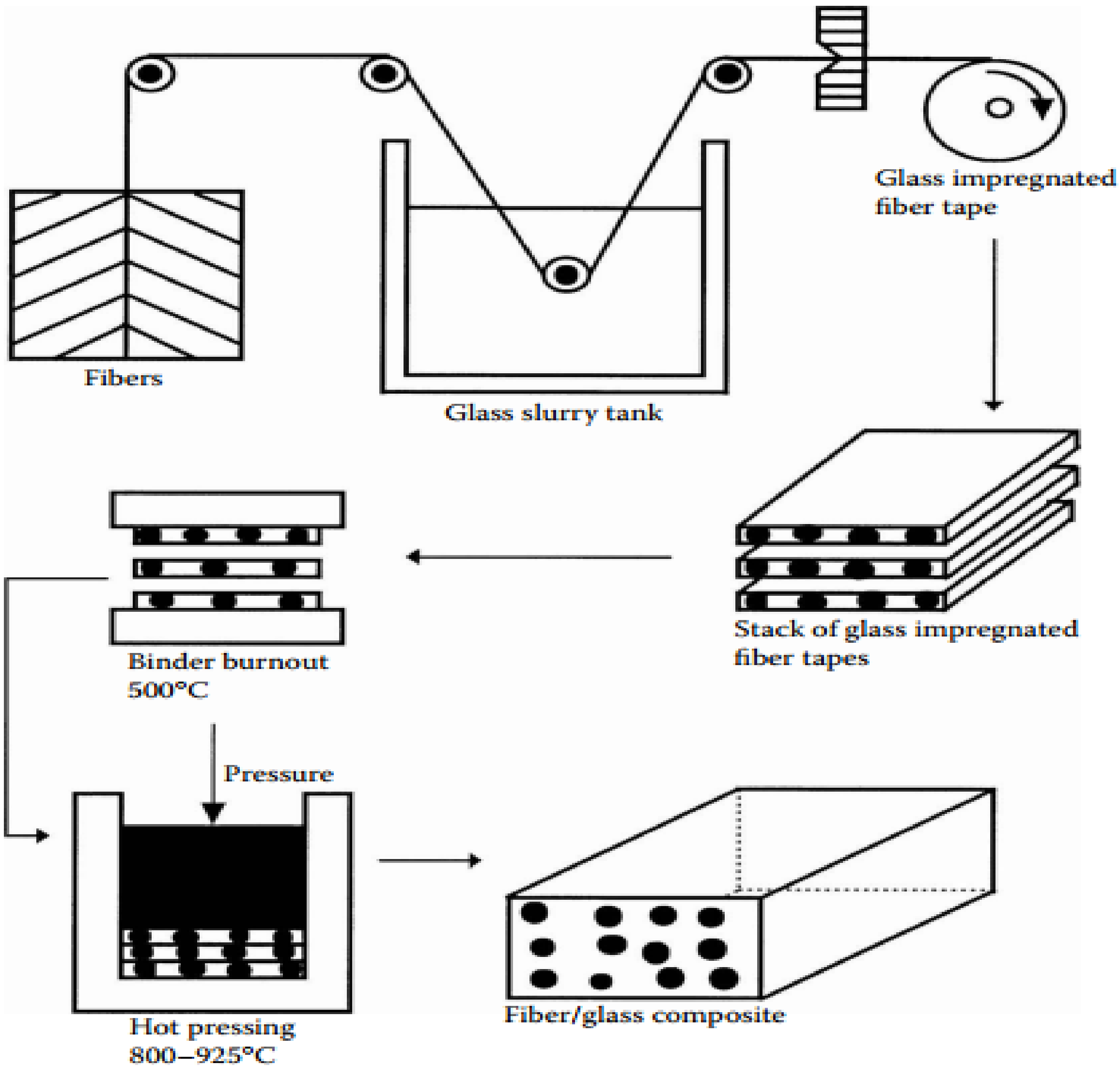

FIGURE 1.31
…tic of slurry infiltration process for ceramic matrix composites. (From Chawla, K.K., …nd Business Media from *Ceramics Matrix Composites*, Kluwer Academic Publishers, Lon-…3, Figure 4.1, p. 128. Reproduced with permission of Springer–Verlag.)

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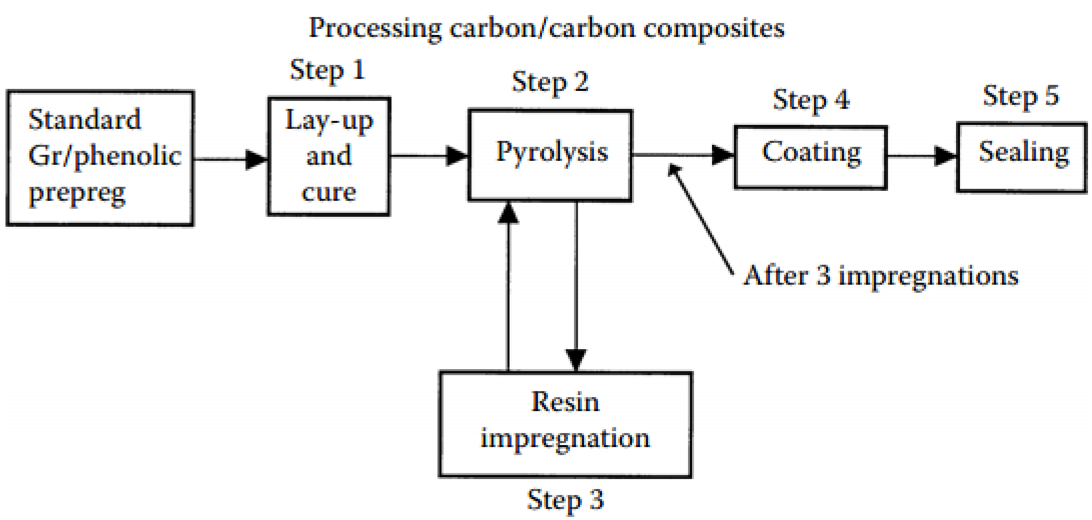

FIGURE 1.33
Schematic of processing carbon–carbon composites. (Reprinted with permission from Klein, A.J., *Adv. Mater. Processes*, 64–68, November 1986, ASM International.)

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- ally a laminate structure made of various laminas stacked on each other. Knowing the macromechanics of a single lamina, one develops the macromechanics of a laminate. Stiffness, strengths, and thermal and moisture expansion coefficients can be found for the whole laminate.
- Laminate failure is based on stresses and application of failure theories to each ply. This knowledge of analysis of composites can then eventually form the basis for the mechanical design of structures made of composites.

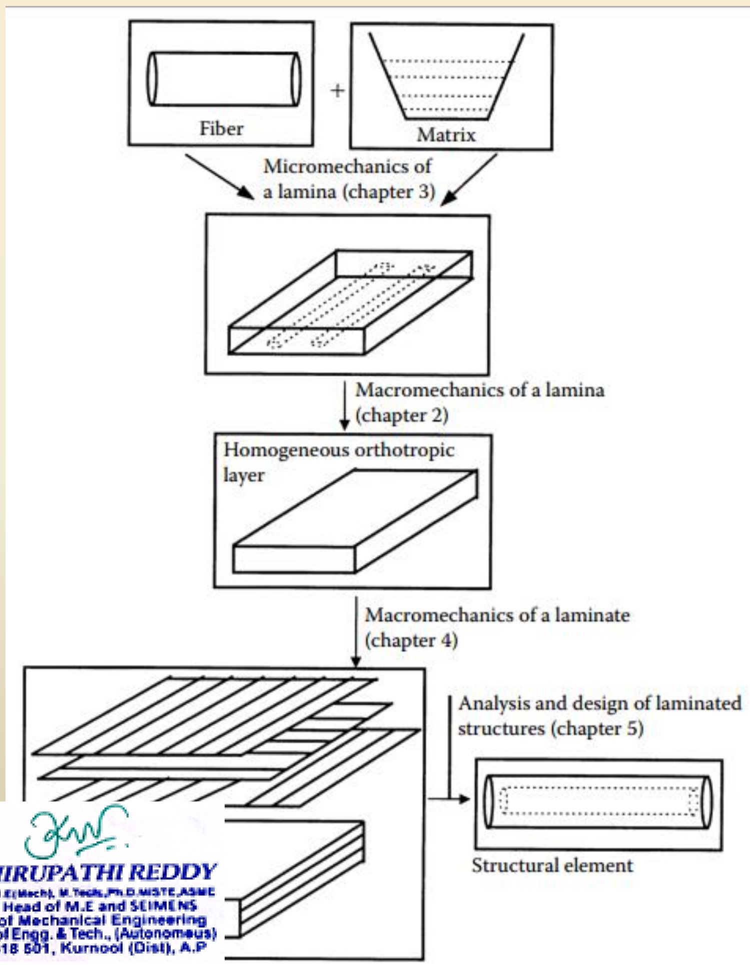

FIGURE 1.35
Schematic of analysis of laminated composites.

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Carbon-Carbon matrix composite (CCMCs)

- Carbon-Carbon composite is a carbon fiber reinforced carbon matrix composite.

- It is a two-phase composite material and as the name implies, both the matrix and reinforcement fiber are carbon.

- Carbon-Carbon can be tailor-made to give a wide variety of products by controlling the choice of fiber-type, fiber presentation and the matrix.

- Compared to other materials such as graphite, ceramics, metal, and plastic, it is lightweight and strong and can withstand temperatures over 2000℃ without any loss in performance.

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CARBON/CARBON MATRIX COMPOSITE

- C/Cs are developed specifically for parts that must operate in extreme temperature ranges. Composed of a carbon matrix reinforced with carbon yarn fabric, 3-D woven fabric, 3-D braiding, etc.
- C/C composites meet applications ranging from rockets to aerospace because of their ability to maintain and even increase their structural properties at extreme temperatures.

Advantages:

- Extremely high temperature resistance (1930°C – 2760°C).
- Strength actually increases at higher temperatures (up to 1930°C).
- High strength and stiffness.
- Good resistance to thermal shock.

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Carbon – Carbon Composites (CCC)

- Carbon Carbon Composites are those special composites in which both the reinforcing fibers and the matrix material are both pure carbon.
- Carbon-Carbon Composites are the woven mesh of Carbon-fibers.
- Carbon-Carbon Composites are used for their high strength and modulus of rigidity.
- Carbon-Carbon Composites are light weight material which can withstand temperatures up to 3000°C.
- Carbon-Carbon Composites' structure can be tailored to meet requirements.

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Properties Of C–C Composites (CCC)

- Excellent Thermal Shock Resistance(Over 2000°C)
- Low Coefficient of Thermal Expansion
- High Modulus of Elasticity (200 GPa)
- High Thermal Conductivity (100 W/m*K)
- Low Density (1830 Kg/m^3)
- High Strength
- Low Coefficient of Friction (in Fiber direction)
- Thermal Resistance in non-oxidizing atmosphere
- High Abrasion Resistance
- High Electrical Conductivity
- Non-Brittle Failure

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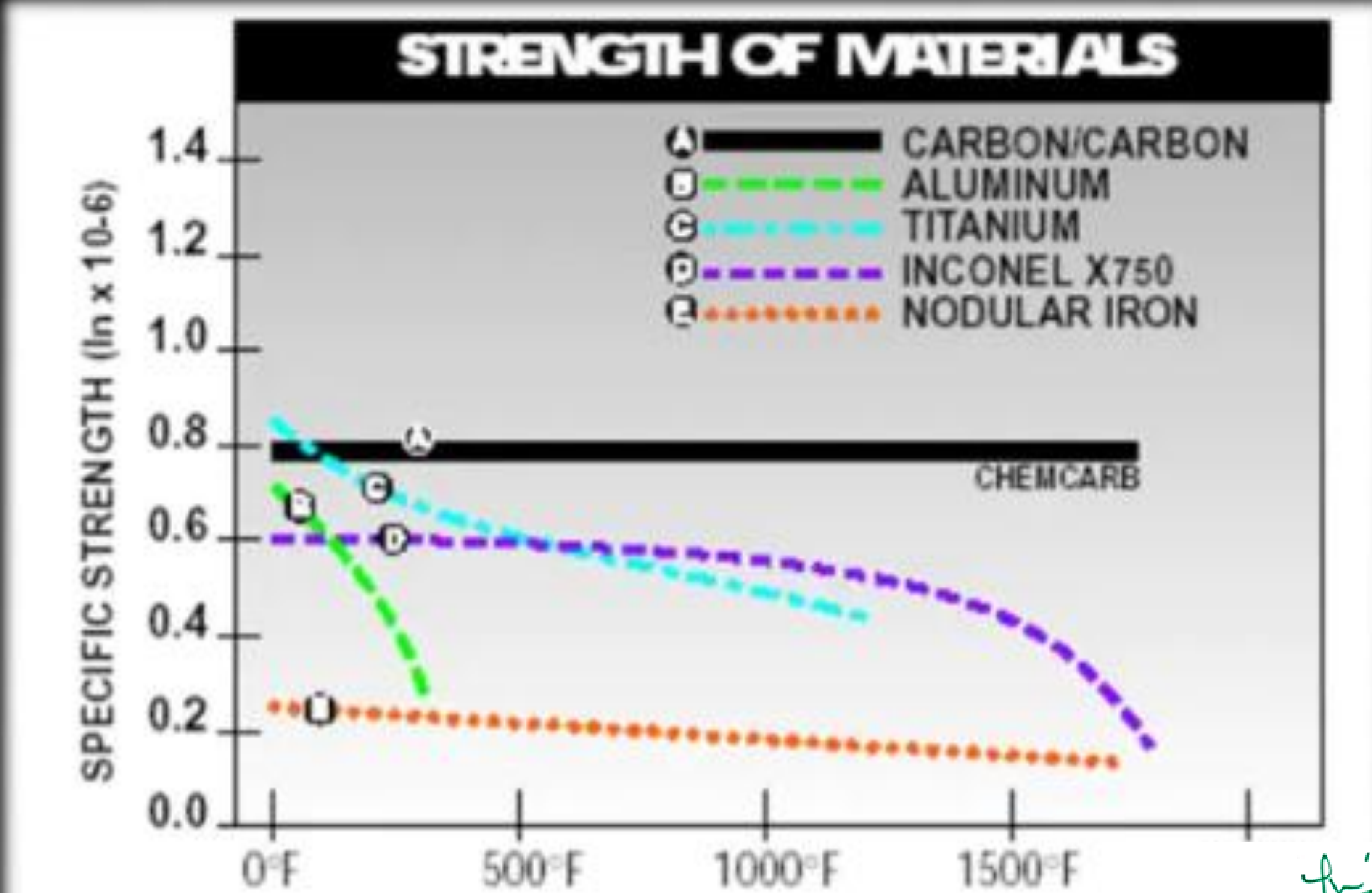
Properties Of C–C Composites (CCC)



CHEMCARB composites retain their physical prop
even at temperatures exceeding 3,700°F.

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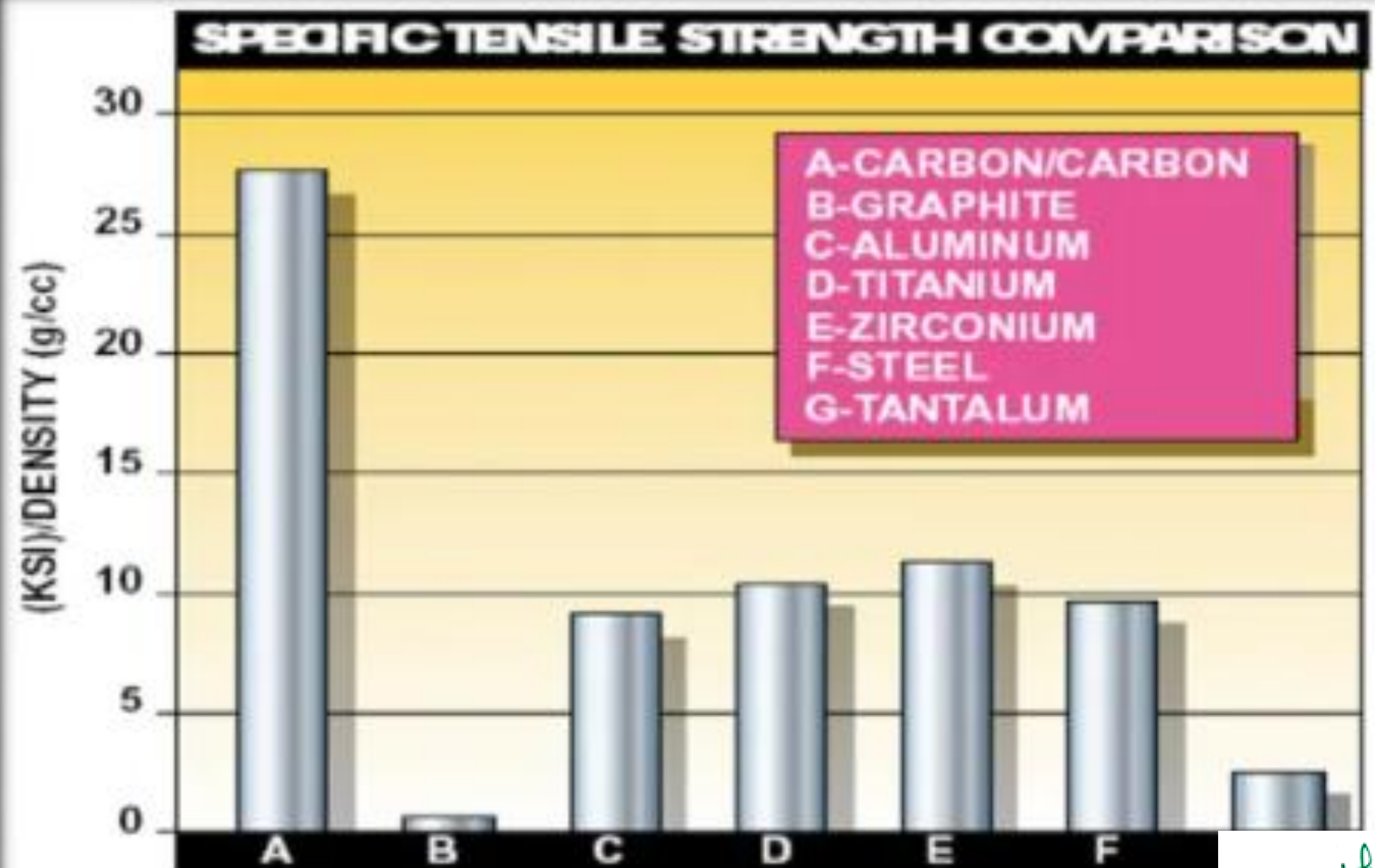
Properties Of C–C Composites (CCC)



For its density, CHEMCARB composites offer outstanding tensile strength, exceeding that of many metals.

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Compared to Metals	High heat resistance
	Low thermal expansion
	Lightweight (1/5 of metal)
	Does not bond
	Excellent resistance to corrosion and radiation
Compared to Graphite	High strength and rigidity
	High resistance to fracture
Compared to Ceramics	High resistance to fracture
	High thermal shock resistance
	Precision machinable
Compared to Plastics	High heat resistance
	Excellent resistance to corrosion and radiation
	High wear resistance

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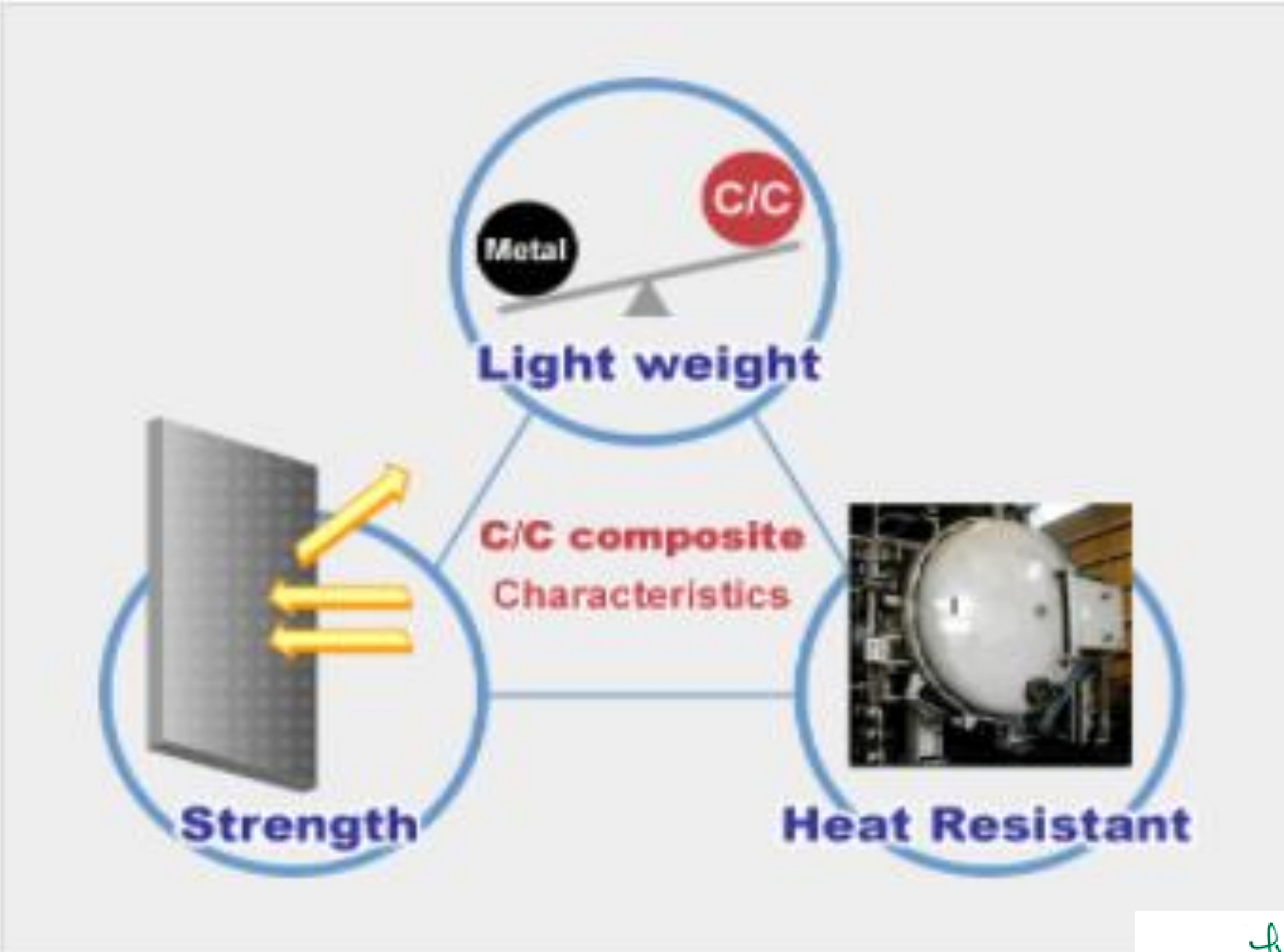

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A Carbon-Carbon composite is a carbon fiber reinforced carbon matrix composite. It is a two-phase composite material and as the name implies, both the matrix and reinforcement fiber are carbon. Carbon-Carbon can be tailor-made to give a wide variety of products by controlling the choice of fiber-type, fiber presentation and the matrix. Carbon-Carbon is primarily used for extreme high temperatures and friction applications.

Carbon-Carbon combines the desirable properties of the two constituent carbon material. The Carbon matrix (Heat resistance, Chemical resistance, Low thermal expansion coefficient, High-thermal conductivity, Low electric resistance, Low specific gravity) and the Carbon Fiber (High-strength, High elastic modulus) are molded together to form a better combination material. The reinforcing fiber is typically either a continuous (long-fiber) or discontinuous (short-fiber) carbon fiber type.

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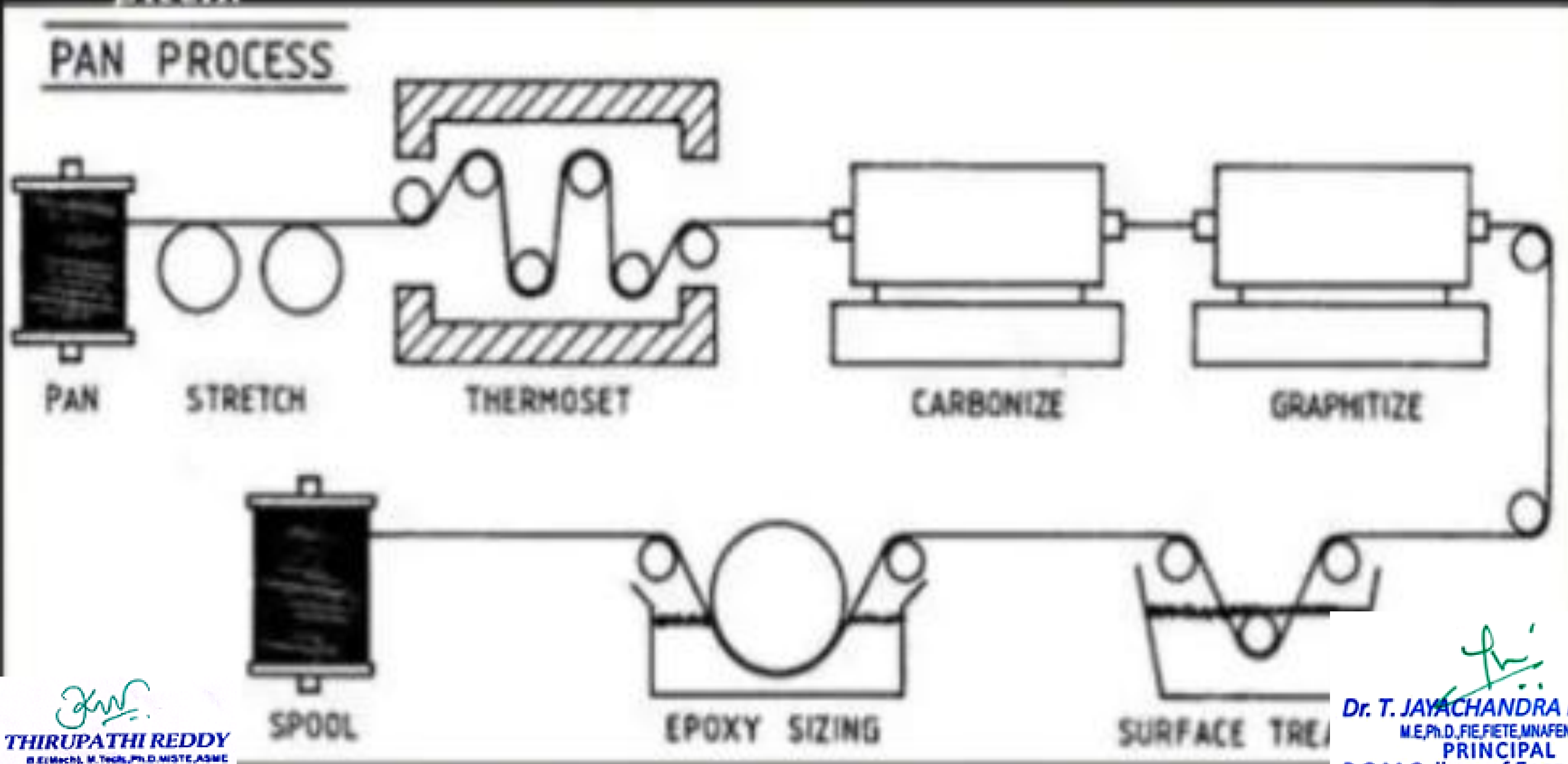
Processing Of Carbon Fiber

- About 90% of the **carbon fibers** produced are made from **polyacrylonitrile (PAN)** process.
- The remaining 10% are made from rayon or petroleum pitch.



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Processing Of Carbon Fiber

PAN-PROSSES

- In this method carbon fibers are produced by conversion of polyacrylonitrile (PAN) precursor through the following stages:
Stretching filaments from polyacrylonitrile precursor and their thermal oxidation at 200°C.

- The filaments are held in tension. Carbonization in Nitrogen atmosphere at a temperature about 1200°C for several hours.

- During this stage non-carbon elements (O,N,H) volatilize resulting in enrichment of the fibers with carbon. Graphitization at about 2500°C.

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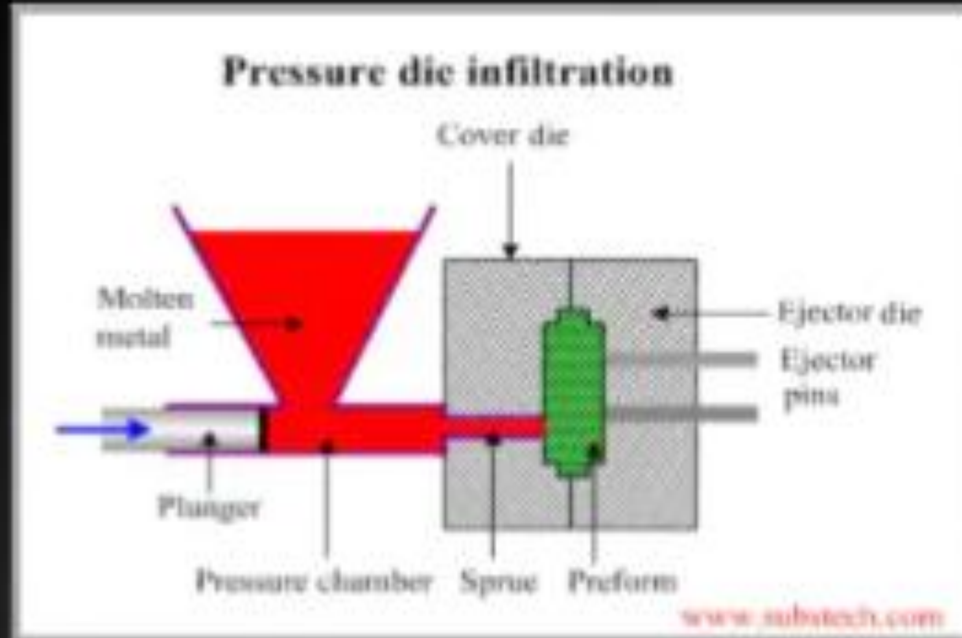
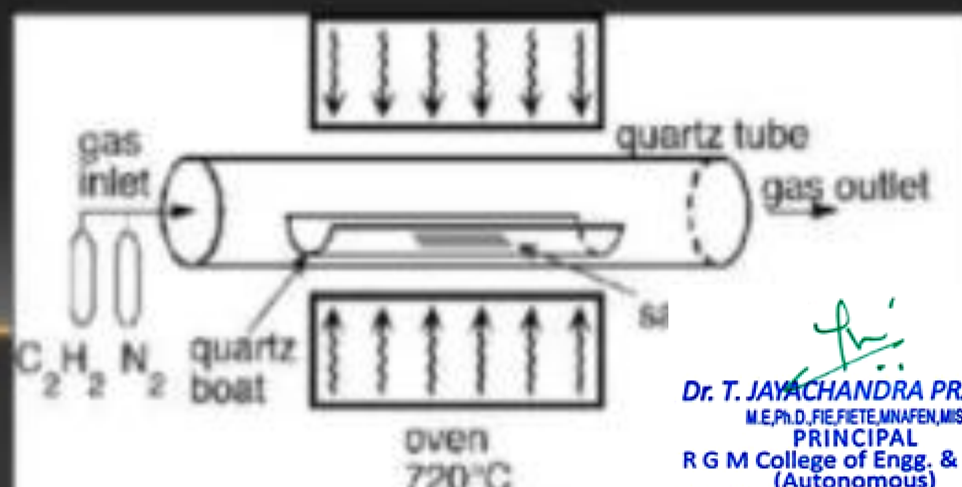
Fabrication Of C-C Composite

- Liquid Phase Infiltration



- Chemical Vapor Deposition



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Liquid Phase Infiltration

- Preparation of C/C fiber pre-form of desired shape and structure.
- Liquid pre-cursor : Petroleum pitch/ Phenolic resin/ Coal tar.
- Pyrolysis (Chemical deposition by heat in absence of O2.
- It is processed at 540–1000°C under high pressure.
- Pyrolysis cycle is repeated 3 to 10 times for desired density.
- Heat Treatment converts amorphous C into crystalline C.
- Temperature range of treatment :1500-3000°C.
- Heat treatment increases Modulus of Elasticity an

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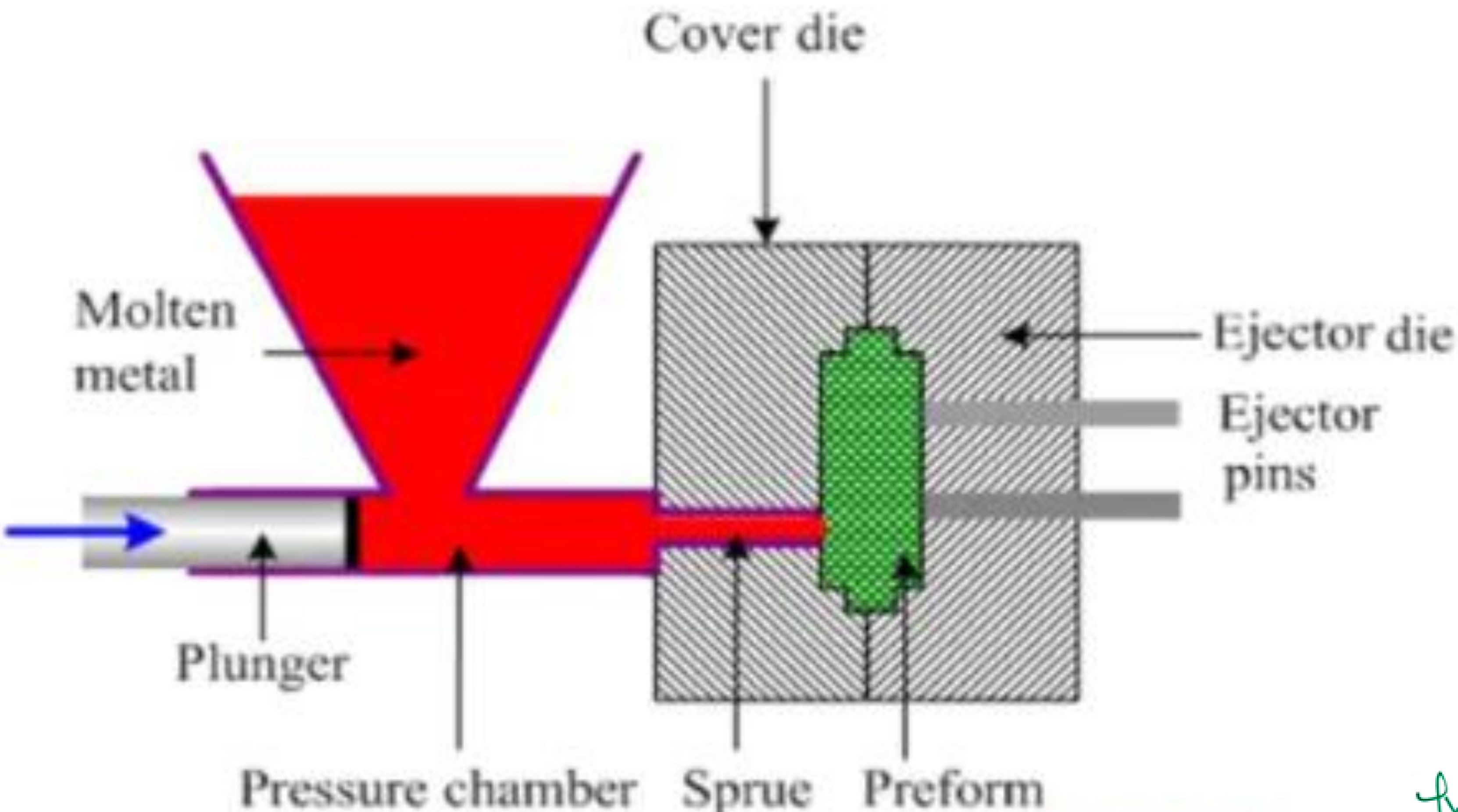
Liquid Phase Infiltration

Pressure die infiltration



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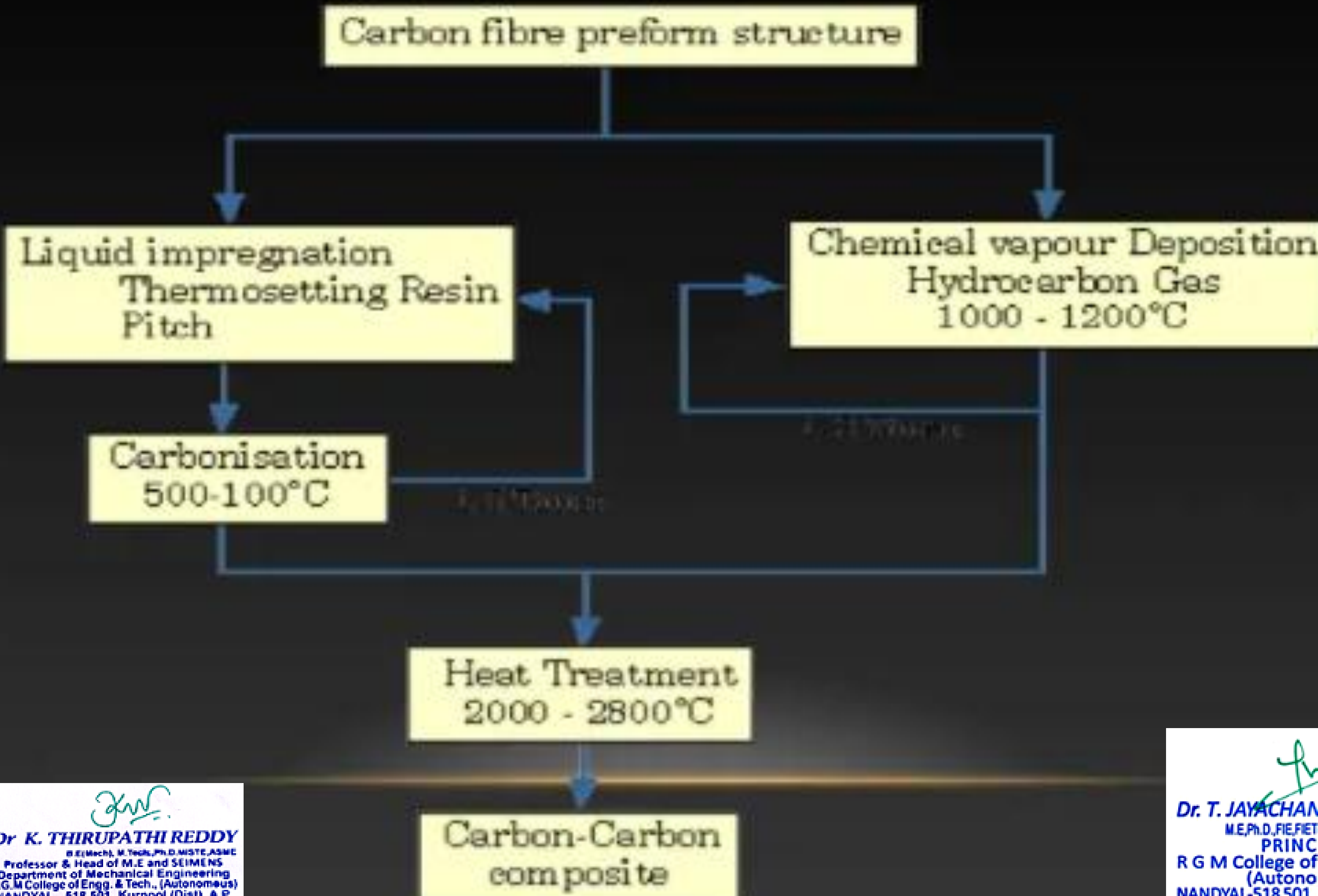
Flow Chart Of Manufacturing Process



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Chemical Vapor Deposition

- Preparation of C/C fiber pre-form of desired shape and structure
- Densification of the composite by CVD technique
- Infiltration from pressurized hydrocarbon gases (Methane /Propane)at 990-1210°C
- Gas is pyrolyzed from deposition on fibre surface
- Process duration depends on thickness of pre-form
- Heat treatment increases Modulus of Elasticity and Strength
- This process gives higher strength and modulus of

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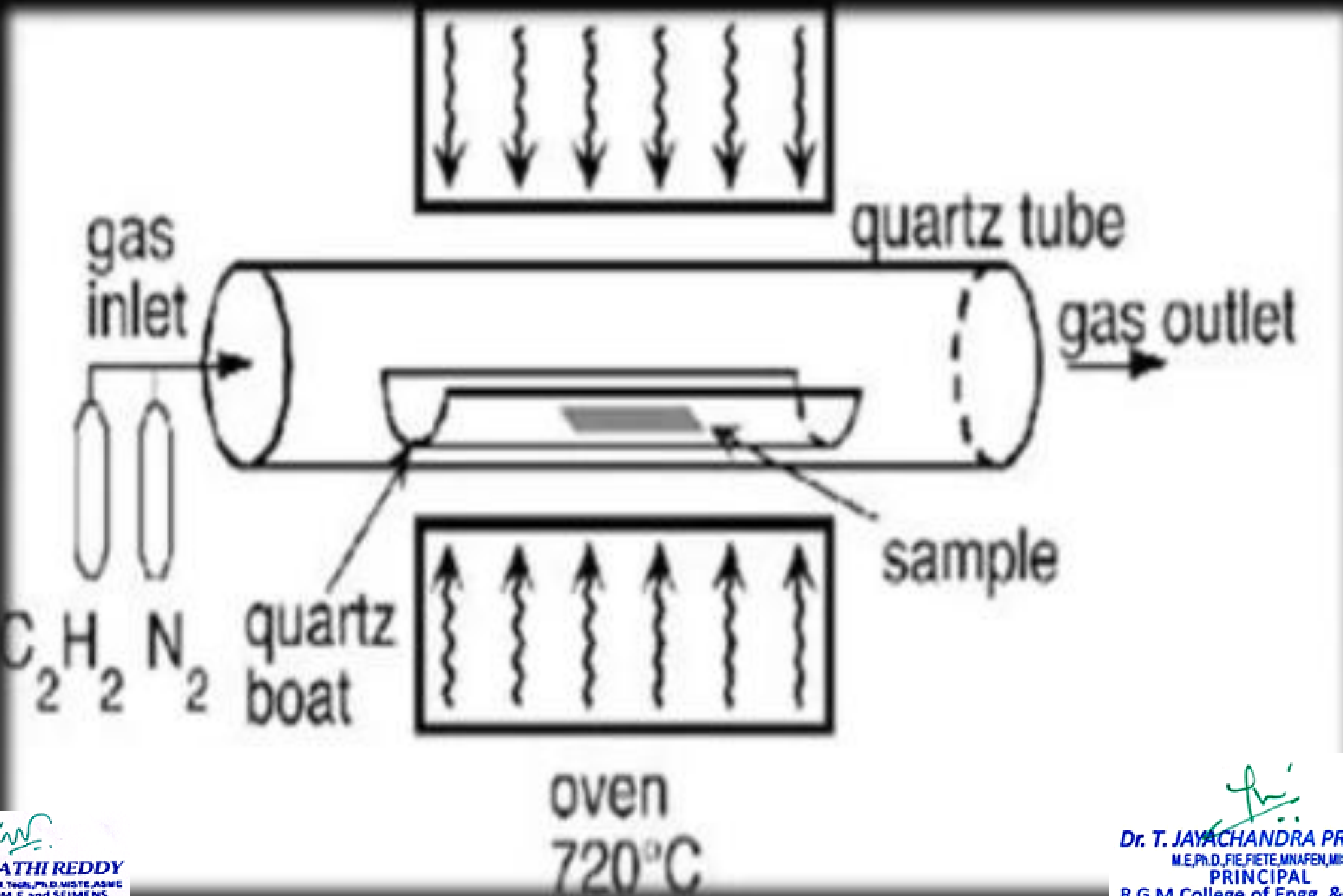
Chemical Vapor Deposition



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Limitation of CVD

- Hydrocarbon Gases Infiltrating into interfilament surfaces and cracks , sometimes these gases deposite on outer cracks and leave lot of pores.

- Reinfiltration and densification required.

- Month long process(for specific applications).

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Carbon – Carbon Composites (CCC)

Advantages

- Light Weight (1.6-2.0g/cm^3)
- High Strength at High Temperature (up to 2000 °C) in non-oxidizing atm.
- Low Coefficient of thermal expansion.
- High thermal conductivity (>Cu & Ag).
- High thermal shock resistance.

Carbon – Carbon Composites (CCC)

Disadvantages

- High fabrication cost.
- Porosity.
- Poor oxidation resistance – formation of gaseous oxides in oxygen atm.
- Poor inter-laminar properties.

Application Of C-C Composite

- High Performance Braking System
- Refractory Material
- Hot-Pressed Dies(brake pads)
- Turbo-Jet Engine Components
- Heating Elements
- Missile Nose-Tips
- Rocket Motor Throats
- Leading Edges(Space Shuttle, Agni missile)
- Heat Shields
- X-Ray Targets
- Aircraft Brakes
- Reentry vehicles
- Biomedical implants
- Engine pistons
- ronic heat sinks
- motive and motorcycle bodies

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Uses of Carbon-Carbon Composites

- Aircraft, F-1 racing cars and train brakes
- Space shuttle nose tip and leading edges
- Rocket nozzles and tips

http://www.fibermaterialsinc.com/frSW.htm

http://www.futureshuttle.com/conference/ThermalProtectionSystem/Curry_73099.pdf

http://www.fibermaterialsinc.com/frSW.htm

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Matrix and reinforcements in composites

PMCs

Matrix materials

Thermoplastics: Polyethylene, polystyrene, polycarbonate, polypropylene, nylon, Acryl butadiene styrene (ABS), Acetals etc

Thermo-sets: epoxy, polyester, polyurethanes, silicones, phenolics etc

Reinforcements: Glass fibers, carbon fiber, Kevlar fibers, aramid fibers are some synthetic materials.

Coir fibers, jute fibers, sisal fibers, banana fibers, bamboo fibers are some natural fibers etc

Elastomers:

matrix: rubber materials

Reinforcements: metal wires

MMCs

Matrix materials: Aluminum, magnesium, Titanium, cobalt, nickel etc

Reinforcements: Alumina, boron carbide, titanium carbide, boron etc

CMCs

Matrix materials: alumina(oxide form), SiC(non oxide form)

Reinforcements: SiC(whiskers), Titanium Boride (TiB_2) Aluminum Nitride(AlN) Zirconium oxide(ZrO_2) etc

...erial: Carbon

...ent: Carbon /graphite fiber

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Applications of composites

Aerospace

gliders
helicopter blades
transmission shafts
elevators
spoilers(aerodynamic device)
rocket boosters
nozzles
antenna covers
fuselages
Doors/sears
food trays
rudders (tail)

Automobiles

leaf springs
car seats & bumpers
body components
Chassis
engine components
Fuel tanks
tire guards
window frames
front grills
Engine bonnet
mud guards
lamp heads & housings
cabins
Instrument panels
cabins
light housings
radiator fans

Marine

fishing boats
life boats
anti marine ships
rescue crafts
hover craft
yachts
naval ships
hulls
Decks
bulk heads
masts propulsion shafts

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Applications of composites

Sport goods

fishing rods
hockey sticks
arrows
javelins
base ball bats
helmets
exercise equipment
shoe soles and heels
golf rackets
pole vault poles

Elec./ Electronics

Switches
Wires
Optical fibers
circuits
Mother boards
sinks
semiconductors

Industrial

Reactors
boiling tubs
tanks
Distillation columns
cooling towers

Construction

window frames
bath room panels
cladding panels
house furniture
roofing panels
pipes and ducts
swimming pools
diving boards
door panels
over head tanks
POP ceiling
pipe lines
flooring

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UNIT-II

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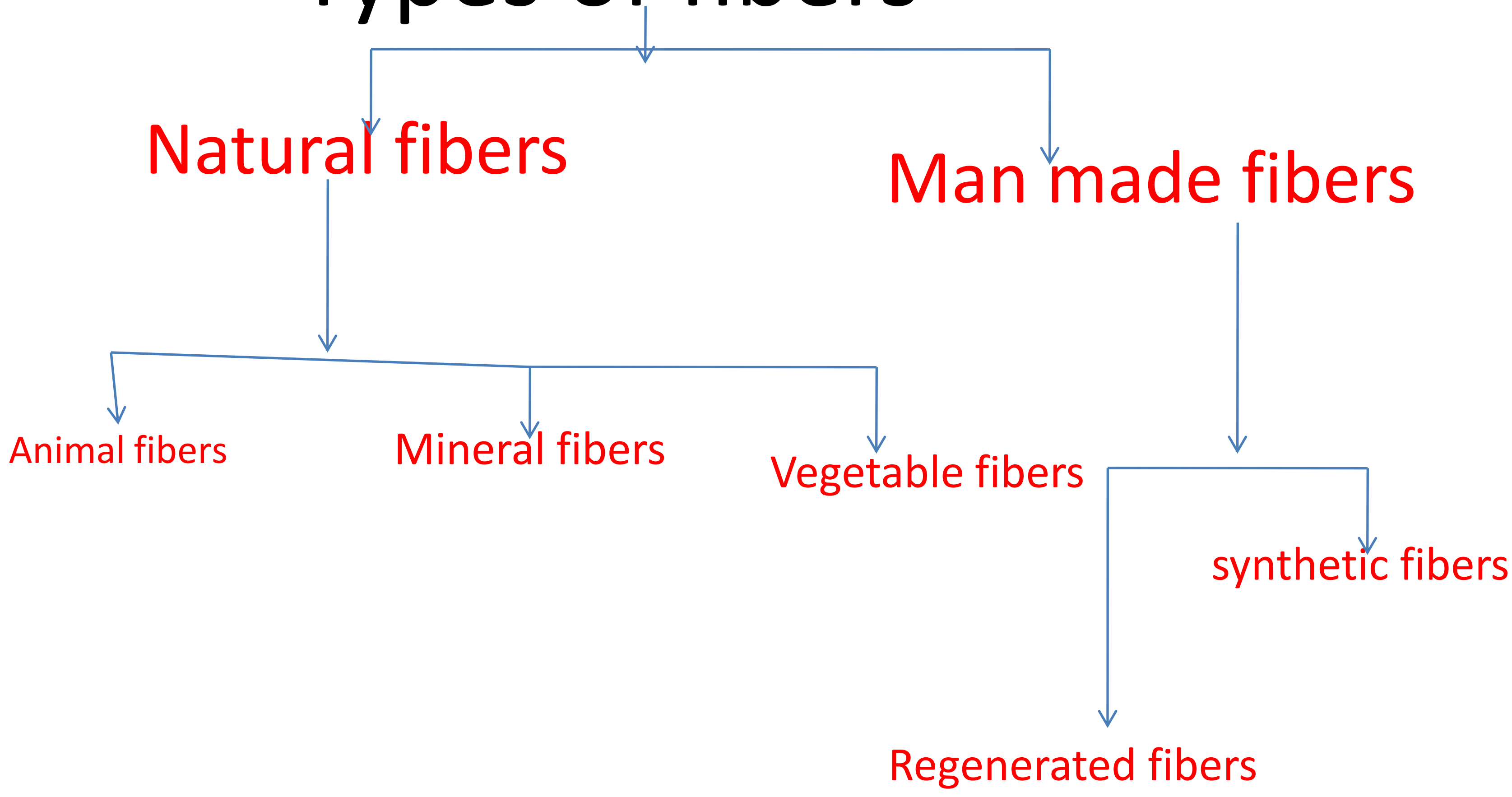
Types of fibers



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Fiber characteristics

- extremely thin and flexible
- one dimension (l>d)
- high modulus and strength
- better default properties
- lateral dimn. Should be in microns
- fiber should be stronger than matrix
- High aspect ration

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SILICA FIBER

Introduction

1. Silica fibers are fibers made of **sodium silicate** (water glass)
2. They can be made such that they are substantially free from non- alkali metal compounds.
3. They are used in heat protection (including asbestos substitution) and in packings and compensators.
4. silica fiber used as a reinforcing the material and yet wet webs and filter linings.
5. Silica fibers are used as a Optical Fibers Optical fiber is used as a medium for telecommunication and computer networking because it is flexible and can be bundled as cables. It is especially advantageous for long-distance communications, because infrared light propagates through the fiber with much lower attenuation compared to electricity in electrical cables.
6. strength can be further improved by providing the polymer jackets.

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Characteristics

- Superb transparency
- good purity,ρ=2.61g/cc
- heat resistance as high as 1700°C
- Excellent chemical inertness
- A silica fiber has an amazingly high mechanical strength against pulling and even bending, provided that the fiber is not too thick.
- Silica glass can be doped with various materials in order to improve various properties.
- Silica has a high damage threshold.

SILICA FIBER

Applications

Applications in rockets, spacecrafts, missiles, heat-fire resistant equipments.

Pressure control devices, expansion joints to reduce heat, counterbalancing the destability, friction lining materials

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Glass fibers

- glass fibre is material consisting of numerous extremely fin fibers of glass.
- it is cheaper and significantly less brittle material.
- used as a reinforcing material in polymer matrix composites

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Types of glass fibers

- E-glass fiber: E stands for electrical application,most common type of glass fiber (alumino- borosilicate glass with less than 1% alkali oxide), mainly usd for glass reinforced plastics
- D-glass fiber: D stands for dielectric suitable for low dielectric constants. (borosilicate glass with less th)
- S-glass fiber: S stands for strength(tensile)(alumino silicate glass without CaO but with high MgO content)
- C-glass fiber: C stands for chemical resistance, used for insulation purpose.(alkali lime glass with high boron oxide content)
- E-CR glass fiber: E-CR stands for electrical and chemical resistance.(It has alumino lime silicate with less than 1% alakli oxide.
- A-glass fiber: A stands for alkali resistance.

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characteristics

- resistance to attack of most of the chemicals.
- it has comparable mechanical properties with carbon fiber.
- it is a durable and light weight material

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Properties

- High tensile strength
- High dimensional stability
- High heat resistance.
- Good thermal conductivity
- Great fire resistance.
- Good chemical resistance.
- Outstanding electrical properties
- Dielectric permeability
- compatible with matrix materials
- great durability
- non-totting
- highly economical

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Disadvantages

- inhale causes lung disease

Applications

- rocket bodies
- exhaust nozzles
- heat shields
- wall panels
- fishing rods
- insulators
- rinforcements

Boron fibre

- Introduction
- It is also called hybrid boron fiber.
- First introduced in the year of 1959.
- Chemical vapor deposition (CVD) deposition process is used to produce these fibers.
- in CVD process material is deposited on a thin filament.
- It is fine, dense deposited material which determines the strength and modulus of fiber.
- in CVD process boron tri-chlorides are mixed with the hydrogen.

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Boron fiber

- Tensile strength (3600MPa)
- Tensile modulus (400GPa)
- compressive strength(6900MPa)
- Fracture strength (17GPa)
- α= 4.5ppm/°C
- $_{\rho}$=2.57 g/cm^3
- Φ = 142μm

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Boron fiber

- ceramic monofilaments used in complex helical structures.
- fiber dia. Ranges from 33-400$^{\mu m}$
- Thermal expansion would mismatch boron and tungsten.
- Boron is a brittle material hence for large diameters results less flexibility
- If boron is coated on SiC fiber and B_4C fiber ,then it protects the surface.
- $_{it}$ exibits linear axial stress strain relationship upto650°C
- it strong in both tension and compression

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Applications

- Bicycle frames
- sports goods
- fishing rods
- space shuttle
- Air craft repairs

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Kevlar fiber

- It is widely used fiber in combination with GF/CF
- it is formed by hydrogen bonds between the polymer chains.
- looks like a long twisted coil.
- yellowish color.
- Strong and heat resistant
- strength is intact at cryogenic temp. -196°C
- At higher temps. Strength is reduced(Ex: at 160°C 10% TS is reduced and also 260°C 50% TS is reduced.
- High shear strength, ρ=1.44 g/cc, TS =3600MPa
- production is similar to nylon fiber

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Applications

- bullet proof vests
- bicycle tires
- racing sails
- personal armors
- Helicopter rotor blades
- combat helmets
- racing car bodies
- field hockey bats

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Boron Carbide fiber (B_4C)

- color is dark grey
- extremely hard ceramic material
- boron-carbon are made with covalent bonds
- Vickers hardness is greater than 30GPa
- it is 3ed hardest material after diamond and boron nitride.
- P=2.52g/cc, E=460GPa, Hardness=38GPa, fracture toughness =3.5MPa/sq.m
- high performance abrasive material
- flexural strength is more than 400Mpa
- $B_2O_3 + 7C \longrightarrow B_4C + 6CO$
- B_2O_3 boron trichloride

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drawbacks

- low thermal conductivity
- susceptible to thermal shock failure.
- extremely brittle

applications

- Nuclear reactors
- MMCs
- solid fuel-Ramjets
- brake lining materials
- armor plating
- cutting tools and dies
- abrasives
- nozzles for slurry pumping

Carbon fiber

- carbon fibers are bonded together to form a long chain
- produced from Poly-acrylonitrile (PAN) or pitch.
- 5X stronger and 2X stiffer than steel
- 2.33X lesser in weight

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Advantages

- High tensile strength
- high extension at break
- High modulus
- good electrical conductivity
- Low α
- low ρ
- high wear resistance
- long working life
- compressive strength is greater than all fibers
- properties are better than other metals
- Insensitive to temperature
- density is lesser than steel

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DISADVANTAGES

- ❑ Costly
- ❑ it causes lung cancer

APPLICATIONS

- ➢ Rackets
- ➢ golf sticks
- ➢ Automotive body parts
- ➢ mobile cases
- ➢ recharge batteries
- ➢ fuel cells
- ➢ Portable power banks
- ➢ music instruments

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Silicon Carbide Fiber

- SiC is a simple compound with carbon atoms attached to silicon through triple bond, leaving both atoms with +ve and –ve charge.
- ‘Si’ is metalloid and the ‘carbon’ is non-metal and properties formed between the metals and nonmetals.
- It is used as a reinforcing/abrasive/ ceramics material
- the grains of SiC can be bonded together by sintering to form very hard material.
- it is a ceramic material widely used in applications require high endurance.
- SiC has diamond like tetrahedral crystal structure formed by covalent bonds
- Just like carbon does in diamond
- It exists in crystalline form.
- $SiO_2 + 3C \longrightarrow SiC + 2CO$ at temp 1600°C-2500°C

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Properties of SiC

- Low density
- high strength and stiffness
- Lowα
- High thermal conductivity
- High hardness
- High elastic modulus
- High thermal shock resistance
- high chemical inertness
- It irritates eyes,skin

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Applications of SiC

- Wear resistance parts for pumps and rockets engines
- LEDs and semiconductors
- car clutches
- car brakes
- refractory lining
- gas flow liners
- bearings
- turbine parts
- heat exchangers
- Grinding wheels
- Jewelry

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PROCESSING OF POLYMER MATRIX COMPOSITES (PMCs)

BY

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TYPES OF MANUFACTURING PROCESSES

- HAND LAY UP
- SPRAY LAYUP
- VACUUM BAGGING
- PULTRUSION
- RESIN TRANSFER MOULDING
- FILAMENT WINDING
- AUTOCLAVE MOULDING

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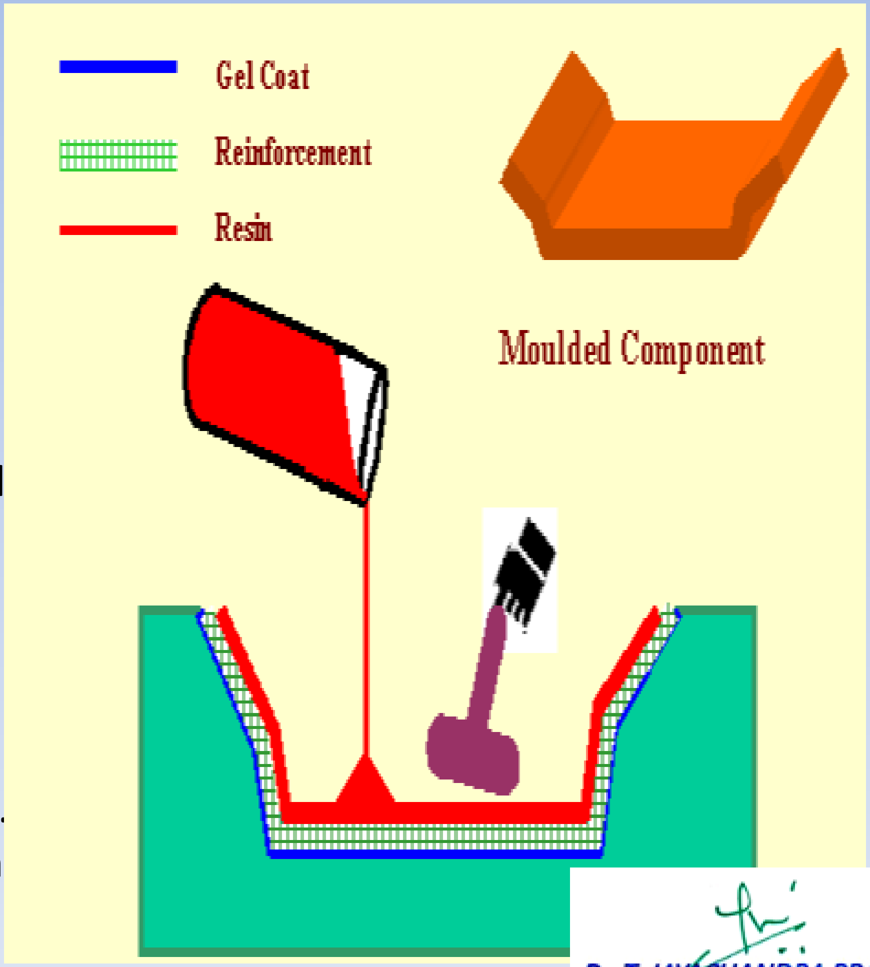
HANDLAY UP PROCCESS

- Composites are made manually.
- It is a slow process and labor consuming
- The largest number of reinforced plastics composite products are produced by the hand lay-up process.
- Mat type or woven/ fabric fiber type fibers are used.
- Mould is prepared based on the final shape of the product.
- Catalyzed resin is used as a matrix which is made up of resin and catalyst.
- Catalyzed resin is prepared based on the stoichiometric ratio s of both.
- Mould is open.
- We get one side only smooth surface.
- Brush and rollers are used in this process.
- Curing is done at room temp.
- post curing parts are removed after keeping some time in the furnace to ensure mould releasing agent to melt.

Fabrication steps

- Mould is coated with mould releasing agent for easy removal of mould after curing.
- Then mould is coated with gel coat to give coloring purpose.
- Fiber fabrics are cut into desired shapes and then stacked into the mould all over.
- pour the some amount of catalyzed resin all over the mould and further we have to spread it all over the mould with brush and roller to ensure wetting.
- We have to add another layer of fiber to be spread all over the mould and then poured some more amount of fiber into the mould.
- We have to put fiber layer plus resin layer alternatively until we get desired thickness.
- we have to finish this process before resin starts gelling.



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Advantages

- Widely used.
- Low tooling cost.
- Custom shape.
- Larger and complex items can be produced.

Disadvantages

- Labour intensive.
- Low-volume process.
- Styrene emission.
- Quality control is entirely dependent on the skill of labourers.

- Only 30% of the fiber can be stacked.
- Emission due to open mould
- air entrapment makes air bubbles formation.

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SPRAY LAY UP PROCESS

- Continuous strand glass roving and initiated resin are then fed through a chopper gun, which deposits the resin-saturated “chop” on the mold.
- This is done by spray gun.
- mould is open mould releasing agent and gel coat is applied before streaming the fiber and resin.
- Here chopped fibers are used where as In hand lay up mat are used as a fibers
- Spray gun injects chopped fiber catalyzed resin on to the mould surface with HP jet.
- Fibers are cut into 25 to 50mm length with the help of adjustable blade in the gun.
- this process is good for automation for high rate of production.
- mechanical properties are moderate due to the not using of continuous fibers.

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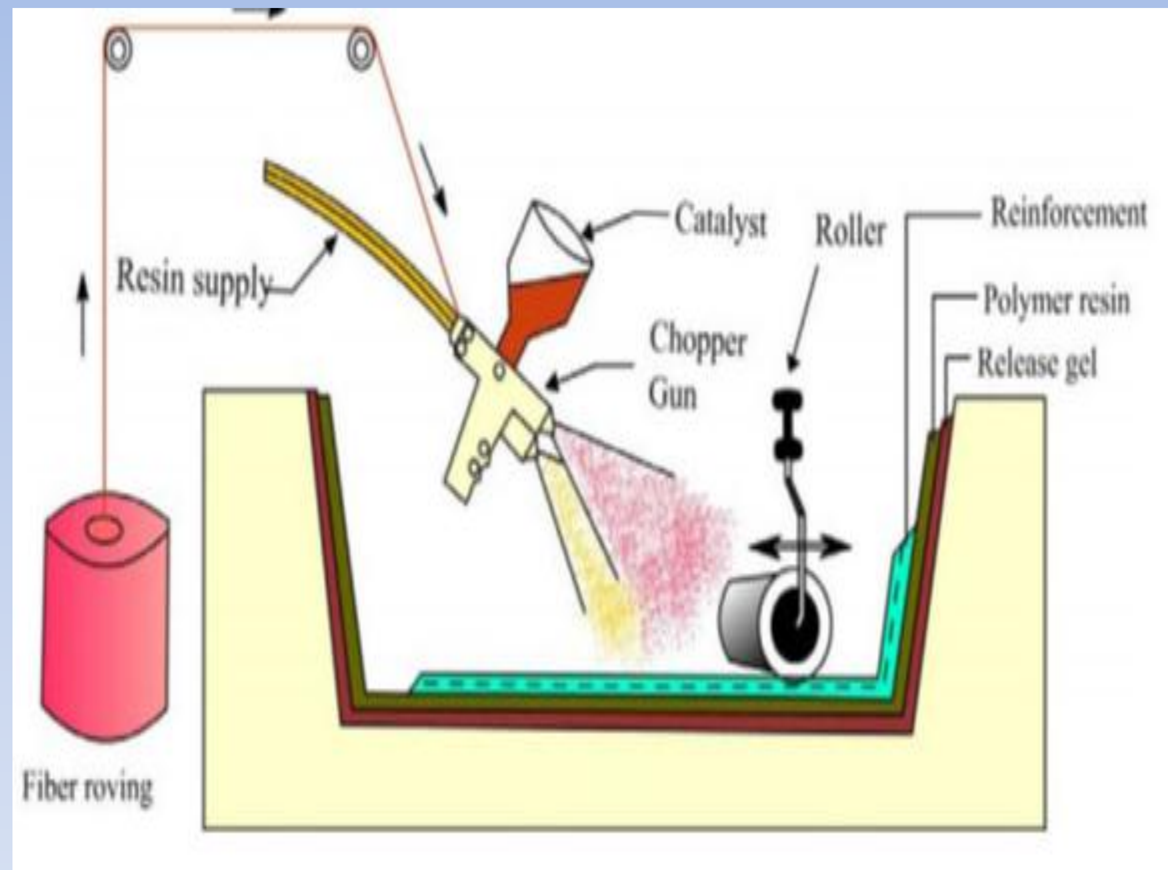
Fabrication steps

1. MRA and gel coats are applied.
2. With the help of the gun chopped fibers and resin are injected on to the mould surface directly.
3. chopped fibers are dressed in proper shape and placed all over the mould to impart desired thickness. This has to be done manually.
4. to reduce defects resin is spread uniformly to ensure bonding between the fiber and matrix.
5. We have to do continuously until we get completed the entire mould with desired thickness.
6. then allow some time for curing. We should remove the casting from the mould.



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Advantages and disadvantages of spray lay up

- Tooling cost is low.
- Semiskilled workers are easily trained.
- Design Flexibility.
- Molded-in inserts and structural changes are possible.
- Sandwich constructions are possible.
- Large and Complex items can be produced.
- Minimum equipment investment is necessary.
- The startup lead time and the cost are minimal.

- Labor Intensive.
- Low volume process.
- Longer curing times.
- Production uniformity is difficult.
- Waste factor is high.

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applications

boats, tanks, transportation components, and tub/shower units in a large variety of shapes and sizes.

What is vacuum bagging?

•Vacuum bagging (or vacuum bag laminating) is a clamping method that uses atmospheric pressure to hold the adhesive or resin-coated components of a lamination in place until the adhesive cures.

•(When discussing composites, "resin" generally refers to the resin system— mixed or cured resin.)

- *Vacuum bag molding*
- *Also known as vacuum bagging.*
- *Open mold techniques for thermoset composites.*

- ***Hand lay-up**: The application of reinforcement along with a polyester or epoxy resin by hand.*
- ***Vacuum bagging**: The use of a vacuum bag ... en pressure over the composite to conso... aterial.*

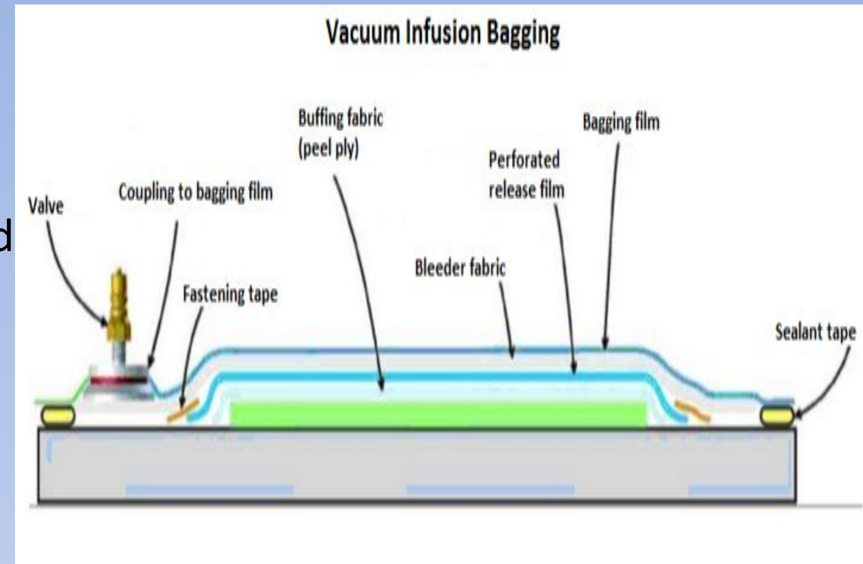
vacuum bagging process

vacuum bagging process utilizes a flexible and transparent film (ie: fabric, nylon, rubberized sheet or plastic) in order to fully enclose and compacting the wet laminate by using atmospheric pressure. this process is also called vacuum bagging.

It uses a vacuum and pump to extract the air from inside the vacuum bag and compress the part under atmospheric pressure in order for the compacting and hardening process to take place.

vacuum bagging is an upgrade of the wet lay-up process and is widely spread in the composite industry because of its clear benefits over this method.

you will most often see the use of fiberglass, carbon fiber and resin materials being [illegible] together using the vacuum bag



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Benefits

Finished product will yield a better strength rating and be lighter.

Parts that are stronger yet lighter

the ratio of glass to resin which is better accomplished.

materials for basic parts are inexpensive and easily obtained.

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Disadvantages

Applied vacuum pressure then removes excess resin; however the amount removed will depend on multiple different and critical variables that may be hard to control.

Removing excess resin, which was first brought in, is a clear waste of money and resources.

In larger projects, it is also necessary to apply the vacuum bagging process a couple of times since the resin pot-life is the limiting factor.

The amount of resin that is removed from part to part can also vary substantially depending on the timing of the vacuum pressure being applied.

The process of bagging can become rushed opening up the opportunity for error if a leak in the vacuum seal occurs and cannot be immediately located.

Unfortunately with bagging, the fiber to volume ratio cannot be successfully calculated as it can with other processes, and over-bleeding or dry laminates can be a large concern.

Bigger and more complex lay-ups also require additional helpers, increasing labor needs and support.

Another imminent disadvantage with hand-lay-up and bagging is that the process must be completed once started, with no option to pause or take a step back.

There is a clear time and forgiveness disadvantage in wetting-out and squeegee processes with a race against the resin pot-life and getting all of the materials in place.

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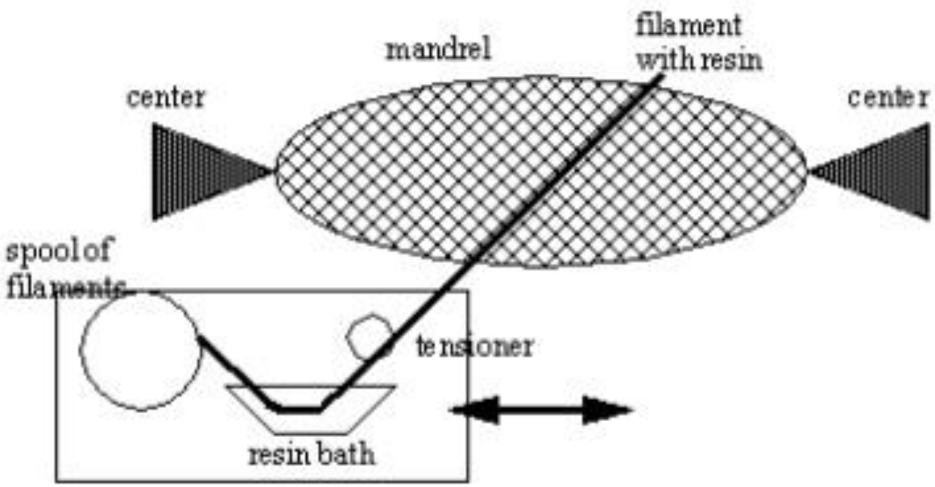
Filament Winding

Filament Winding method involves a continuous filament of reinforcing material wound onto a rotating mandrel in layers at different layers. If a liquid thermosetting resin is applied on the filament prior to winding the, process is called Wet Filament Winding. If the resin is sprayed onto the mandrel with wound filament, the process is called Dry Filament Winding.
Besides conventional curing of molded parts at room temperature, Autoclave curing may be used.



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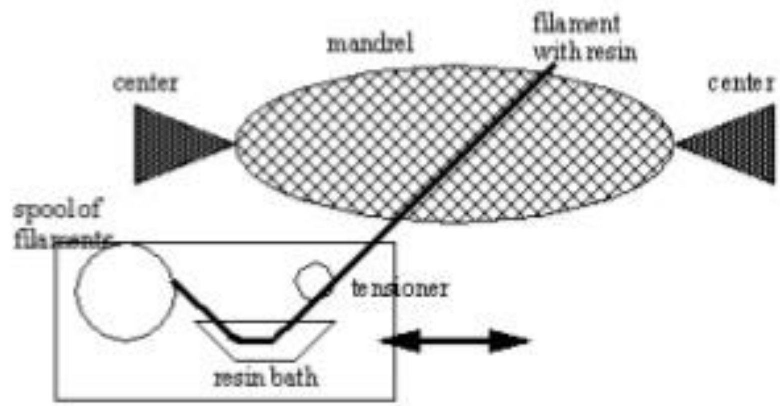
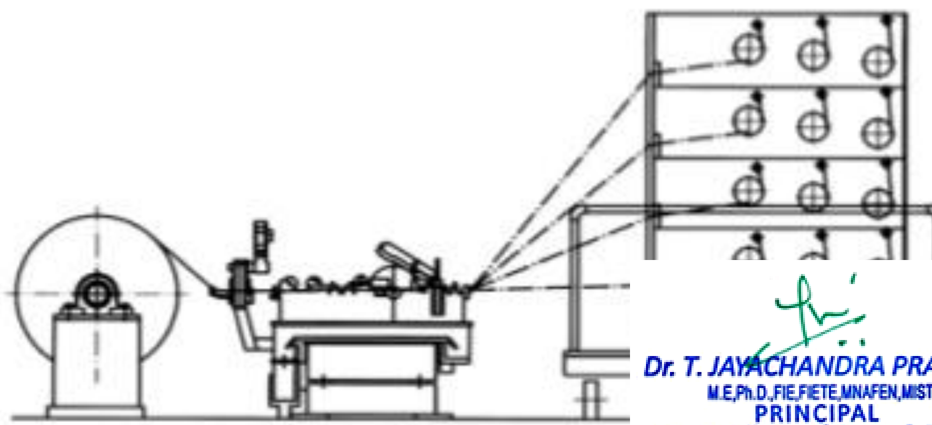
Filament Winding

- **Filament Winding Process**
  - For Round or Cylindrical parts
  - A tape of resin impregnated fibers is wrapped over a rotating mandrel to form a part.
  - These windings can be helical or hooped.
  - There are also processes that use dry fibres with resin application later, or prepregs are used.
  - Parts vary in size from 1" to 20'
  - Winding direction
    - Hoop/helical layers
    - Layers of different material
  - High strengths are possible due to winding designs in various direction
  - Winding speeds are typically 100 m/min and typical winding tensions are 0.1 to 0.5 kg.





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Filament Winding

- Demolding
  - To remove the mandrel, the ends of the parts are cut off when appropriate, or a collapsible mandrel (e.g., low melt temperature alloys) is used.
  - Curing in done in an Autoclave for thermoset resins (polyester, epoxy, phenolic, silicone) and some thermoplastics (PEEK)
  - Fibers are E-glass, S-glass, carbon fiber and aramids (toughness and lightweight) .
  - Inflatable mandrels can also be used to produce parts that are designed for high pressure applications, or parts that need a liner, and they can be easily removed.
- Advantages
  - Good for wide variety of part sizes
  - Parts can be made with strength in several different directions
  - Very low scrap rate
  - Non-cyclindrical parts can be formed after winding
  - Flexible mandrels can be left in as tank liners
  - Reinforcement panels, and fittings can be inserted during winding
  - Due to high hoop stress, parts with high pressure ratings can be made
- Disadvantages
  - Viscosity and pot life of resin must be carefully chosen
  - NC programming can be difficult
  - Some shapes can't be made with filament winding
  - Factors such as filament tension must be controlled

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Filament Winding

The filament winding process has the following advantages:

1. The process may be automated and provides high production rates.
2. Highest-strength products are obtained because of fiber placement control.
3. There is versatility of sizes.
4. Control of strength in different directions possible.

The following are limitations of filament winding:

1. Winding reverse curvatures is difficult.
2. Winding at low angles (parallel to rotational axis) is difficult.
3. Complex (double-curvature) shapes are difficult to obtain.
4. There is poor external surface.

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Filament winding - applications

- pressure vessels, storage tanks and pipes
- rocket motors, launch tubes
  - Light Anti-armour Weapon (LAW)
    - Hunting Engineering made a nesting pair in 4 minutes with ~20 mandrels circulated through the machine and a continuous curing oven.
- drive shafts
- Entec "the world's largest five-axis filament winding machine" for wind turbine blades
  - length 45.7 m, diameter 8.2 m, weight > 36 tonnes.

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Filament winding

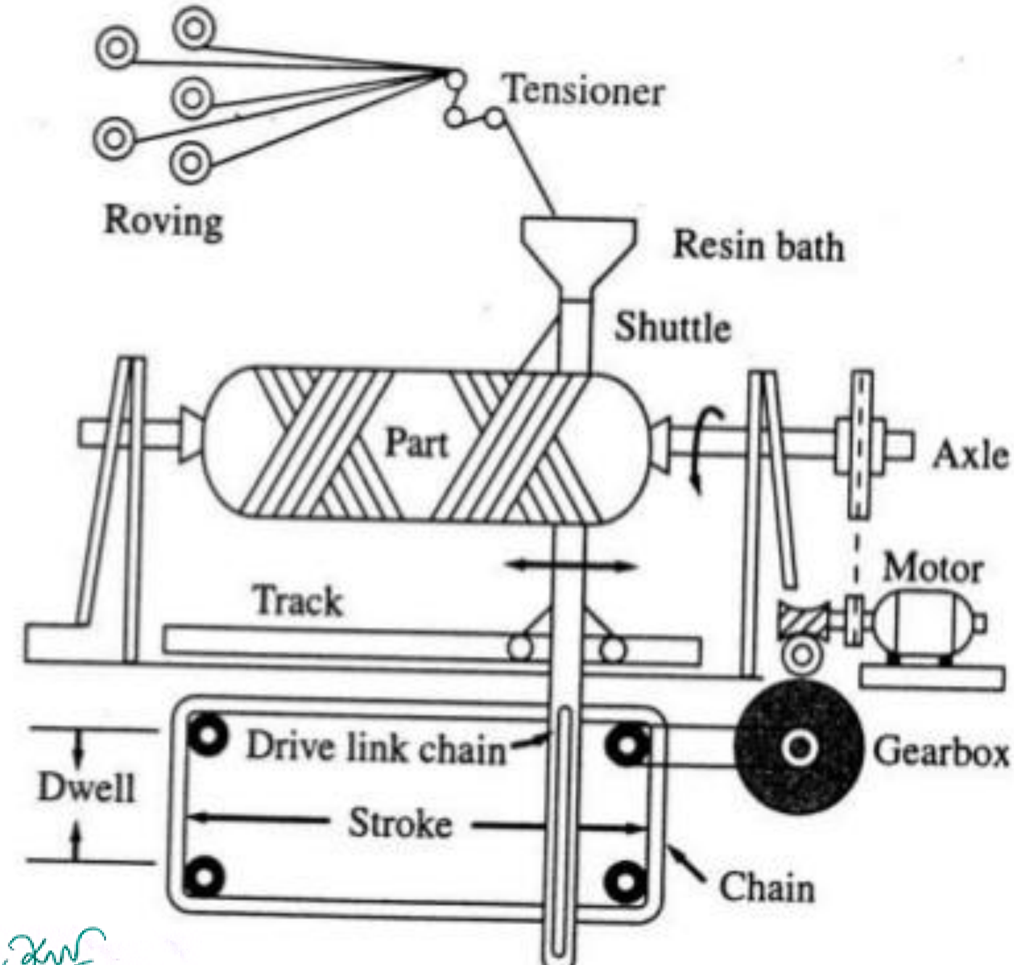

Figure 1.5 The wet process.

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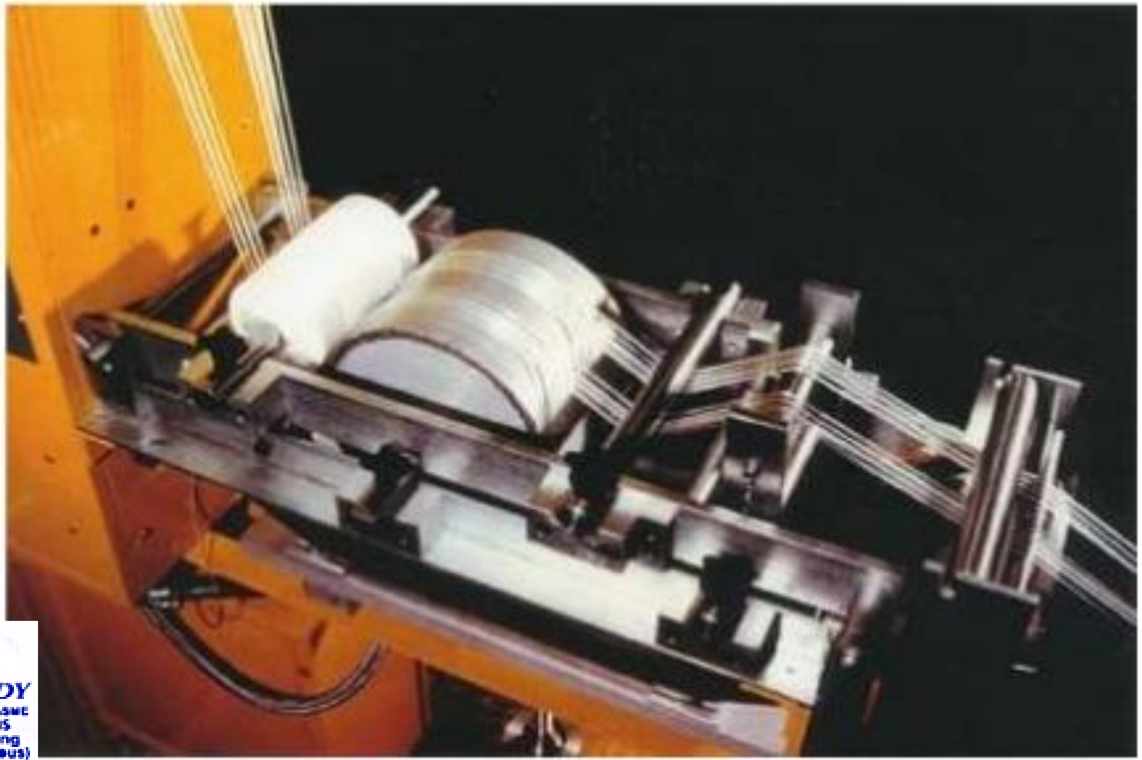
FILAMENT WINDING CHARACTERISTICS

- The cost is about half that of tape laying
- Productivity is high (50 kg/h).
- Applications include: fabrication of composite pipes, tanks, and pressure vessels. Carbon fiber reinforced rocket motor cases used for Space Shuttle and other rockets are made this way.



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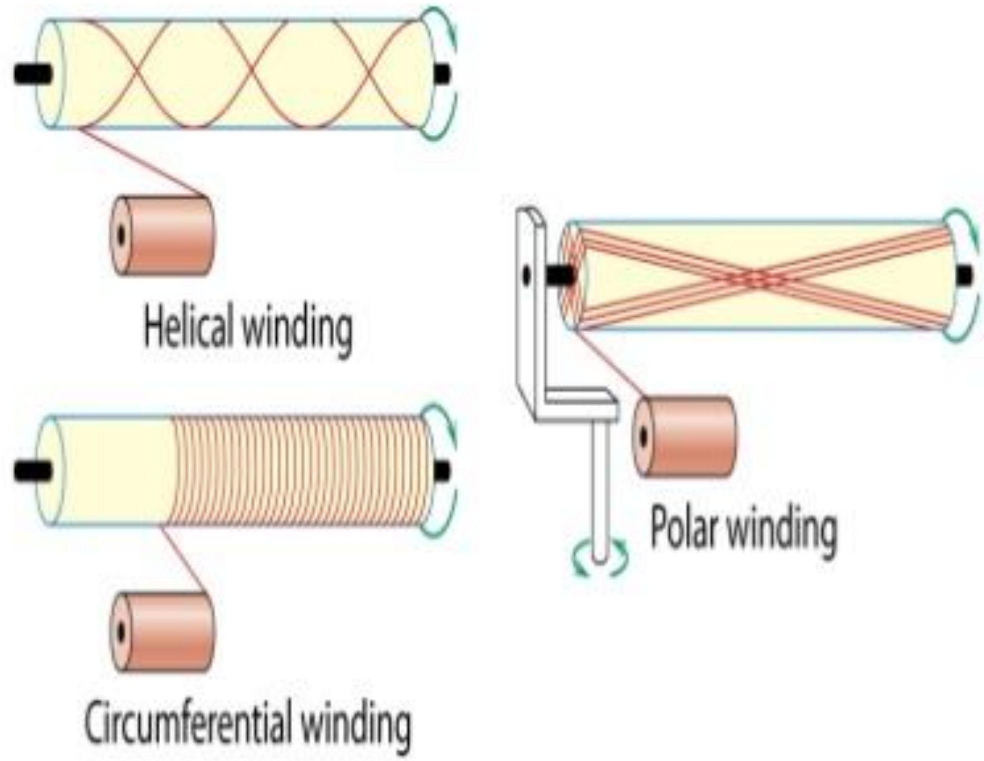
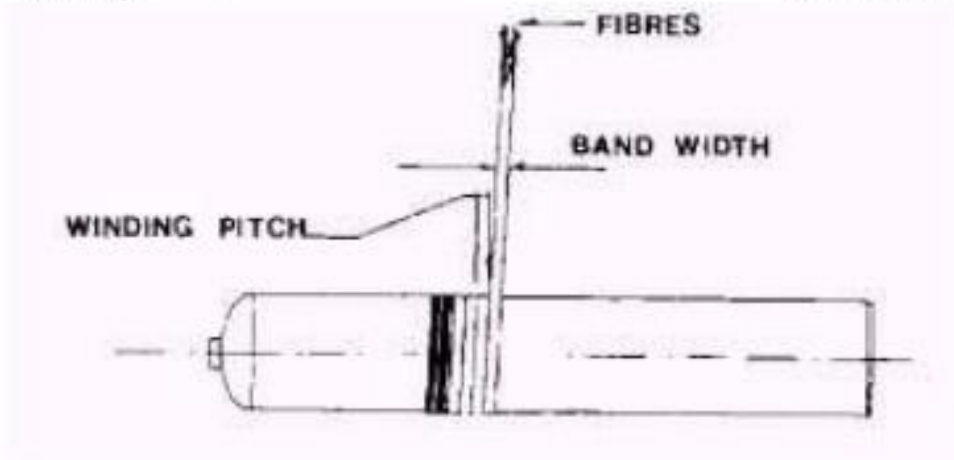
Filament winding - winding patterns

- **hoop** (90°) - girth or circumferential winding
  - angle is normally just below 90° degrees
  - each complete rotation of the mandrel shifts the fibre band to lie alongside the previous band.
- **helical**
  - complete fibre coverage without the band having to lie adjacent to that previously laid.
- **polar**
  - domed ends or spherical components
  - fibres constrained by bosses on each pole of the component.
- **axial** (0°)
  - **beware:** difficult to maintain nsion

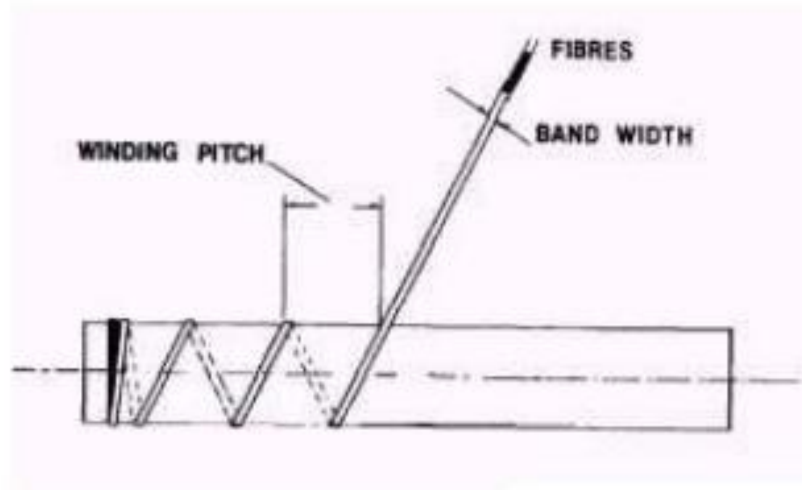


Filament winding patterns

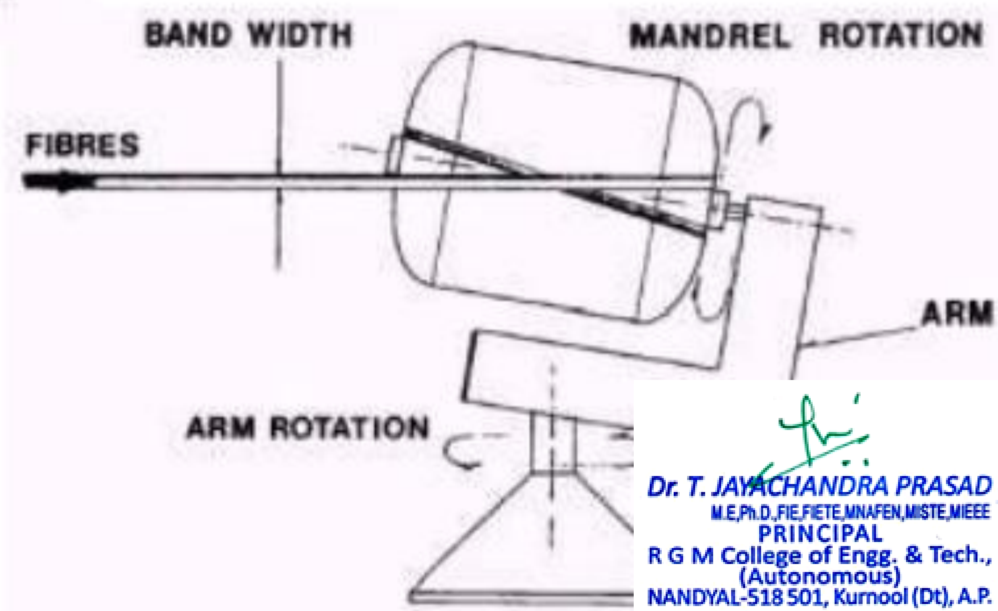
hoop :



helical:



polar:



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Applications of filament winding:

hollow and circular or oval sectioned components, such as pipes and tanks.

Pressure vessels, pipes and drive shafts.

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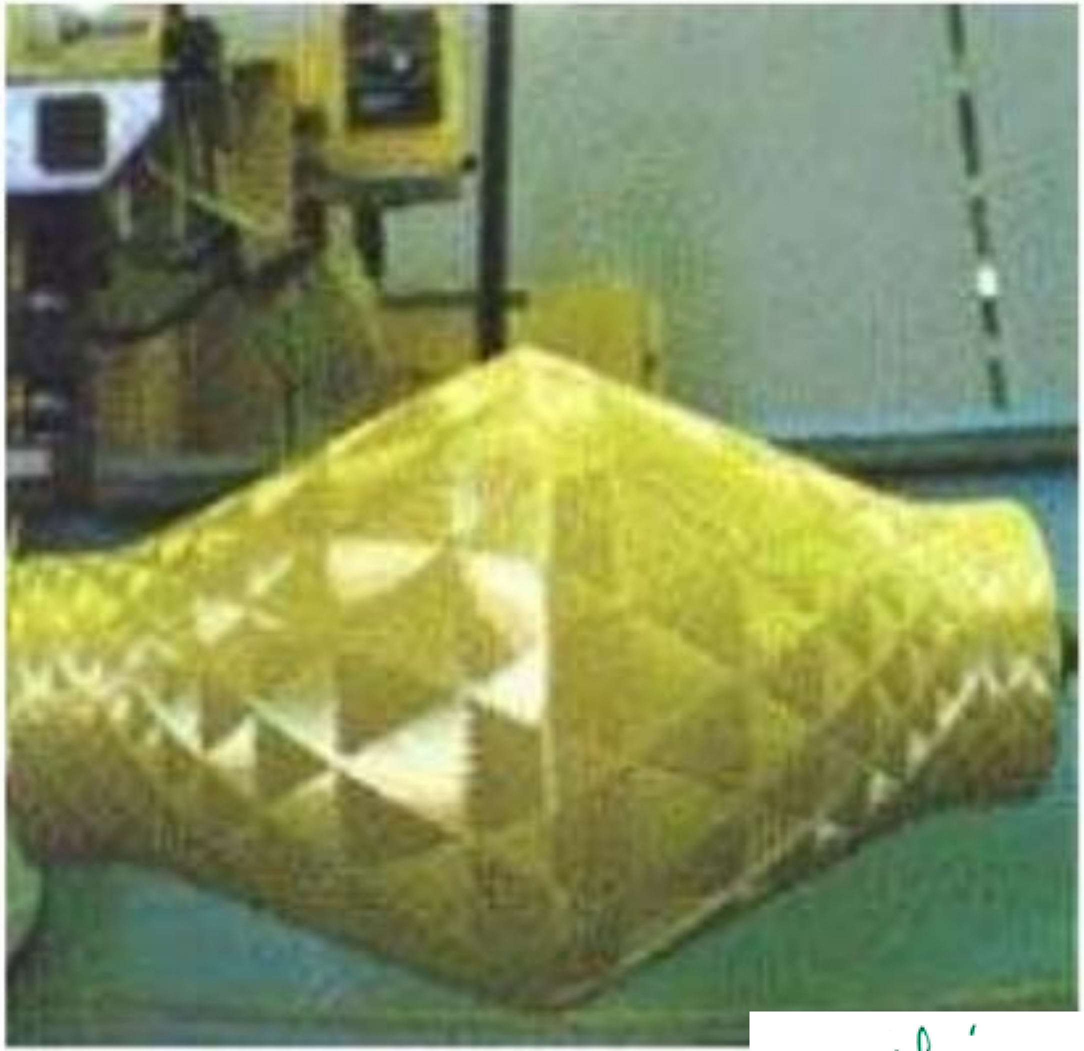
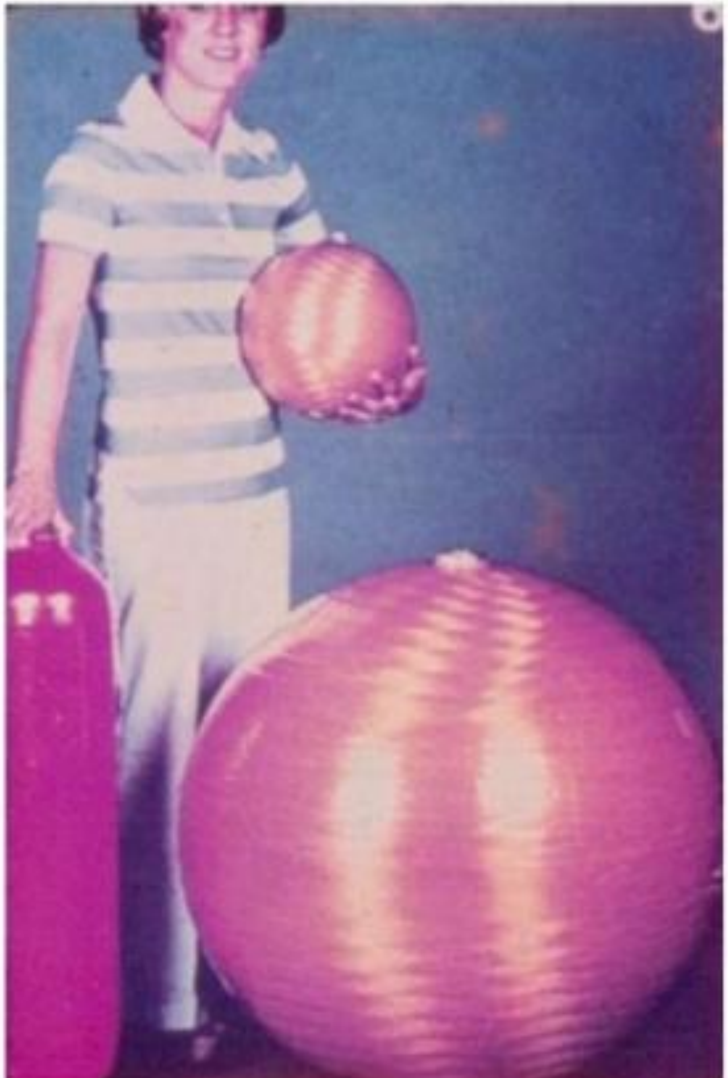
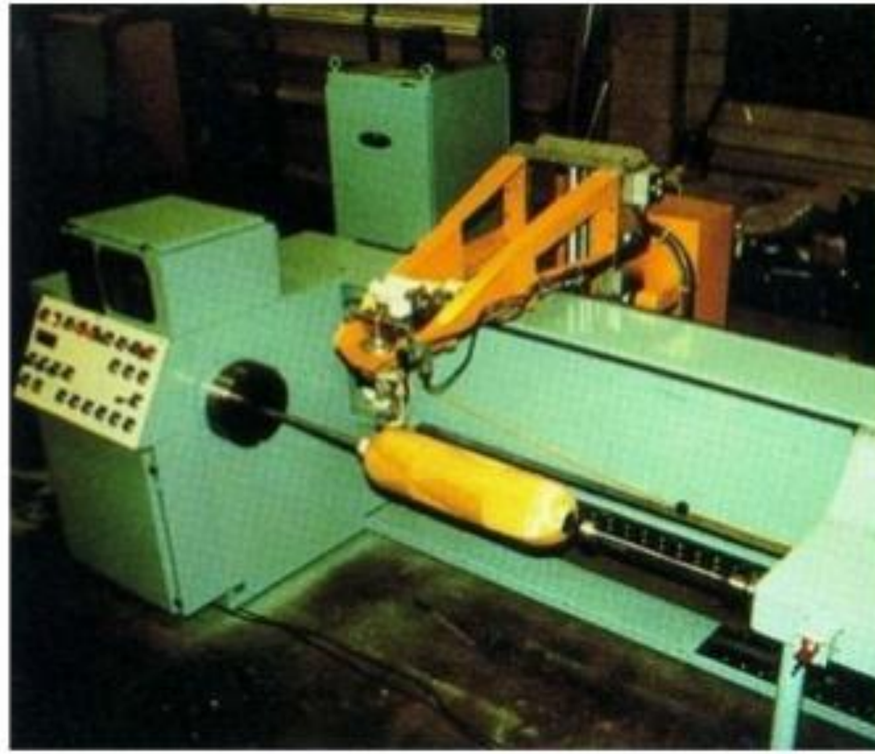
- Kevlar component



Filament wound pressure bottles for gas storage





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❑ Manufacturing Process of thermosetting polymers:

Pultrusion:

Pultrusion is a process where composite parts are manufactured by pulling layers of fibers/fabrics, bathed with resin, through a heated die, thus forming the desired cross-sectional shape with no length limitation.

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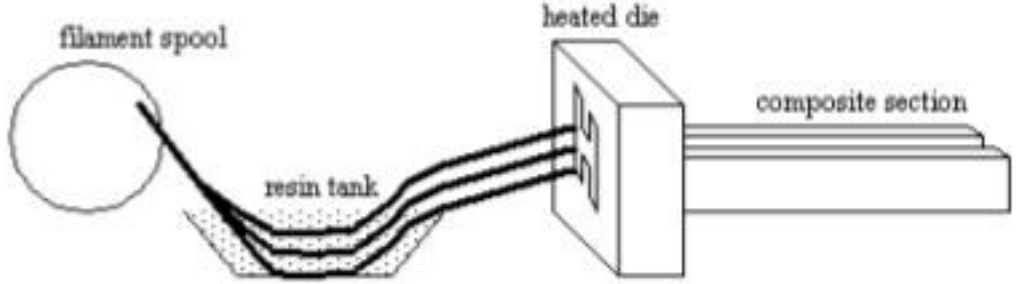
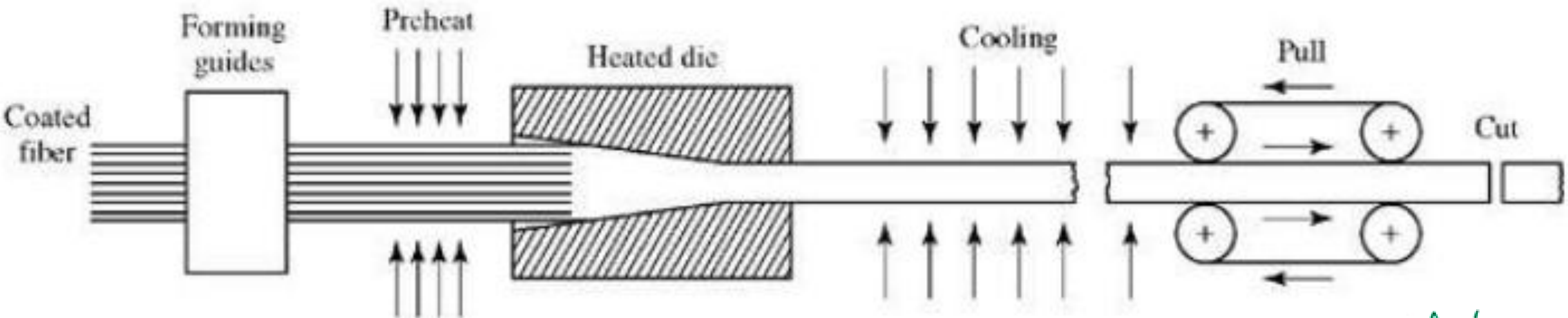
Pultrusion

- Manufacturing
  - Fibers are brought together over rollers, dipped in resin and drawn through a heated die. A continuous cross section composite part emerges on the other side.





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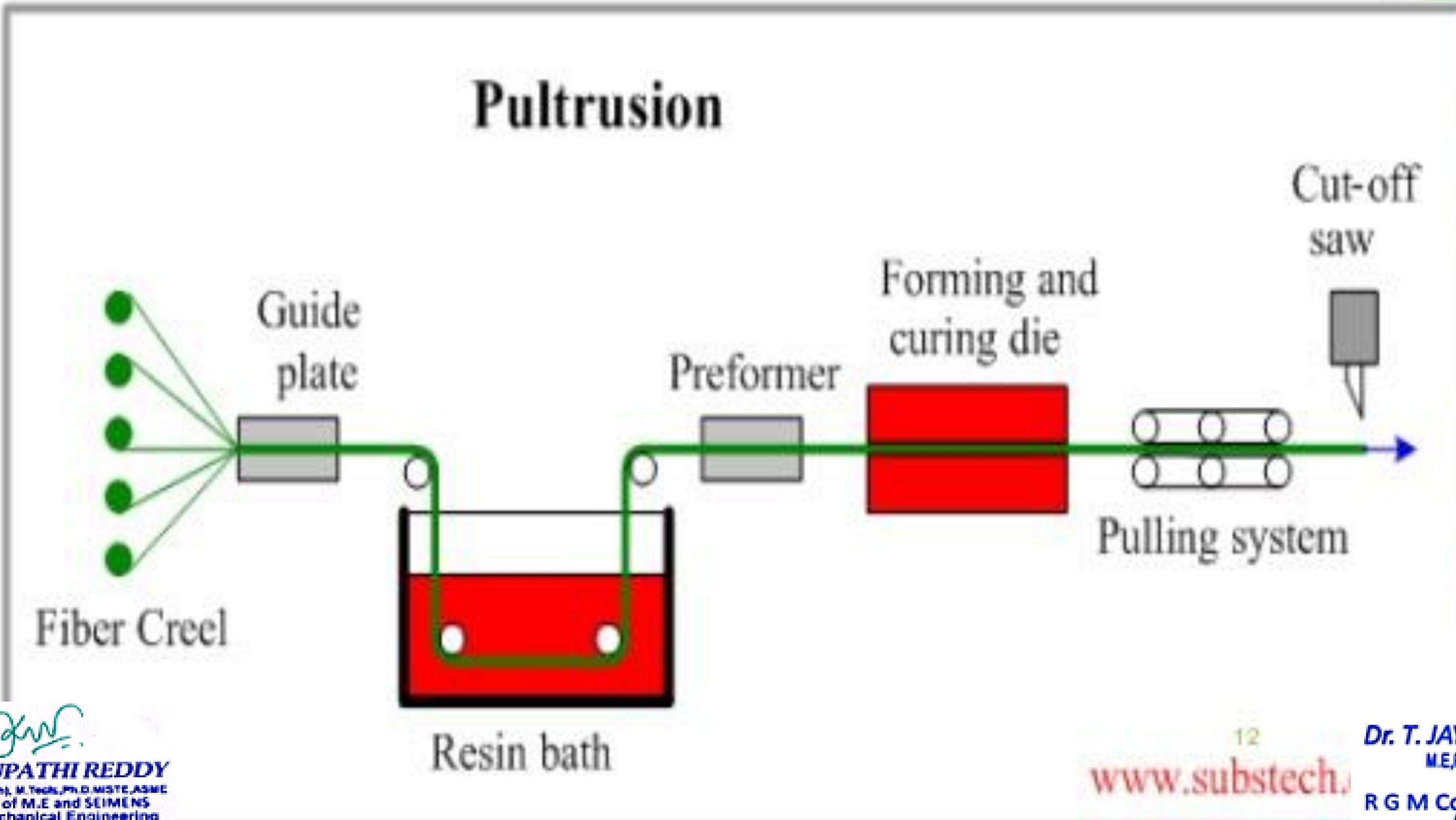
➢ Pultrusion process:

- Fibers are **pulled from** a set of fiber creels and through a **resin bath**. It then pass through a **performer** which gives it required cross sectional shape. The **F & C dies** finalize the required shape & **remove** excess resin & **cure** the **composite** so that it can be cut into required length.



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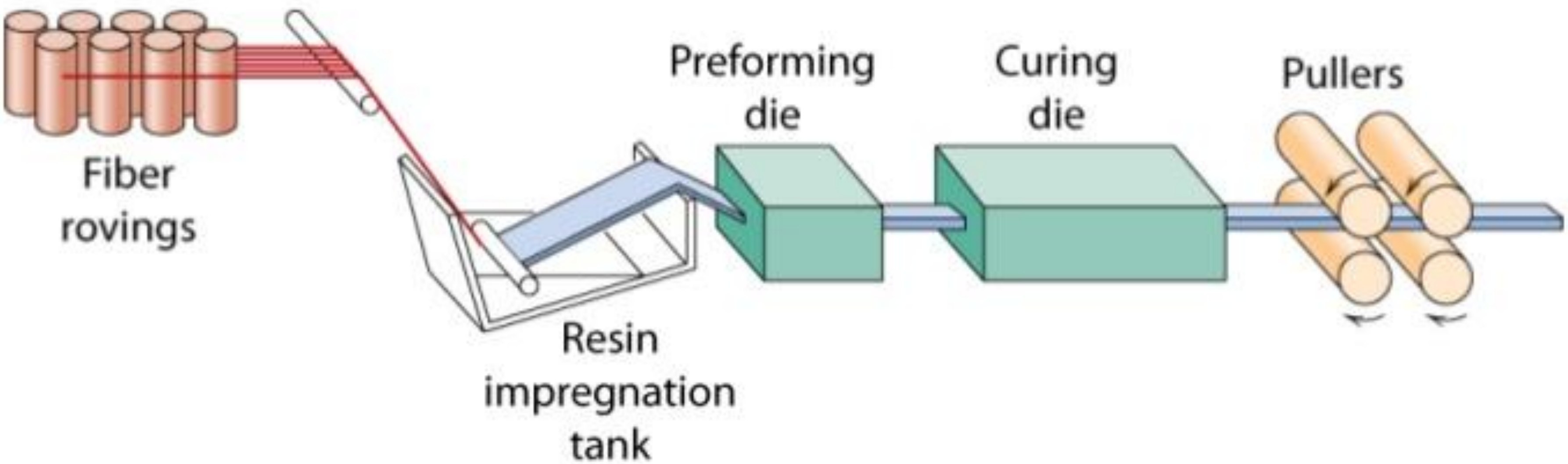
Pultrusion



- Design
  - Hollow parts can be made using a mandrel that extends out the exit side of the die.
  - Variable cross section parts are possible using dies with sliding parts.
  - Two main types of dies are used, fixed and floating. Fixed dies can generate large forces to wet fiber. Floating dies require an external power source to create the hydraulic forces in the res
    are used when curing is to be done by the heated di

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· Very low scrap. Up to 95% utilization of materials (75% for layup).

· Rollers are used to ensure proper resin impregnation of the fiber.

· Material forms can also be used at the inlet to the die when materials such as mats, weaves, or stitched material is used.

· For curing, tunnel ovens can be used. After the part is formed and gelled in the die, it emerges, enters a tunnel oven where curing is completed.

· Another method is, the process runs intermittently with sections emerging from the die, and the pull is stopped, split dies are brought up to the sections to cure it, they then retract, and the pull continues. (Typical lengths for curing are 6" to 24")

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- Materials
  - Most fibers are used (carbon, glass, aramids) and Resins must be fast curing because of process speeds. (polyester and epoxy)
- Processing
  - speeds are 0.6 to 1 m/min; thickness are 1 to 76 mm; diameters are 3 mm to 150mm
  - double clamps, or belts/chains can be used to pull the part through. The best designs allow for continuous operation for production.
  - diamond or carbide saws are used to cut sections of the final part. The saw is designed to track the part as it moves.
  - these parts have good axial properties.
- Advantages
  - good material usage compared to layup
  - high throughput and higher resin contents are possible
- Disadvantages
  - part cross section should be uniform.
  - Fiber and resin might accumulate at the die opening, leading to increased friction causing jamming, and breakage.
  - when excess resin is used, part strength will decrease
  - void can result if the die does not conform well to the fibers being pulled
  - quick curing systems decrease strength

Pultrusion -characteristics

- seek uniform thickness in order to achieve uniform cooling and hence minimise residual stress.
- hollow profiles require a cantilevered mandrel to enter the die from the fibre-feed end.
- continuous constant cross-section profile
- normally thermoset (thermoplastic possible)
  - impregnate with resin
  - pull through a heated die
    - resin shrinkage reduces friction in the die
    - polyester easier to process than epoxy
- tension control as in filament winding
- post-die, profile air-cooled before gripped
  - hand-over-hand hydraulic clamps
  - conveyor belt/caterpillar track systems.
- moving cut-off machine ("flying cutter"). The solid laminate will be cut to the desired length
- Inside the metal die, precise temperature control activates the curi... t resin.

- Shapes such as rods, channels, angle and flat stocks can be easily produced.
- Production rate is 10 to 200 cm/min.
- Profiles as wide as 1.25 m with more than 60% fiber volume fraction can be made routinely.
- No bends or tapers allowed (continuous molding cycle)

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➢ Pultrusion process:

Advantages:

- High volume productivity
- Rapid processing
- Low material scrap rate
- Good quality control

Potential Problems:

- Improper fiber wet-out
- Fiber breakage
- Die jamming
- Complex die design

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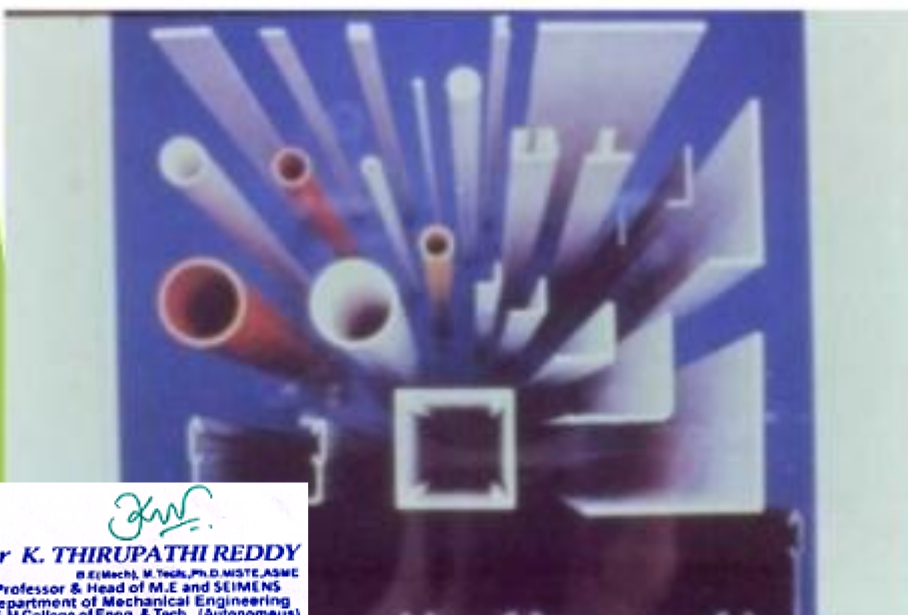
➢ Pultrusion process:

Applications

- Uses as Panels, Beams, Ladders
- Tool Handles,
- Electrical Insulators,
- Light poles, Hand rails, Roll-up doors etc



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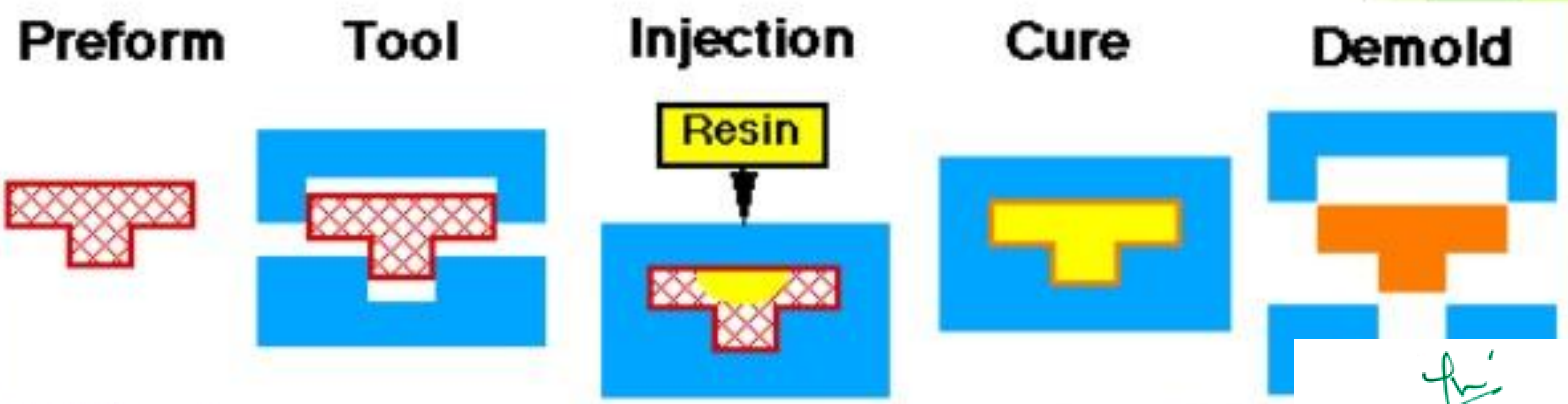
➢ Resin Transfer Molding

- In the RTM process, dry reinforcement is pre-shaped and oriented into skeleton of the actual part known as the **preform** which is **inserted into** a heated **matched die mold** .
- The **heated mold** is **closed** and the liquid resin is **injected**.
- The part is **cured in mold**.
- Finally mold is opened and part is removed from mold.



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➢ Resin Transfer Molding

Advantages

- Large complex shapes and curvatures can be made easily.
- High level of automation.
- Simpler than in manual operations.
- Takes less time to produce.
- Low volatile emission
- Cost effective.
- Low skill labor required

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➢ Resin Transfer Molding

Disadvantages

- Mold design is **complex**.
- Requires Mold-filling **Analysis**.
- Fiber reinforcement may **Move** during resin transfer.

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➢ Resin Transfer Molding

Application:

- Wind Turbine blade.
- Ship body.
- Car body.
- Truck panel.

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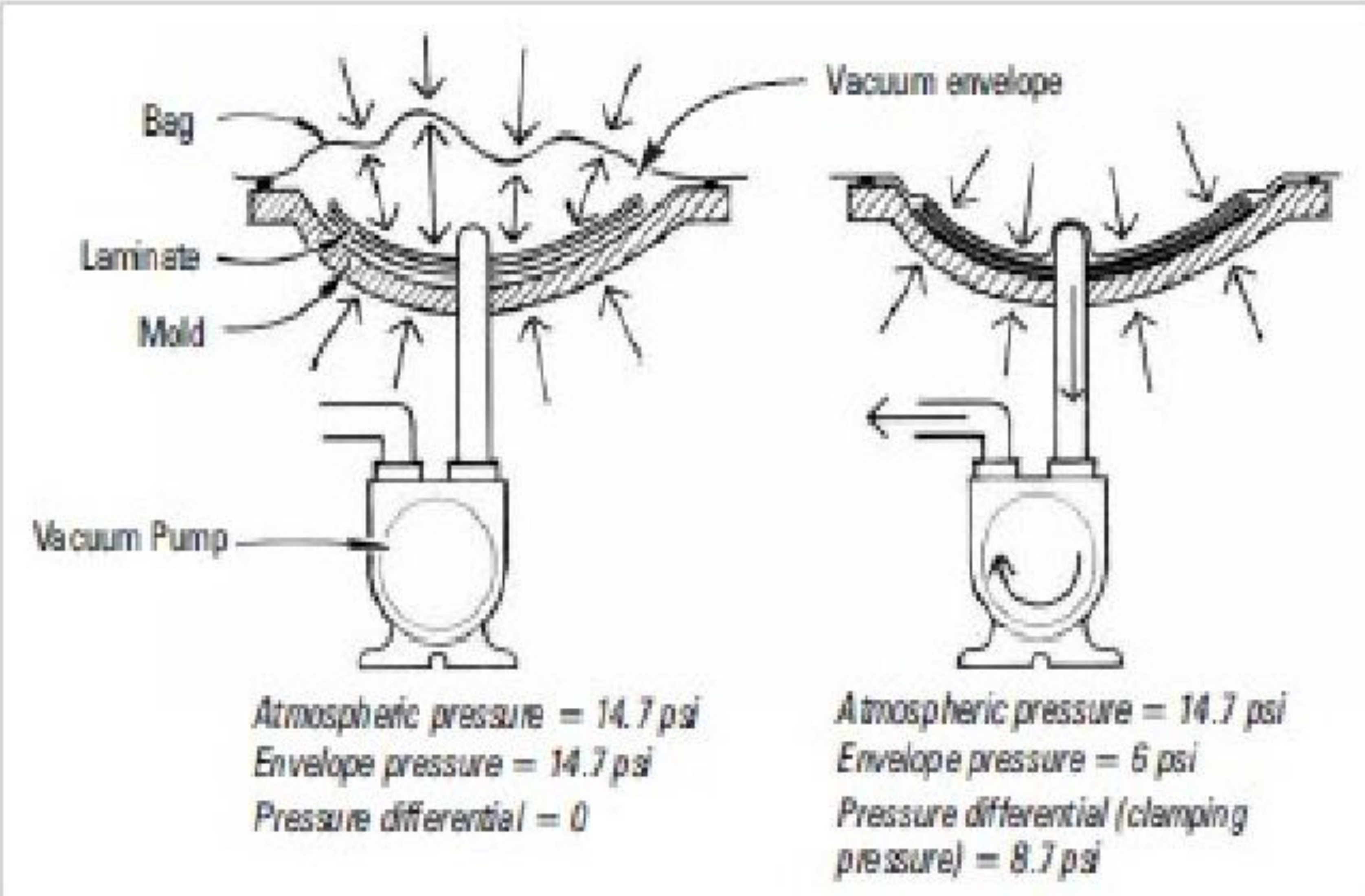

Figure 1-1 *A typical vacuum bagging lay-up before and after vacuum is app*

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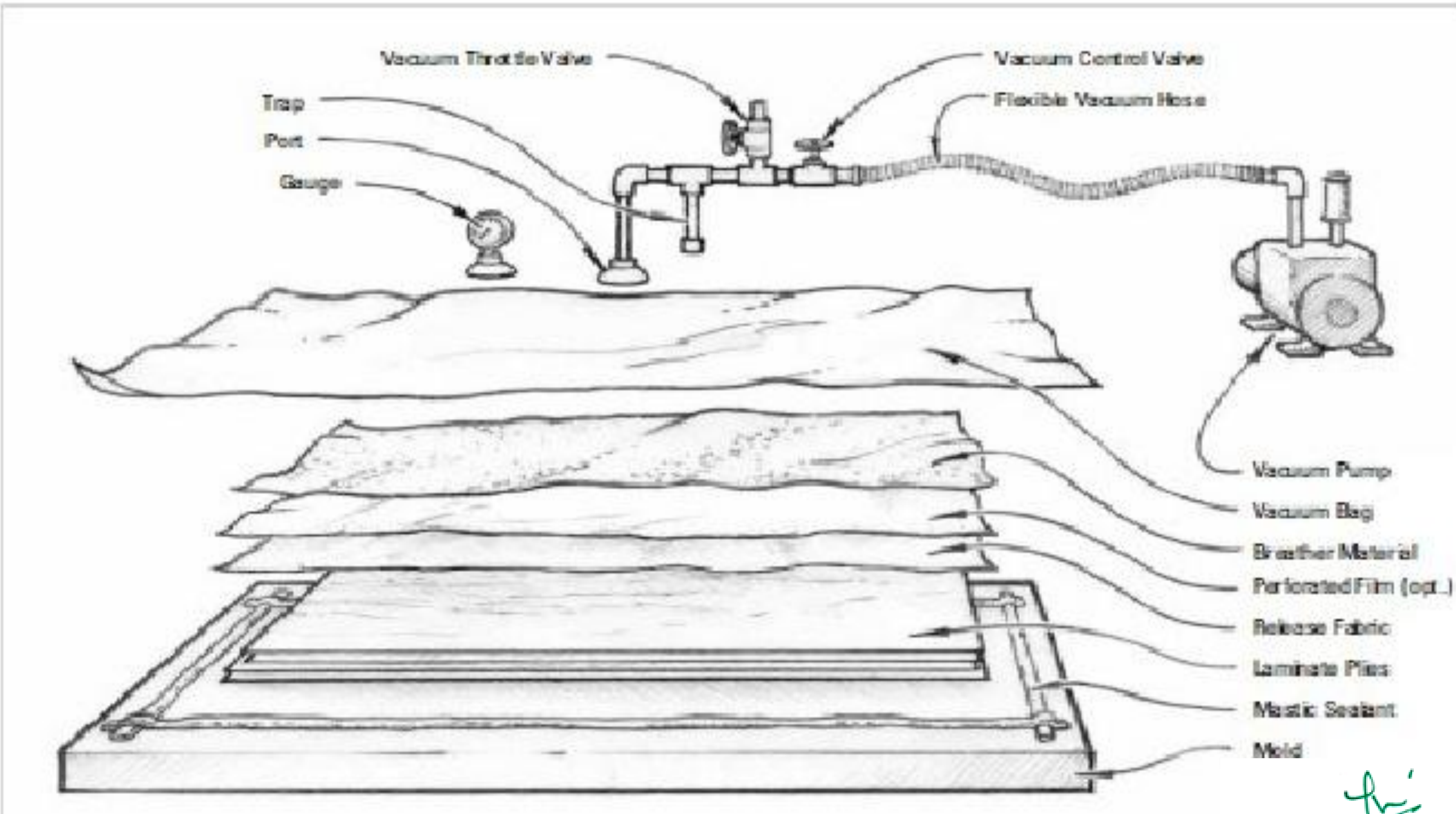
Vacuum bagging -Equipment



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Vacuum Bagging - Materials

- *Release fabric*
- *Perforated film*
- *Breather Material*
- *Vacuum bag*
- *Mastic sealant*
- *Plumbing system*
- *Mold release*

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Release fabric:

- *Smooth woven fabric – not bond to epoxy.*
- *Used to separate breather and laminate.*
- *Excess epoxy can wick through release fabric.*

Perforated film:

- *Used in conjunction with release fabric.*
- *This film helps to hold the resin in laminate, when high vacuum pressure is used with slow curing resin system (or) thin laminates.*

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Breather material:

- *(or) bleeder cloth.*
- *Allows air from all parts of the envelope – to be drawn to a port (or) manifold by providing slight air space between the bag and laminate.*

Vacuum bag:

- *If vacuum pressure less than 5psi(10 hg) at room temperature – 6mil polyethylene plastic can be used.*
- *Clear plastic material is preferable as compared to opaque - for easy inspection*

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- *For high pressure and temperature applications – specially manufactured vacuum bags can be used.*
- *Vacuum bag should always larger than mould.*

Mastic sealant:

- *Provide a continuous air tight sealant between bag and mold.*
- *Also used to seal the point where the manifold enters the bag and to repair leaks in the bag.*

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Plumbing system:

- *Provides an airtight passage from vacuum envelope to vacuum pump – allowing pump to remove air.*
- *A basic system consists of flexible (or) rigid hose pipe, a trap, a port that connects pipe to the envelope.*
- ***Vacuum hose** designed specially for this.*

- ***Vacuum port** connects the exhaust tubing to vacuum bag.*
- ***Control valve** – control of airflow at the envelope.*

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- ***Trap*** *incorporated into the line as close as possible to the envelope.*
- ***Vacuum gauge*** *– is necessary to monitor the level.*

  The reading of negative pressure inside the bag is equal to the net pressure of the atmospheric pressing on the outside of the bag.

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Vacuum Bagging - Advantages

- *Even Clamping pressure*
- *Control of resin content*
- *Custom shapes*
- *Efficient laminating*

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Types of processing discussed at a glance

Forming process of Thermosetting matrix composites

- **Pultrusion process**
  - **Application**
    - Motor vehicles
    - Spots goods such as fishing rods, ski-poles . Boats and marine
- **Resin transfer molding**
  - **Application**
    - Truck panels, Wind turbine blades
    - Medical composites
    - Bathroom fixtures, Car body , helmet, etc

Forming process of thermoplastic matrix composites

- **Injection molding**
  - **Application**
    - Quality plastic molding at the lowest possible price
    - Electronic product , kitchen product, water cooler parts, plastic handle
- **Diaphragm forming**
  - **Application**
    - Engine cover,
    - Double curvature component

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Thank You

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UNIT-IV

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2

Macromechanical Analysis of a Lamina

Chapter Objectives

- Review definitions of stress, strain, elastic moduli, and strain energy.
- Develop stress–strain relationships for different types of materials.
- Develop stress–strain relationships for a unidirectional/bidirectional lamina.
- Find the engineering constants of a unidirectional/bidirectional lamina in terms of the stiffness and compliance parameters of the lamina.
- Develop stress–strain relationships, elastic moduli, strengths, and thermal and moisture expansion coefficients of an angle ply based on those of a unidirectional/bidirectional lamina and the angle of the ply.

2.1 Introduction

A lamina is a thin layer of a composite material that is generally of a thickness on the order of 0.005 in. (0.125 mm). A laminate is constructed by stacking a number of such laminae in the direction of the lamina thickness (Figure 2.1). Mechanical structures made of these laminates, such as a leaf spring suspension system in an automobile, are subjected to various loads, such as bending and twisting. The design and analysis of such laminated structures demands knowledge of the stresses and strains in the laminate. Also, design tools, such as failure theories, stiffness models, and optimization algorithms, need the values of these laminate stresses and strains.

However, the building blocks of a laminate are single lamina, so understanding the mechanical analysis of a lamina precedes understanding that of a laminate. A lamina is unlike an isotropic homog
example, if the lamina is made of isotropic homoge

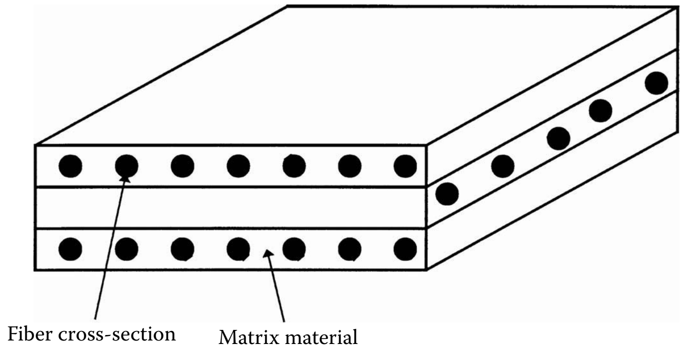

FIGURE 2.1
Typical laminate made of three laminae.

isotropic homogeneous matrix, the stiffness of the lamina varies from point to point depending on whether the point is in the fiber, the matrix, or the fiber–matrix interface. Accounting for these variations will make any kind of mechanical modeling of the lamina very complicated. For this reason, the macromechanical analysis of a lamina is based on average properties and considering the lamina to be homogeneous. Methods to find these average properties based on the individual mechanical properties of the fiber and the matrix, as well as the content, packing geometry, and shape of fibers are discussed in Chapter 3.

Even with the homogenization of a lamina, the mechanical behavior is still different from that of a homogeneous isotropic material. For example, take a square plate of length and width w and thickness t out of a large isotropic plate of thickness t (Figure 2.2) and conduct the following experiments.

Case A: Subject the square plate to a pure normal load P in direction 1. Measure the normal deformations in directions 1 and 2, δ_{1A} and δ_{2A}, respectively.

Case B: Apply the same pure normal load P as in case A, but now in direction 2. Measure the normal deformations in directions 1 and 2, δ_{1B} and δ_{2B}, respectively.

Note that

$$\delta_{1A} = \delta_{2B},$$
$$\delta_{2A} = \delta_{1B}. \tag{2.1a,b}$$

However, taking a unidirectional square plate (Fig
dimensions $w \times w \times t$ out of a large composite lamir
me case A and B experiments, note t

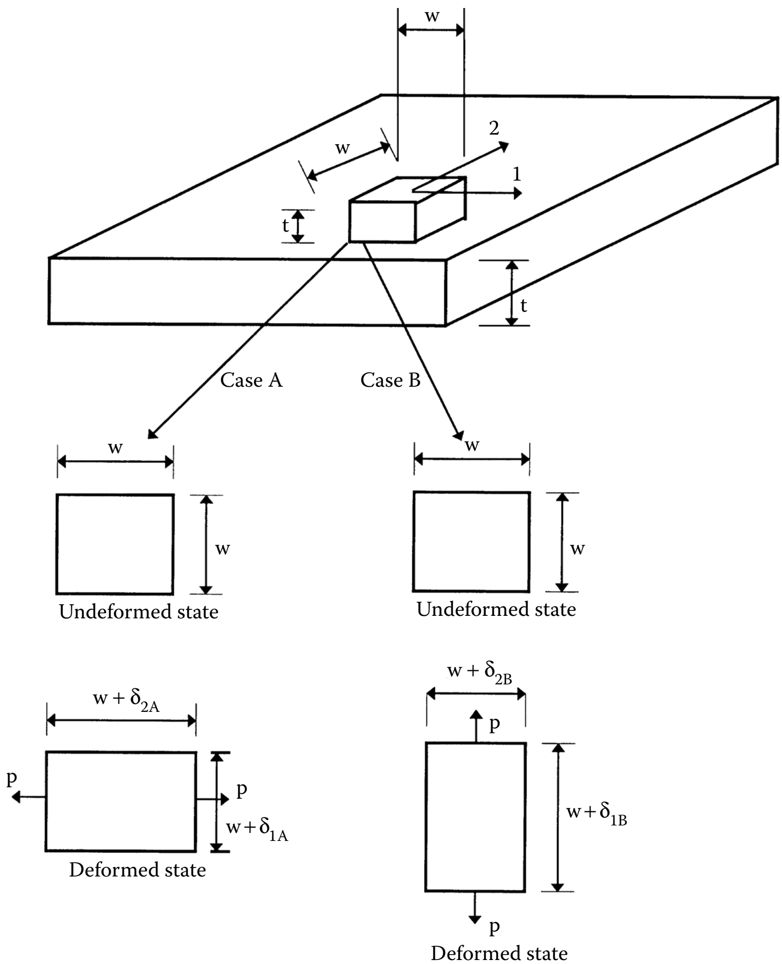

FIGURE 2.2
Deformation of square plate taken from an isotropic plate under normal loads.

$$\delta_{1A} \neq \delta_{2B} ,$$
$$\delta_{2A} \neq \delta_{1B} . \quad (2.2a,b)$$

because the stiffness of the unidirectional lamina in the direction of fibers is much larger than the stiffness in the direction perpendicular to the fibers. Thus, the mechanical characterization of a unidirectional lamina will require more parameters than it will for an isotropic lamina.

Also, note that if the square plate (Figure 2.4) taken … to the sides of the square plate, the … rent angles. In fact, the square plate …

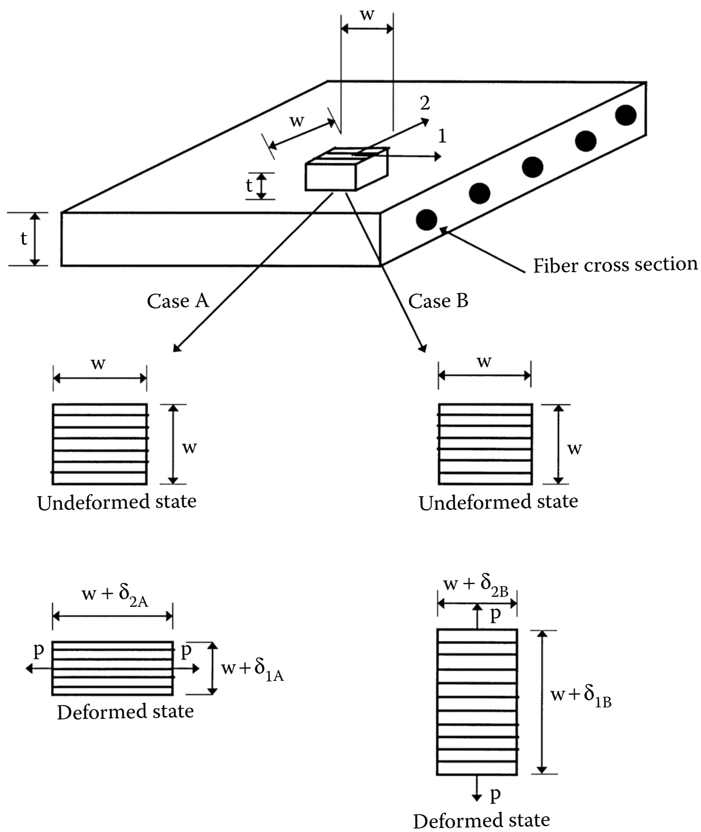

FIGURE 2.3
Deformation of a square plate taken from a unidirectional lamina with fibers at zero angle under normal loads.

deformations in the normal directions but would also distort. This suggests that the mechanical characterization of an angle lamina is further complicated.

Mechanical characterization of materials generally requires costly and time-consuming experimentation and/or theoretical modeling. Therefore, the goal is to find the minimum number of parameters required for the mechanical characterization of a lamina.

Also, a composite laminate may be subjected to a temperature change and may absorb moisture during processing and operation. These changes in temperature and moisture result in residual stresses and strains in the laminate. The calculation of these stresses and strains in a laminate depends on the response of each lamina to these two environment... chapter, the stress–strain relationships based on tem... moisture content will also be developed for a single l... moisture on a laminate are discussed

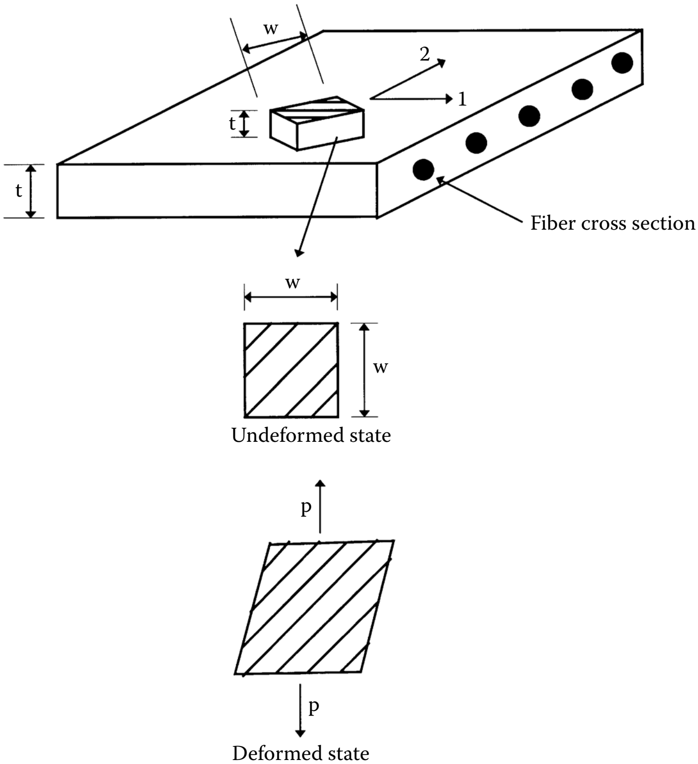

FIGURE 2.4
Deformation of a square plate taken from a unidirectional lamina with fibers at an angle under normal loads.

2.2 Review of Definitions

2.2.1 Stress

A mechanical structure takes external forces, which act upon a body as surface forces (for example, bending a stick) and body forces (for example, the weight of a standing vertical telephone pole on itself). These forces result in internal forces inside the body. Knowledge of the internal forces at all points in the body is essential because these forces need to be less than the strength of the material used in the structure. Stress, which is defined as the intensity of the load per unit area, determines this knowledge because the strengths of a material are intrinsically known in term

Imagine a body (Figure 2.5) in equilibrium under vari

ection, forces will need to be applied

intains equilibrium as in the origina

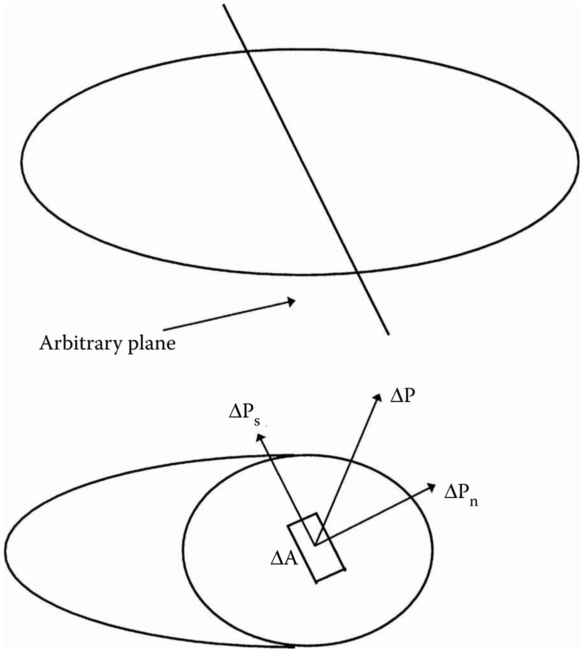

FIGURE 2.5
Stresses on an infinitesimal area on an arbitrary plane.

section, a force ΔP is acting on an area of ΔA. This force vector has a component normal to the surface, ΔP_n, and one parallel to the surface, ΔP_s. The definition of stress then gives

$$\sigma_n = \lim_{\Delta A \to 0} \frac{\Delta P_n}{\Delta A},$$

$$\tau_s = \lim_{\Delta A \to 0} \frac{\Delta P_s}{\Delta A}. \qquad (2.3a,b)$$

The component of the stress normal to the surface, σ_n, is called the normal stress and the stress parallel to the surface, τ_s, is called the shear stress. If one takes a different cross-section through the same point, the stress remains unchanged but the two components of stress, normal stress, σ_n, and shear stress, τ_s, will change. However, it has been proved that a complete definition of stress at a point only needs use of any three mutuall
nate systems, such as a Cartesian coordinate system.

and coordinate system x–y–z. Take a
the body as shown in Figure 2.6. Th

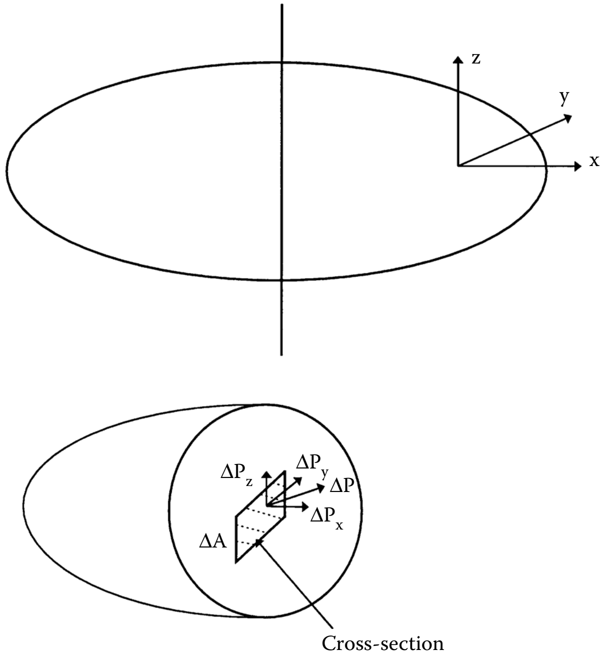

FIGURE 2.6
Forces on an infinitesimal area on the y–z plane.

on an area ΔA. The component ΔP_x is normal to the surface. The force vector ΔP_s is parallel to the surface and can be further resolved into components along the y and z axes: ΔP_y and ΔP_z. The definition of the various stresses then is

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta P_x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta P_y}{\Delta A},$$

$$\tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta P_z}{\Delta A}. \qquad (2.4a–c)$$

Similarly, stresses can be defined for cross-sections p
xz planes. For defining all these stresses, the stress a
g an infinitesimal cuboid in a right-ha

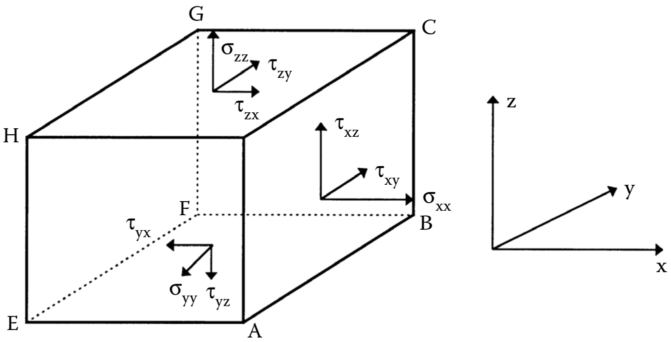

FIGURE 2.7
Stresses on an infinitesimal cuboid.

and finding the stresses on each of its faces. Nine different stresses act at a point in the body as shown in Figure 2.7. The six shear stresses are related as

$$\tau_{xy} = \tau_{yx} \, ,$$

$$\tau_{yz} = \tau_{zy} \, ,$$

$$\tau_{zx} = \tau_{xz} \, . \tag{2.5a–c}$$

The preceding three relations are found by equilibrium of moments of the infinitesimal cube. There are thus six independent stresses. The stresses σ_x, σ_y, and σ_z are normal to the surfaces of the cuboid and the stresses τ_{yz}, τ_{zx}, and τ_{xy} are along the surfaces of the cuboid.

A tensile normal stress is positive, and a compressive normal stress is negative. A shear stress is positive, if its direction and the direction of the normal to the face on which it is acting are both in positive or negative direction; otherwise, the shear stress is negative.

2.2.2 Strain

Similar to the need for knowledge of forces inside a body, knowing the deformations because of the external forces is also important. For example, a piston in an internal combustion engine may not develop larger stresses than the failure strengths, but its excessive deformation [illegible]
Also, finding stresses in a body generally requires findir[illegible]
is because a stress state at a point has six component[illegible]
[illegible]orium equations (one in each directio[illegible]

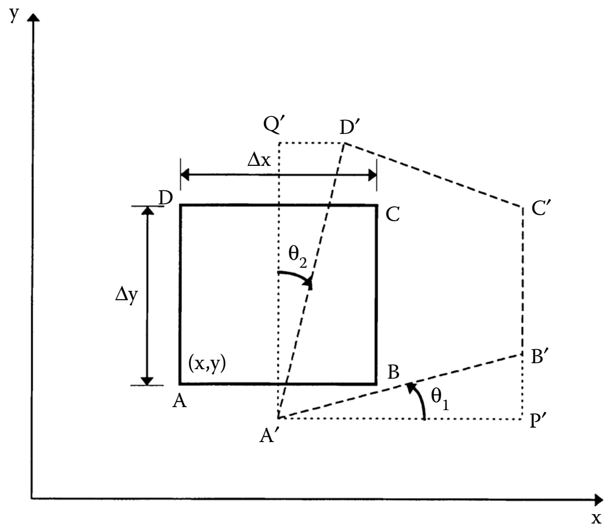

FIGURE 2.8
Normal and shearing strains on an infinitesimal area in the *x*–*y* plane.

The knowledge of deformations is specified in terms of strains — that is, the relative change in the size and shape of the body. The strain at a point is also defined generally on an infinitesimal cuboid in a right-hand coordinate system. Under loads, the lengths of the sides of the infinitesimal cuboid change. The faces of the cube also get distorted. The change in length corresponds to a normal strain and the distortion corresponds to the shearing strain. Figure 2.8 shows the strains on one of the faces, *ABCD*, of the cuboid.

The strains and displacements are related to each other. Take the two perpendicular lines *AB* and *AD*. When the body is loaded, the two lines become *A′B′* and *A′D′*. Define the displacements of a point (x,y,z) as

$u = u(x,y,z)$ = displacement in x-direction at point (x,y,z)

$v = v(x,y,z)$ = displacement in y-direction at point (x,y,z)

$w = w(x,y,z)$ = displacement in z-direction at point (x,y,z)

The normal strain in the x-direction, ε_x, is defined as the change of length of line *AB* per unit length of *AB* as

$$\varepsilon_x = \lim_{AB \to 0} \frac{A'B' - AB}{AB}, \quad (2.6)$$

$$A'B' = \sqrt{(A'P')^2 + (B'P')^2},$$

$$= \sqrt{[\Delta x + u(x+\Delta x, y) - u(x,y)]^2 + [v(x+\Delta x, y) - v(x,y)]^2},$$

$$AB = \Delta x. \tag{2.7a,b}$$

Substituting the preceding expressions of Equation (2.7) in Equation (2.6),

$$\varepsilon_x = \lim_{\Delta x \to 0} \left\{ \left[1 + \frac{u(x+\Delta x, y) - u(x,y)}{\Delta x} \right]^2 + \left[\frac{v(x+\Delta x, y) - v(x,y)}{\Delta x} \right]^2 \right\}^{1/2} - 1 .$$

Using definitions of partial derivatives

$$\varepsilon_x = \left[\left(1 + \frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 \right]^{1/2} - 1$$

$$\varepsilon_x = \frac{\partial u}{\partial x}, \tag{2.8}$$

because

$$\frac{\partial u}{\partial x} << 1 ,$$

$$\frac{\partial v}{\partial x} << 1 ,$$

for small displacements.

The normal strain in the y-direction, ε_y is defined as the change in the length of line AD per unit length of AD as

$$\varepsilon_y = \lim_{AD \to 0} \frac{A'D' - AD}{AD} , \tag{2.9}$$

where

$$A'D' = \sqrt{(A'Q')^2 + (Q'D')^2},$$

$$A'D' = \sqrt{[\Delta y + v(x, y+\Delta y) - v(x,y)]^2 + [u(x, y+\Delta y) - u(x,y)]^2},$$

$$AD = \Delta y. \tag{2.10a,b}$$

Substituting the preceding expressions of Equation (2.10) in Equation (2.9),

$$\varepsilon_y = \lim_{\Delta y \to 0} \left\{ \left[1 + \frac{v(x, y+\Delta y) - v(x,y)\}}{\Delta y} \right]^2 + \left[\frac{u(x, y+\Delta y) - u(x,y)}{\Delta y} \right]^2 \right\}^{1/2} - 1 .$$

Using definitions of partial derivatives,

$$\varepsilon_y = \left[\left(1 + \frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 \right]^{1/2} - 1$$

$$\varepsilon_y = \frac{\partial v}{\partial y}, \tag{2.11}$$

because

$$\frac{\partial u}{\partial y} << 1,$$

$$\frac{\partial v}{\partial y} << 1,$$

for small displacements.

A normal strain is positive if the corresponding length increases; a normal strain is negative if the corresponding length decreases.

The shearing strain in the *x–y* plane, γ_{xy} is defined as the change in the angle between sides *AB* and *AD* from 90°. This angular change takes place by the inclining of sides *AB* and *AD*. The shearing strain is thus defined as

$$\gamma_{xy} = \theta_1 + \theta_2,$$

where

$$q_1 = \lim_{AB \to 0} \frac{P'B'}{A'P'},$$

$$P'B' = v(x + \Delta x, y) - v(x, y),$$

$$A'P' = u(x + \Delta x, y) + \Delta x - u(x, y), \qquad (2.13a–c)$$

$$\theta_2 = \lim_{AD \to 0} \frac{Q'D'}{A'Q'},$$

$$Q'D' = u(x, y + \Delta y) - u(x, y),$$

$$A'Q' = v(x, y + \Delta y) + \Delta y - v(x, y). \qquad (2.14a–c)$$

Substituting Equation (2.13) and Equation (2.14) in Equation (2.12),

$$\gamma_{xy} = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \frac{\dfrac{v(x+\Delta x, y) - v(x, y)}{\Delta x}}{\dfrac{u(x+\Delta x, y) + \Delta x - u(x, y)}{\Delta x}} + \frac{\dfrac{u(x, y+\Delta y) - u(x, y)}{\Delta y}}{\dfrac{v(x, y+\Delta y) + \Delta y - v(x, y)}{\Delta y}}$$

$$= \frac{\dfrac{\partial v}{\partial x}}{1 + \dfrac{\partial u}{\partial x}} + \frac{\dfrac{\partial u}{\partial y}}{1 + \dfrac{\partial u}{\partial y}}$$

$$= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}, \qquad (2.15)$$

because

$$\frac{\partial u}{\partial x} << 1,$$

$$\frac{\partial v}{\partial y} << 1,$$

...ments.

The shearing strain is positive when the angle between the sides *AD* and *AB* decreases; otherwise, the shearing strain is negative.

The definitions of the remaining normal and shearing strains can be found by noting the change in size and shape of the other sides of the infinitesimal cuboid in Figure 2.7 as

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y},$$

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z},$$

$$\varepsilon_z = \frac{\partial w}{\partial z}. \qquad (2.16a–c)$$

Example 2.1

A displacement field in a body is given by

$u = 10^{-5}(x^2 + 6y + 7xy)$

$v = 10^{-5}(yz)$

$w = 10^{-5}(xy + yz^2)$

Find the state of strain at $(x,y,z) = (1,2,3)$.

Solution

From Equation (2.8),

$$\in_x = \frac{\partial u}{\partial x}$$

$$= \frac{\partial}{\partial x}\left(10^{-5}\left(x^2 + 6y + 7xz\right)\right)$$

$$= 10^{-5}\left(2x + 7z\right)$$

$$= 10^{-5}\left(2 \times 1 + 7 \times 3\right)$$

$$= 2.300 \times 10^{-4}.$$

From Equation (2.11),

$$\epsilon_y = \frac{\partial v}{\partial y}$$

$$= \frac{\partial}{\partial y}\left(10^{-5}\left(yz\right)\right)$$

$$= 10^{-5}\left(z\right)$$

$$= 10^{-5}\left(3\right)$$

$$= 3.000 \times 10^{-5} .$$

From Equation (2.16c),

$$\epsilon_z = \frac{\partial w}{\partial z}$$

$$= \frac{\partial}{\partial z}\left(10^{-5}\left(xy + yz^2\right)\right)$$

$$= 10^{-5}\left(2yz\right)$$

$$= 10^{-5}(2 \times 2 \times 3)$$

$$= 1.2 \times 10^{-4} .$$

From Equation (2.15),

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$= \frac{\partial}{\partial y}\left(10^{-5}\left(x^2 + 6y + 7xz\right)\right) + \frac{\partial}{\partial x}\left(10^{-5}\left(\right.\right.$$

$$= 10^{-5}(6) + 10^{-5}(0)$$

$$= 6.000 \times 10^{-5}.$$

From Equation (2.16a),

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$= \frac{\partial}{\partial z}\left(10^{-5}(yz)\right) + \frac{\partial}{\partial y}\left(10^{-5}(xy + yz^2)\right)$$

$$= 10^{-5}(y) + 10^{-5}(x + z^2)$$

$$= 10^{-5}(2) + 10^{-5}(1 + 3^2)$$

$$= 1.2 \times 10^{-4}.$$

From Equation (2.16b),

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$= \frac{\partial}{\partial x}\left(10^{-5}(xy + yz^2)\right) + \frac{\partial}{\partial z}\left(10^{-5}(x^2 + 6y + 7xz)\right)$$

$$= 10^{-5}(y) + 10^{-5}(7x)$$

$$= 10^{-5}(2) + 10^{-5}(7 \times 1)$$

$$= 9.000 \times 10^{-5}.$$

2.2.3 Elastic Moduli

Section 2.2.2, three equilibrium equa
x stress components at a point. For a

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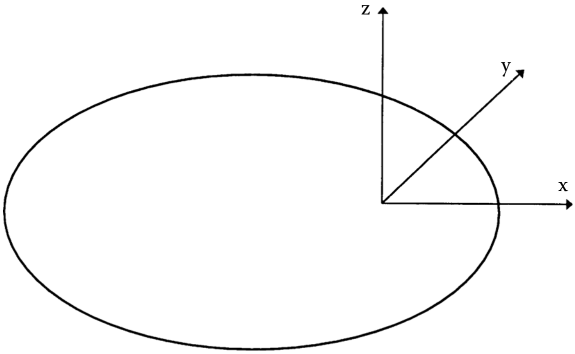

FIGURE 2.9
Cartesian coordinates in a three-dimensional body.

elastic and has small deformations, stresses and strains at a point are related through six simultaneous linear equations called Hooke's law. Note that 15 unknown parameters are at a point: six stresses, six strains, and three displacements. Combined with six simultaneous linear equations of Hooke's law, six strain-displacement relations — given by Equation (2.8), Equation (2.11), Equation (2.15), and Equation (2.16) — and three equilibrium equations give 15 equations for the solution of 15 unknowns.[1] Because strain-displacement and equilibrium equations are differential equations, they are subject to knowing boundary conditions for complete solutions.

For a linear isotropic material in a three-dimensional stress state, the Hooke's law stress–strain relationships at a point in an x–y–z orthogonal system (Figure 2.9) in matrix form are

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix}, \quad (2.17)$$

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\frac{E(1-\nu)}{(1-2\nu)(1+\nu)} & \frac{\nu E}{(1-2\nu)(1+\nu)} & \frac{\nu E}{(1-2\nu)(1+\nu)} & 0 & 0 & 0\\ \frac{\nu E}{(1-2\nu)(1+\nu)} & \frac{E(1-\nu)}{(1-2\nu)(1+\nu)} & \frac{\nu E}{(1-2\nu)(1+\nu)} & 0 & 0 & 0\\ \frac{\nu E}{(1-2\nu)(1+\nu)} & \frac{\nu E}{(1-2\nu)(1+\nu)} & \frac{E(1-\nu)}{(1-2\nu)(1+\nu)} & 0 & 0 & 0\\ 0 & 0 & 0 & G & 0 & 0\\ 0 & 0 & 0 & 0 & G & 0\\ 0 & 0 & 0 & 0 & 0 & G\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}, \tag{2.18}$$

where ν is the Poisson's ratio. The shear modulus G is a function of two elastic constants, E and ν, as

$$G=\frac{E}{2(1+\nu)}. \tag{2.19}$$

The 6 × 6 matrix in Equation (2.17) is called the compliance matrix [S] of an isotropic material. The 6 × 6 matrix in Equation (2.18), obtained by inverting the compliance matrix in Equation (2.17), is called the stiffness matrix [C] of an isotropic material.

2.2.4 Strain Energy

Energy is defined as the capacity to do work. In solid, deformable, elastic bodies under loads, the work done by external loads is stored as recoverable strain energy. The strain energy stored in the body per unit volume is then defined as

$$W=\frac{1}{2}(\sigma_x\varepsilon_x+\sigma_y\varepsilon_y+\sigma_z\varepsilon_z+\tau_{xy}\gamma_{xy}+\tau_{yz}\gamma_{yz}+\tau_{zx}\gamma_{zx}). \tag{2.20}$$

Example 2.2

Consider a bar of cross-section A and length L (Figure 2.10). A uniform tensile load P is applied to the two ends of the rod; find the stat
and strain energy per unit volume of the body. Assume
of a homogeneous isotropic material of Young's modu

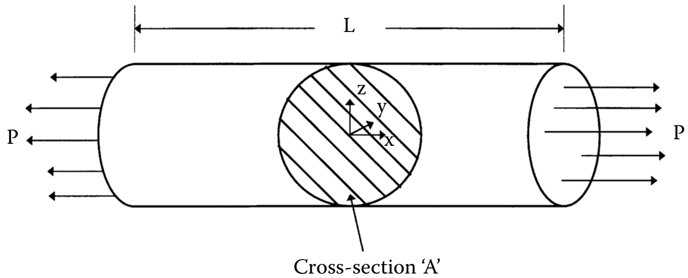

FIGURE 2.10
Cylindrical rod under uniform uniaxial load, *P*.

Solution

The stress state at any point is given by

$$\sigma_x = \frac{P}{A}, \sigma_y = 0, \sigma_z = 0, \tau_{yz} = 0, \tau_{zx} = 0, \tau_{xy} = 0. \tag{2.21}$$

If the circular rod is made of an isotropic, homogeneous, and linearly elastic material, then the stress–strain at any point is related as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \frac{P}{A} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{2.22}$$

$$\varepsilon_x = \frac{P}{AE}, \varepsilon_y = -\frac{\nu P}{AE}, \varepsilon_z = -\frac{\nu P}{AE},$$
$$\gamma_{yz} = 0, \gamma_{zx} = 0, \gamma_{xy} = 0. \tag{2.23}$$

The strain energy stored per unit volume in the rod, p

$$W = \frac{1}{2}\left[\left(\frac{P}{A}\right)\left(\frac{P}{AE}\right) + (0)\left(-\frac{\nu P}{AE}\right) + (0)\left(-\frac{\nu P}{AE}\right) + (0)\,(0) + (0)(0) + (0)(0)\right]$$

$$= \frac{1}{2}\frac{P^2}{A^2E}$$

$$= \frac{1}{2}\frac{\sigma_x^2}{E}. \tag{2.24}$$

2.3 Hooke's Law for Different Types of Materials

The stress–strain relationship for a general material that is not linearly elastic and isotropic is more complicated than Equation (2.17) and Equation (2.18). Assuming linear and elastic behavior for a composite is acceptable; however, assuming it to be isotropic is generally unacceptable. Thus, the stress–strain relationships follow Hooke's law, but the constants relating stress and strain are more in number than seen in Equation (2.17) and Equation (2.18). The most general stress–strain relationship is given as follows for a three-dimensional body in a 1–2–3 orthogonal Cartesian coordinate system:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}, \tag{2.25}$$

where the 6×6 $[C]$ matrix is called the stiffness matrix. The stiffness matrix has 36 constants.

What happens if one changes the system of coordinates from an orthogonal system 1–2–3 to some other orthogonal system, 1′–2′–3′? Then, new stiffness and compliance constants will be required to relate stresses and strains in the new coordinate system 1′–2′–3′. However, the new stiffness and compliance matrices in the 1′–2′–3′ system will be a functior compliance matrices in the 1–2–3 system and the angle the 1′–2′–3′ system and the 1–2–3 system.

Inverting Equation (2.25), the general strain–stress relationship for a three-dimensional body in a 1–2–3 orthogonal Cartesian coordinate system is

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix}. \qquad (2.26)$$

In the case of an isotropic material, relating the preceding strain–stress equation to Equation (2.17), one finds that the compliance matrix is related directly to engineering constants as

$$S_{11} = \frac{1}{E} = S_{22} = S_{33}$$

$$S_{12} = -\frac{\nu}{E} = S_{13} = S_{21} = S_{23} = S_{31} = S_{32}\,, \qquad (2.27)$$

$$S_{44} = \frac{1}{G} = S_{55} = S_{66}\,,$$

and S_{ij}, other than in the preceding, are zero.

It can be shown that the 36 constants in Equation (2.25) actually reduce to 21 constants due to the symmetry of the stiffness matrix [C] as follows. The stress–strain relationship (2.25) can also be written as

$$\sigma_i = \sum_{j=1}^{6} C_{ij}\varepsilon_j, \quad i = 1 \ldots 6\,, \qquad (2.28)$$

where, in a contracted notation,

$$\sigma_4 = \tau_{23},\ \sigma_5 = \tau_{31},\ \sigma_6 = \tau_{12},$$

$$\varepsilon_4 = \gamma_{23},\ \varepsilon_5 = \gamma_{31},\ \varepsilon_6 = \gamma_{12}.$$

The strain energy in the body per unit volume, per Equation (2.20), is expressed as

$$W = \frac{1}{2}\sum_{i=1}^{6}\sigma_i\varepsilon_i. \tag{2.30}$$

Substituting Hooke's law, Equation (2.28), in Equation (2.30),

$$W = \frac{1}{2}\sum_{i=1}^{6}\sum_{j=1}^{6}C_{ij}\varepsilon_j\varepsilon_i. \tag{2.31}$$

Now, by partial differentiation of Equation (2.31),

$$\frac{\partial W}{\partial\varepsilon_i\partial\varepsilon_j} = C_{ij}, \tag{2.32}$$

and

$$\frac{\partial W}{\partial\varepsilon_j\partial\varepsilon_i} = C_{ji}. \tag{2.33}$$

Because the differentiation does not necessarily need to be in either order,

$$C_{ij} = C_{ji}. \tag{2.34}$$

Equation (2.34) can also be proved by realizing that

$$\sigma_i = \frac{\partial W}{\partial\varepsilon_i}.$$

Thus, only 21 independent elastic constants are in the general stiffness matrix [C] of Equation (2.25). This also implies that only 21 independent constants are in the general compliance matrix [S] of Equation (2.26).

2.3.1 Anisotropic Material

The material that has 21 independent elastic constants ... ial. Once these constants are found f... ain relationship can be developed at

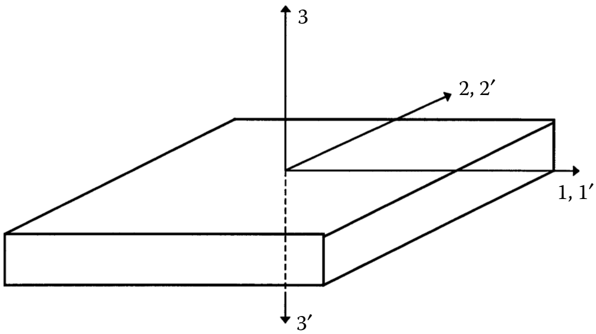

FIGURE 2.11
Transformation of coordinate axes for 1–2 plane of symmetry for a monoclinic material.

these constants can vary from point to point if the material is nonhomogeneous. Even if the material is homogeneous (or assumed to be), one needs to find these 21 elastic constants analytically or experimentally. However, many natural and synthetic materials do possess material symmetry — that is, elastic properties are identical in directions of symmetry because symmetry is present in the internal structure. Fortunately, this symmetry reduces the number of the independent elastic constants by zeroing out or relating some of the constants within the 6×6 stiffness $[C]$ and 6×6 compliance $[S]$ matrices. This simplifies the Hooke's law relationships for various types of elastic symmetry.

2.3.2 Monoclinic Material

If, in one plane of material symmetry* (Figure 2.11), for example, direction 3 is normal to the plane of material symmetry, then the stiffness matrix reduces to

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix}. \tag{2.35}$$

as

mplies that the material and its mirror image ab

$$C_{14} = 0, C_{15} = 0, C_{24} = 0, C_{25} = 0, C_{34} = 0, C_{35} = 0, C_{46} = 0, C_{56} = 0.$$

The direction perpendicular to the plane of symmetry is called the *principal direction*. Note that there are 13 independent elastic constants. Feldspar is an example of a monoclinic material.

The compliance matrix correspondingly reduces to

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{12} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{13} & S_{23} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{45} & S_{55} & 0 \\ S_{16} & S_{26} & S_{36} & 0 & 0 & S_{66} \end{bmatrix}. \quad (2.36)$$

Modifying an excellent example[2] of demonstrating the meaning of elastic symmetry for a monoclinic material given, consider a cubic element of Figure 2.12 taken out of a monoclinic material, in which 3 is the direction perpendicular to the 1–2 plane of symmetry. Apply a normal stress, σ_3, to the element. Then using the Hooke's law Equation (2.26) and the compliance matrix (Equation 2.36) for the monoclinic material, one gets

$$\varepsilon_1 = S_{13}\sigma_3$$

$$\varepsilon_2 = S_{23}\sigma_3$$

$$\varepsilon_3 = S_{33}\sigma_3$$

$$\gamma_{23} = 0$$

$$\gamma_{31} = 0$$

$$\gamma_{12} = S_{36}\sigma_3 . \quad (2.37a–f)$$

The cube will deform in all directions as determined ... ear strains in the 2–3 and 3–1 plane a... not change shape in those planes. Ho...

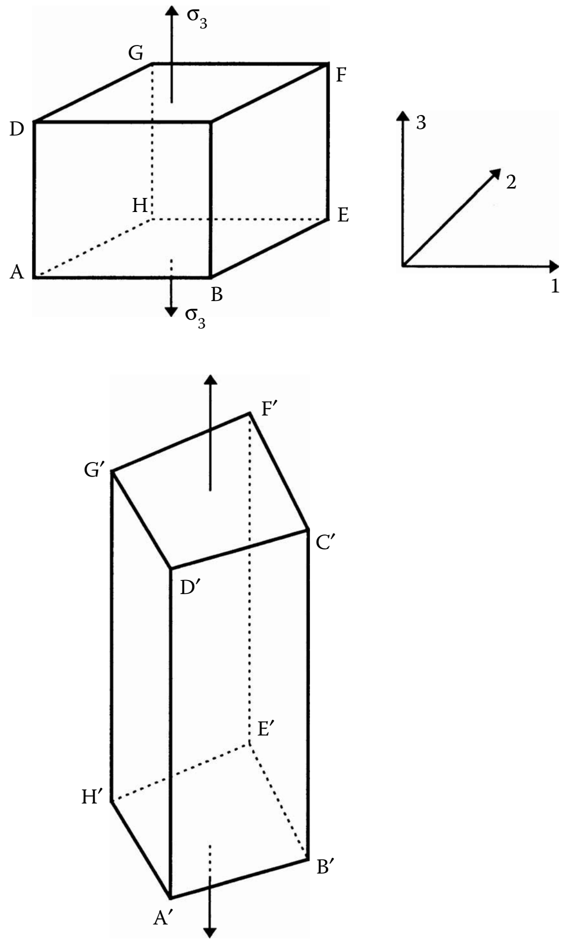

FIGURE 2.12
Deformation of a cubic element made of monoclinic material.

shape in the 1–2 plane. Thus, the faces *ABEH* and *CDFG* perpendicular to the 3 direction will change from rectangles to parallelograms, while the other four faces *ABCD*, *BEFC*, *GFEH*, and *AHGD* will stay as rectangles. This is unlike anisotropic behavior, in which all faces will be deformed in shape, and also unlike isotropic behavior, in which all faces will remain undeformed in shape.

2.3.3 Orthotropic Material (Orthogonally Anisotropic)/Specially Orthotropic

If ... three mutually perpendicular planes c... matrix is given by

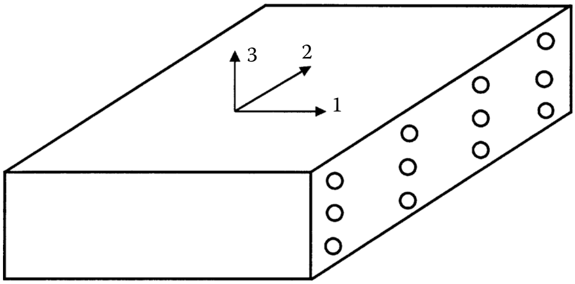

FIGURE 2.13
A unidirectional lamina as a monoclinic material with fibers, arranged in a rectangular array.

$$[C]=\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}. \tag{2.38}$$

The preceding stiffness matrix can be derived by starting from the stiffness matrix $[C]$ for the monoclinic material (Equation 2.35). With two more planes of symmetry, it gives

$$C_{16}=0, C_{26}=0, C_{36}=0, C_{45}=0 .$$

Three mutually perpendicular planes of material symmetry also imply three mutually perpendicular planes of elastic symmetry. Note that nine independent elastic constants are present. This is a commonly found material symmetry unlike anisotropic and monoclinic materials. Examples of an orthotropic material include a single lamina of continuous fiber composite, arranged in a rectangular array (Figure 2.13), a wooden bar, and rolled steel.

The compliance matrix reduces to

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & \\ 0 & 0 & 0 & 0 & S_{55} & \\ 0 & 0 & 0 & 0 & 0 & S_{(} \end{bmatrix} \quad (2.39)$$

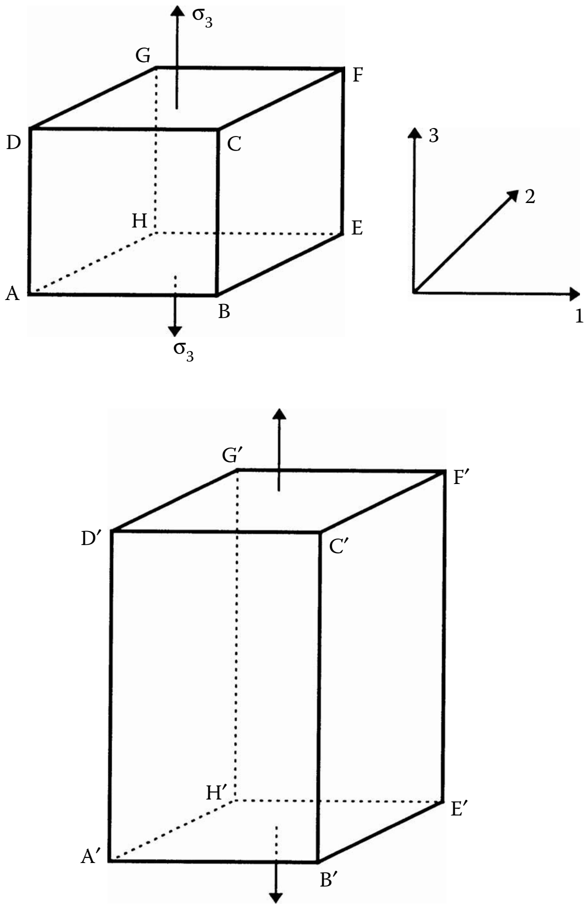

FIGURE 2.14
Deformation of a cubic element made of orthotropic material.

Demonstrating the meaning of elastic symmetry for an orthotropic material is similar to the approach taken for a monoclinic material (Section 2.3.2). Consider a cubic element (Figure 2.14) taken out of the orthotropic material, where 1, 2, and 3 are the principal directions or 1–2, 2–3, and 3–1 are the three mutually orthogonal planes of symmetry. Apply a normal stress, σ_3, to the element. Then, using the Hooke's law Equation (2.26) and the compliance matrix (Equation 2.39) for the orthotropic material, one gets

$$\varepsilon_1 = S_{13}\sigma_3$$

$$\varepsilon_2 = S_{23}\sigma_3$$

$$\varepsilon_3 = S_{33}\sigma_3$$

$$\gamma_{23} = 0$$

$$\gamma_{31} = 0 \quad (2.40a–f)$$

$$\gamma_{12} = 0.$$

The cube will deform in all directions as determined by the normal strain equations. However, the shear strains in all three planes (1–2, 2–3, and 3–1) are zero, showing that the element will not change shape in those planes. Thus, the cube will not deform in shape under any normal load applied in the principal directions. This is unlike the monoclinic material, in which two out of the six faces of the cube changed shape.

A cube made of isotropic material would not change its shape either; however, the normal strains, ε_1 and ε_2, will be different in an orthotropic material and identical in an isotropic material.

2.3.4 Transversely Isotropic Material

Consider a plane of material isotropy in one of the planes of an orthotropic body. If direction 1 is normal to that plane (2–3) of isotropy, then the stiffness matrix is given by

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{C_{22}-C_{23}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55} \end{bmatrix}. \quad (2.41)$$

Transverse isotropy results in the following relations:

$$C_{22} = C_{33}, C_{12} = C_{13}, C_{55} = C_{66}, C_{44} = \frac{C_{22}-C_{23}}{2}.$$

... independent elastic constants. An exa...
...nina in which the fibers are arranged

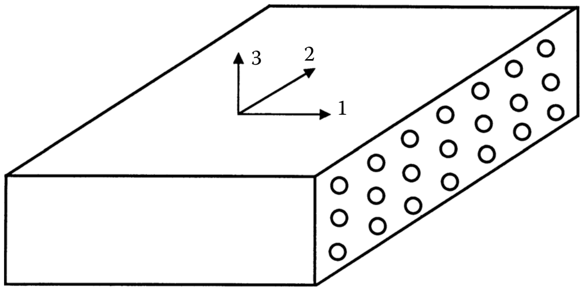

FIGURE 2.15
A unidirectional lamina as a transversely isotropic material with fibers arranged in a square array.

a hexagonal array. One may consider the elastic properties in the two directions perpendicular to the fibers to be the same. In Figure 2.15, the fibers are in direction 1, so plane 2–3 will be considered as the plane of isotropy.

The compliance matrix reduces to

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{12} & S_{23} & S_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(S_{22}-S_{23}) & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{55} \end{bmatrix}. \quad (2.42)$$

2.3.5 Isotropic Material

If all planes in an orthotropic body are identical, it is an isotropic material; then, the stiffness matrix is given by

$$[C]=\begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \end{bmatrix}. \quad (2.43)$$

in the following additional relations

$$C_{11} = C_{22}, C_{12} = C_{23}, C_{66} = \frac{C_{22} - C_{23}}{2} = \frac{C_{11} - C_{12}}{2} .$$

This also implies infinite principal planes of symmetry. Note the two independent constants. This is the most common material symmetry available. Examples of isotropic bodies include steel, iron, and aluminum. Relating Equation (2.43) to Equation (2.18) shows that

$$C_{11} = \frac{E(1-\nu)}{(1-2\nu)(1+\nu)},$$

$$C_{12} = \frac{\nu E}{(1-2\nu)(1+\nu)}. \quad (2.44a\text{–}b)$$

Note that

$$\frac{C_{11} - C_{12}}{2}$$

$$= \frac{1}{2}\left[\frac{E(1-\nu)}{(1-2\nu)(1+\nu)} - \frac{\nu E}{(1-2\nu)(1+\nu)}\right]$$

$$= \frac{E}{2(1+\nu)}$$

$$= G.$$

The compliance matrix reduces to

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(S_{11}-S_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(S_{11}-S_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \end{bmatrix}. \quad (2.45)$$

the number of independent elastic

s:

LLC

- Anisotropic: 21
- Monoclinic: 13
- Orthotropic: 9
- Transversely isotropic: 5
- Isotropic: 2

Example 2.3

Show the reduction of anisotropic material stress–strain Equation (2.25) to those of a monoclinic material stress–strain Equation (2.35).

Solution

Assume direction 3 is perpendicular to the plane of symmetry. Now in the coordinate system 1–2–3, Equation (2.25) with $C_{ij} = C_{ji}$ from Equation (2.34) is

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix}, \tag{2.46}$$

Also, in the coordinate system 1′–2′–3′ (Figure 2.11),

$$\begin{bmatrix} \sigma_{1'} \\ \sigma_{2'} \\ \sigma_{3'} \\ \tau_{2'3'} \\ \tau_{3'1'} \\ \tau_{1'2'} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{1'} \\ \varepsilon_{2'} \\ \varepsilon_{3'} \\ \gamma_{2'3'} \\ \gamma_{3'1'} \\ \gamma_{1'2'} \end{bmatrix}, \tag{2.47}$$

Because there is a plane of symmetry normal to direction 3, the stresses and strains in the 1–2–3 and 1′–2′–3′ coordinate systems are related by

$$\sigma_1 = \sigma_{1'}, \sigma_2 = \sigma_{2'}, \sigma_3 = \sigma_{3'}$$

$$\tau_{23} - \tau_{2'3'}, \tau_{31} = -\tau_{3'1'}, \tau_{12} = \tau_{1'2'},$$

$$\varepsilon_1 = \varepsilon_{1'}, \varepsilon_2 = \varepsilon_{2'}, \varepsilon_3 = \varepsilon_{3'},$$

$$\gamma_{23} = -\gamma_{2'3'}, \gamma_{31} = -\gamma_{3'1'}, \gamma_{12} = \gamma_{1'2'}. \quad (2.49a\text{–}f)$$

The terms in the first equation of Equation (2.46) and Equation (2.47) can be written as

$$\sigma_1 = C_{11}\varepsilon_1 + C_{12}\varepsilon_2 + C_{13}\varepsilon_3 + C_{14}\gamma_{23} + C_{15}\gamma_{31} + C_{16}\gamma_{12},$$

$$\sigma_{1'} = C_{11}\varepsilon_{1'} + C_{12}\varepsilon_{2'} + C_{13}\varepsilon_{3'} + C_{1'4}\gamma_{2'3'} + C_{15}\gamma_{3'1'}, + C_{16}\gamma_{1'2'} \quad (2.50a\text{–}b)$$

Substituting Equation (2.48) and Equation (2.49) in Equation (2.50b),

$$\sigma_1 = C_{11}\varepsilon_1 + C_{12}\varepsilon_2 + C_{13}\varepsilon_3 - C_{14}\gamma_{23} - C_{15}\gamma_{31} + C_{16}\gamma_{12}\,. \quad (2.51)$$

Subtracting Equation (2.51) from Equation (2.50a) gives

$$0 = 2C_{14}\gamma_{23} + 2C_{15}\gamma_{31}\,. \quad (2.52)$$

Because γ_{23} and γ_{31} are arbitrary,

$$C_{14} = C_{15} = 0. \quad (2.53a)$$

Similarly, one can show that

$$C_{24} = C_{25} = 0,$$

$$C_{34} = C_{35} = 0,$$

$$C_{46} = C_{56} = 0. \quad (2.54b\text{-}d)$$

Thus, only 13 independent elastic constants are present in a monoclinic material.

Example 2.4

The stress–strain relation is given in terms of comp
rial in Equation (2.26) and Equatio
x equations in terms of the nine engi

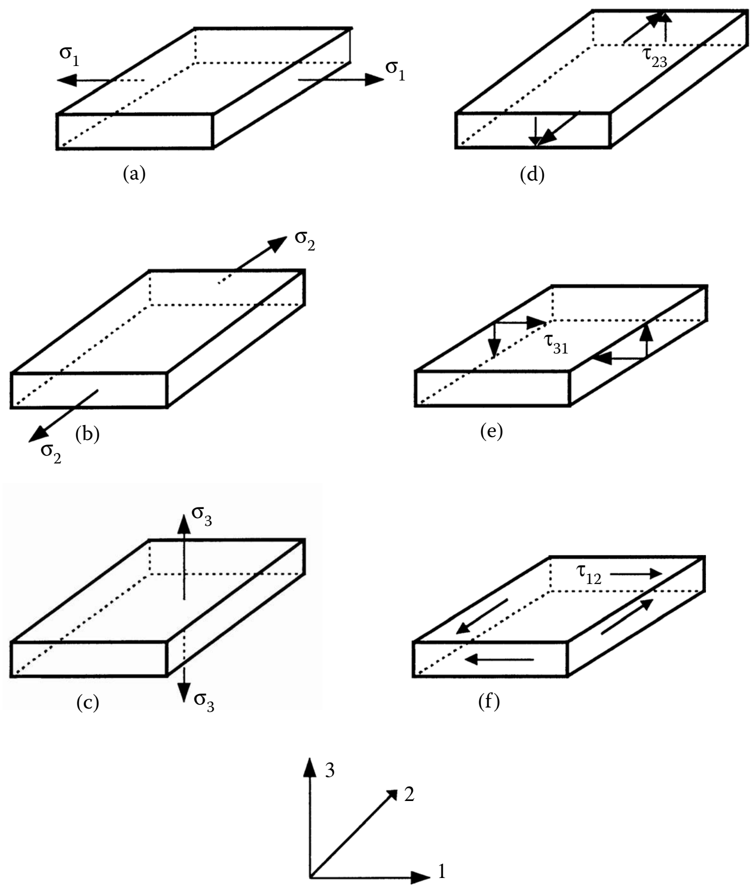

FIGURE 2.16
Application of stresses to find engineering constants of a three-dimensional orthotropic body.

an orthotropic material. What is the stiffness matrix in terms of the engineering constants?

Solution

Let us see how the compliance matrix and engineering constants of an orthotropic material are related. As shown in Figure 2.16a, apply $\sigma_1 \neq 0$, $\sigma_2 = 0$, $\sigma_3 = 0$, $\tau_{23} = 0$, $\tau_{31} = 0$, $\tau_{12} = 0$. Then, from Equation (2.26) and Equation (2.39):

$$\varepsilon_1 = S_{11}\sigma_1$$

$$\varepsilon_2 = S_{12}\sigma_1$$

$$\varepsilon_3 = S_{13}\sigma_1$$

$$\gamma_{23} = 0$$

$$\gamma_{31} = 0$$

$$\gamma_{12} = 0.$$

The Young's modulus in direction 1, E_1, is defined as

$$E_1 \equiv \frac{\sigma_1}{\varepsilon_1} = \frac{1}{S_{11}} . \tag{2.55}$$

The Poisson's ratio, ν_{12}, is defined as

$$\nu_{12} \equiv -\frac{\varepsilon_2}{\varepsilon_1} = -\frac{S_{12}}{S_{11}} . \tag{2.56}$$

In general terms, ν_{ij} is defined as the ratio of the negative of the normal strain in direction *j* to the normal strain in direction *i*, when the load is applied in the normal direction *i*.

The Poisson's ratio ν_{13} is defined as

$$\nu_{13} \equiv -\frac{\varepsilon_3}{\varepsilon_1} = -\frac{S_{13}}{S_{11}} . \tag{2.57}$$

Similarly, as shown in Figure 2.16b, apply $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 \neq 0$, $\tau_{23} = 0$, $\tau_{31} = 0$, $\tau_{12} = 0$. Then, from Equation (2.26) and Equation (2.39),

$$E_2 = \frac{1}{S_{22}} \tag{2.58}$$

$$\nu_{21} = -\frac{S_{12}}{S_{22}} \tag{2.59}$$

$$\nu_{23} = -\frac{S_{23}}{S_{22}} . \tag{2.60}$$

Similarly, as shown in Figure 2.16c, apply $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 \neq 0$, $\tau_{23} = 0$, $\tau_{31} = 0$, $\tau_{12} = 0$. From Equation (2.26) and Equation (2.39),

$$E_3 = \frac{1}{S_{33}}$$

$$\nu_{31} = -\frac{S_{13}}{S_{33}} \tag{2.62}$$

$$\nu_{32} = -\frac{S_{23}}{S_{33}} . \tag{2.63}$$

Apply, as shown in Figure 2.16d, $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 = 0$, $\tau_{23} \neq 0$, $\tau_{31} = 0$, $\tau_{12} = 0$. Then, from Equation (2.26) and Equation (2.39),

$$\varepsilon_1 = 0$$

$$\varepsilon_2 = 0$$

$$\varepsilon_3 = 0$$

$$\gamma_{23} = S_{44}\tau_{23}$$

$$\gamma_{31} = 0$$

$$\gamma_{12} = 0$$

The shear modulus in plane 2–3 is defined as

$$G_{23} \equiv \frac{\tau_{23}}{\gamma_{23}} = \frac{1}{S_{44}} . \tag{2.64}$$

Similarly, as shown in Figure 2.16e, apply $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 = 0$, $\tau_{23} = 0$, $\tau_{31} \neq 0$, $\tau_{12} = 0$. Then, from Equation (2.26) and Equation (2.39),

$$G_{31} = \frac{1}{S_{55}} . \tag{2.65}$$

Similarly, as shown in Figure 2.16f, apply $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 = 0$, $\tau_{23} = 0$, $\tau_{31} = 0$, $\tau_{12} \neq 0$. Then, from Equation (2.26) and Equation (2.39),

$$G_{12} = \frac{1}{S_{66}} . \tag{2.66}$$

In Equation (2.55) through Equation (2.66), 12 engine
been defined as follows:

moduli, E_1, E_2, and E_3, one in each ma

Six Poisson's ratios, ν_{12}, ν_{13}, ν_{21}, ν_{23}, ν_{31}, and ν_{32}, two for each plane
Three shear moduli, G_{23}, G_{31}, and G_{12}, one for each plane

However, the six Poisson's ratios are not independent of each other. For example, from Equation (2.55), Equation (2.56), Equation (2.58), and Equation (2.59),

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}. \tag{2.67}$$

Similarly, from Equation (2.55), Equation (2.57), Equation (2.61), and Equation (2.62),

$$\frac{\nu_{13}}{E_1} = \frac{\nu_{31}}{E_3}, \tag{2.68}$$

and from Equation (2.58), Equation (2.60), Equation (2.61), and Equation (2.63),

$$\frac{\nu_{23}}{E_2} = \frac{\nu_{32}}{E_3}. \tag{2.69}$$

Equation (2.67), Equation (2.68), and Equation (2.69) are called reciprocal Poisson's ratio equations. These relations reduce the total independent engineering constants to nine. This is the same number as the number of independent constants in the stiffness or the compliance matrix.

Rewriting the compliance matrix in terms of the engineering constants gives

$$[S] = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{13}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{21}}{E_2} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{31}}{E_3} & -\frac{\nu_{32}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \end{bmatrix}. \tag{2.70}$$

Inversion of Equation (2.70) would be the compliance matrix [*C*] and is given by

$$[C]=\begin{bmatrix} \frac{1-\nu_{23}\nu_{32}}{E_2E_3\Delta} & \frac{\nu_{21}+\nu_{23}\nu_{31}}{E_2E_3\Delta} & \frac{\nu_{31}+\nu_{21}\nu_{32}}{E_2E_3\Delta} & 0 & 0 & 0 \\ \frac{\nu_{21}+\nu_{23}\nu_{31}}{E_2E_3\Delta} & \frac{1-\nu_{13}\nu_{31}}{E_1E_3\Delta} & \frac{\nu_{32}+\nu_{12}\nu_{31}}{E_1E_3\Delta} & 0 & 0 & 0 \\ \frac{\nu_{31}+\nu_{21}\nu_{32}}{E_2E_3\Delta} & \frac{\nu_{32}+\nu_{12}\nu_{31}}{E_1E_3\Delta} & \frac{1-\nu_{12}\nu_{21}}{E_1E_2\Delta} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{12} \end{bmatrix}, \quad (2.71)$$

where

$$\Delta=\left(1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{13}\nu_{31}-2\nu_{21}\nu_{32}\nu_{13}\right)/\left(E_1E_2E_3\right). \quad (2.72)$$

Although nine independent elastic constants are in the compliance matrix [S] and, correspondingly, in the stiffness matrix [*C*] for orthotropic materials, constraints on the values of these constants exist. Based on the first law of thermodynamics, the stiffness and compliance matrices must be positive definite. Thus, the diagonal terms of [C] and [S] in Equation (2.71) and Equation (2.70), respectively, need to be positive. From the diagonal elements of the compliance matrix [S], this gives

$$E_1>0\ ,\ E_2>0\ ,\ E_3>0\ ,\ G_{12}>0\ ,\ G_{23}>0\ ,\ G_{31}>0 \quad (2.73)$$

and, from the diagonal elements of the stiffness matrix [*C*], gives

$$1-\nu_{23}\nu_{32}>0\ ,\ 1-\nu_{31}\nu_{13}>0\ ,\ 1-\nu_{12}\nu_{21}>0\ , \quad (2.74)$$

$$\Delta=1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{31}\nu_{13}-2\nu_{13}\nu_{21}\nu_{32}>0$$

Using the reciprocal relations given by Equation (2.67) through Equation (2.69),

$$\frac{\nu_{ij}}{E_i}=\frac{\nu_{ji}}{E_j}\quad \text{for } i\neq j \text{ and } i,j=1,2,3,$$

e inequalities as follows.

For example, because

$$1-\nu_{12}\nu_{21}>0\ ,$$

then

$$\nu_{12}<\frac{1}{\nu_{21}}=\frac{E_1}{E_2}\frac{1}{\nu_{12}}$$

$$\left|\nu_{12}\right|<\left|\frac{E_1}{E_2}\frac{1}{\nu_{12}}\right|$$

$$\left|\nu_{12}\right|<\sqrt{\frac{E_1}{E_2}}\ . \tag{2.75a}$$

Similarly, five other such relationships can be developed to give

$$\left|\nu_{21}\right|<\sqrt{\frac{E_2}{E_1}} \tag{2.75b}$$

$$\left|\nu_{32}\right|<\sqrt{\frac{E_3}{E_2}} \tag{2.75c}$$

$$\left|\nu_{23}\right|<\sqrt{\frac{E_2}{E_3}} \tag{2.75d}$$

$$\left|\nu_{31}\right|<\sqrt{\frac{E_3}{E_1}} \tag{2.75e}$$

$$\left|\nu_{13}\right|<\sqrt{\frac{E_1}{E_3}}\ . \tag{2.75f}$$

These restrictions on the elastic moduli are importan
ite because they show that the nine ind
without influencing the limits of the

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Example 2.5

Find the compliance and stiffness matrix for a graphite/epoxy lamina. The material properties are given as

$$E_1 = 181 GPa \text{ , } E_2 = 10.3 GPa \text{ , } E_3 = 10.3 GPa$$

$$\nu_{12} = 0.28 \text{ , } \nu_{23} = 0.60 \text{ , } \nu_{13} = 0.27$$

$$G_{12} = 7.17 GPa \text{ , } G_{23} = 3.0 GPa \text{ , } G_{31} = 7.00 GPa \text{ .}$$

Solution

$$S_{11} = \frac{1}{E_1} = \frac{1}{181 \times 10^9} = 5.525 \times 10^{-12} Pa^{-1}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{10.3 \times 10^9} = 9.709 \times 10^{-11} Pa^{-1}$$

$$S_{33} = \frac{1}{E_3} = \frac{1}{10.3 \times 10^9} = 9.709 \times 10^{-11} Pa^{-1}$$

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{0.28}{181 \times 10^9} = -1.547 \times 10^{-12} Pa^{-1}$$

$$S_{13} = -\frac{\nu_{13}}{E_1} = -\frac{0.27}{181 \times 10^9} = -1.492 \times 10^{-12} Pa^{-1}$$

$$S_{23} = -\frac{\nu_{23}}{E_2} = -\frac{0.6}{10.3 \times 10^9} = -5.825 \times 10^{-11} Pa^{-1}$$

$$S_{44} = \frac{1}{G_{23}} = \frac{1}{3 \times 10^9} = 3.333 \times 10^{-10} Pa^{-1}$$

$$S_{55} = \frac{1}{G_{31}} = \frac{1}{7 \times 10^9} = 1.429 \times 10^{-10} Pa$$

$$S_{66} = \frac{1}{G_{12}} = \frac{1}{7.17 \times 10^{9}} = 1.395 \times 10^{-10}\, Pa^{-1}.$$

Thus, the compliance matrix for the orthotropic lamina is given by

$$[S] =$$

$$\begin{bmatrix} 5.525\times10^{-12} & -1.547\times10^{-12} & -1.492\times10^{-12} & 0 & 0 & 0 \\ -1.547\times10^{-12} & 9.709\times10^{-11} & -5.825\times10^{-11} & 0 & 0 & 0 \\ -1.492\times10^{-12} & -5.825\times10^{-11} & 9.709\times10^{-11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.333\times10^{-10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.429\times10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.395\times10^{-10} \end{bmatrix} Pa^{-1}$$

The stiffness matrix can be found by inverting the compliance matrix and is given by

$$[C] = [S]^{-1}$$

$$[C] =$$

$$\begin{bmatrix} 0.1850\times10^{12} & 0.7269\times10^{10} & 0.7204\times10^{10} & 0 & 0 & 0 \\ 0.7269\times10^{10} & 0.1638\times10^{11} & 0.9938\times10^{10} & 0 & 0 & 0 \\ 0.7204\times10^{10} & 0.9938\times10^{10} & 0.1637\times10^{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3000\times10^{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6998\times10^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.7168\times10^{10} \end{bmatrix} Pa$$

The preceding stiffness matrix $[C]$ can also be found directly by using Equation (2.71).

2.4 Hooke's Law for a Two-Dimensional Unidirectional Lamina

2.4.1 Plane Stress Assumption

A thin plate is a prismatic member having a small thickness, and it is the case for a typical lamina. If a plate is thin and there are r
it can be considered to be under plane stress (Figure 2.
the plate are free from external loads
se the plate is thin, these three stresse

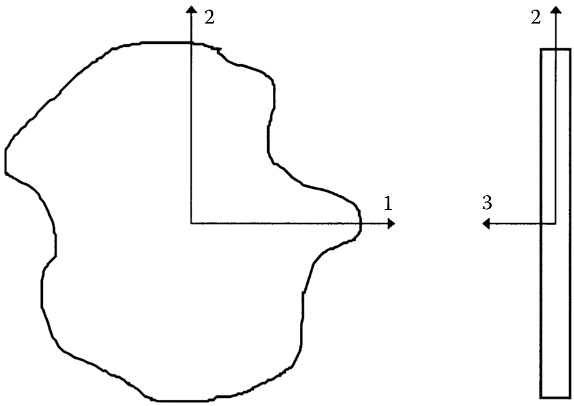

FIGURE 2.17
Plane stress conditions for a thin plate.

assumed to vary little from the magnitude of stresses at the top and the bottom surfaces. Thus, they can be assumed to be zero within the plate also. A lamina is thin and, if no out-of-plane loads are applied, one can assume that it is under plane stress. This assumption then reduces the three-dimensional stress–strain equations to two-dimensional stress–strain equations.

2.4.2 Reduction of Hooke's Law in Three Dimensions to Two Dimensions

A unidirectional lamina falls under the orthotropic material category. If the lamina is thin and does not carry any out-of-plane loads, one can assume plane stress conditions for the lamina. Therefore, taking Equation (2.26) and Equation (2.39) and assuming $\sigma_3 = 0$, $\tau_{23} = 0$, and $\tau_{31} = 0$, then

$$\varepsilon_3 = S_{13}\sigma_1 + S_{23}\sigma_2,$$

$$\gamma_{23} = \gamma_{31} = 0. \tag{2.76a,b}$$

The normal strain, ε_3, is not an independent strain because it is a function of the other two normal strains, ε_1 and ε_2. Therefore, the normal strain, ε_3, can be omitted from the stress–strain relationship (2.39). Also, the shearing strains, γ_{23} and γ_{31}, can be omitted because they are zero. Equation (2.39) for an orthotropic plane stress problem can then be written as

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix},$$

where S_{ij} are the elements of the compliance matrix. Note the four independent compliance elements in the matrix.

Inverting Equation (2.77) gives the stress–strain relationship as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}, \tag{2.78}$$

where Q_{ij} are the reduced stiffness coefficients, which are related to the compliance coefficients as

$$Q_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2},$$

$$Q_{12} = -\frac{S_{12}}{S_{11}S_{22} - S_{12}^2}, \tag{2.79a–d}$$

$$Q_{22} = \frac{S_{11}}{S_{11}S_{22} - S_{12}^2},$$

$$Q_{66} = \frac{1}{S_{66}}.$$

Note that the elements of the reduced stiffness matrix, Q_{ij}, are not the same as the elements of the stiffness matrix, C_{ij} (see Exercise 2.13).

2.4.3 Relationship of Compliance and Stiffness Matrix to Engineering Elastic Constants of a Lamina

Equation (2.77) and Equation (2.78) show the relationship of stress and strain through the compliance [S] and reduced stiffness [Q] matrices. However, stress and strains are generally related through engineering elastic constants. For a unidirectional lamina, these engineering elastics constants are

E_1 = longitudinal Young's modulus (in direction 1)

E_2 = transverse Young's modulus (in direction 2)

ν_{12} = major Poisson's ratio, where the general Poisson's ratio, ν_{ij} is defined as the ratio of the negative of the normal strain in direction j to the normal strain in direction i, when the or applied in direction i

hear modulus (in plane 1–2)

UNIT-V

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$$S_{66} = \frac{1}{G_{12}} = \frac{1}{7.17 \times 10^{9}} = 1.395 \times 10^{-10}\, Pa^{-1}.$$

Thus, the compliance matrix for the orthotropic lamina is given by

$$[S] =$$

$$\begin{bmatrix} 5.525\times10^{-12} & -1.547\times10^{-12} & -1.492\times10^{-12} & 0 & 0 & 0 \\ -1.547\times10^{-12} & 9.709\times10^{-11} & -5.825\times10^{-11} & 0 & 0 & 0 \\ -1.492\times10^{-12} & -5.825\times10^{-11} & 9.709\times10^{-11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.333\times10^{-10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.429\times10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.395\times10^{-10} \end{bmatrix} Pa^{-1}$$

The stiffness matrix can be found by inverting the compliance matrix and is given by

$$[C] = [S]^{-1}$$

$$[C] =$$

$$\begin{bmatrix} 0.1850\times10^{12} & 0.7269\times10^{10} & 0.7204\times10^{10} & 0 & 0 & 0 \\ 0.7269\times10^{10} & 0.1638\times10^{11} & 0.9938\times10^{10} & 0 & 0 & 0 \\ 0.7204\times10^{10} & 0.9938\times10^{10} & 0.1637\times10^{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3000\times10^{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6998\times10^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.7168\times10^{10} \end{bmatrix} Pa$$

The preceding stiffness matrix $[C]$ can also be found directly by using Equation (2.71).

2.4 Hooke's Law for a Two-Dimensional Unidirectional Lamina

2.4.1 Plane Stress Assumption

A thin plate is a prismatic member having a small thickness, and it is the case for a typical lamina. If a plate is thin and there are r
it can be considered to be under plane stress (Figure 2.
the plate are free from external loads
se the plate is thin, these three stresse

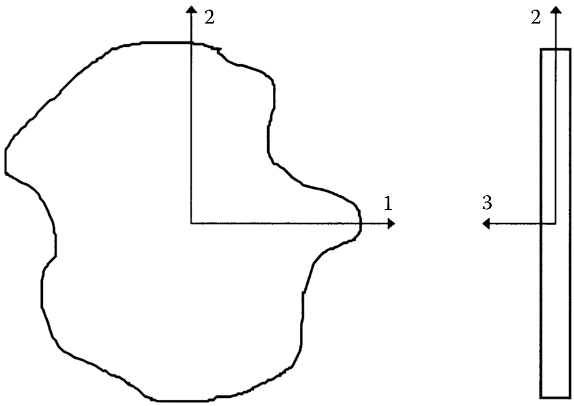

FIGURE 2.17
Plane stress conditions for a thin plate.

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$$\gamma_{23} = \gamma_{31} = 0. \qquad (2.76a,b)$$

The normal strain, ε_3, is not an independent strain because it is a function of the other two normal strains, ε_1 and ε_2. Therefore, the normal strain, ε_3, can be omitted from the stress–strain relationship (2.39). Also, the shearing strains, γ_{23} and γ_{31}, can be omitted because they are zero. Equation (2.39) for an orthotropic plane stress problem can then be written as

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where S_{ij} are the elements of the compliance matrix. Note the four independent compliance elements in the matrix.

Inverting Equation (2.77) gives the stress–strain relationship as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}, \tag{2.78}$$

where Q_{ij} are the reduced stiffness coefficients, which are related to the compliance coefficients as

$$Q_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2},$$

$$Q_{12} = -\frac{S_{12}}{S_{11}S_{22} - S_{12}^2}, \tag{2.79a–d}$$

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$$Q_{66} = \frac{1}{S_{66}}.$$

Note that the elements of the reduced stiffness matrix, Q_{ij}, are not the same as the elements of the stiffness matrix, C_{ij} (see Exercise 2.13).

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ν_{12} = major Poisson's ratio, where the general Poisson's ratio, ν_{ij} is defined as the ratio of the negative of the normal strain in direction j to the normal strain in direction i, when the or applied in direction i

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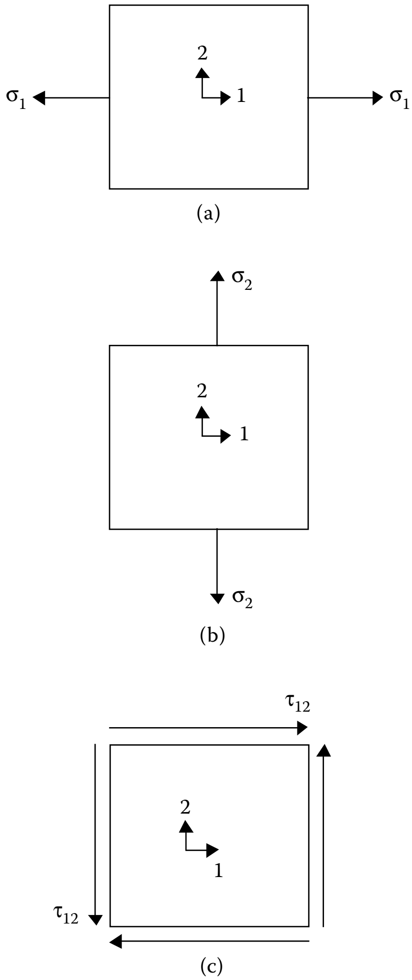


FIGURE 2.18
Application of stresses to find engineering constants of a unidirectional lamina.

Experimentally, the four independent engineering elastic constants are measured as follows and can be related to the four independent elements of the compliance matrix [S] of Equation (2.77).

- Apply a pure tensile load in direction 1 (Figure 2.18a), that is,

$$\sigma_1 \neq 0, \sigma_2 = 0, \tau_{12} = 0.$$

Equation (2.77),

$$\varepsilon_1 = S_{11}\sigma_1,$$
$$\varepsilon_2 = S_{12}\sigma_1, \qquad (2.81a\text{–}c)$$
$$\gamma_{12} = 0.$$

By definition, if the only nonzero stress is σ_1, as is the case here, then

$$E_1 \equiv \frac{\sigma_1}{\varepsilon_1} = \frac{1}{S_{11}}, \qquad (2.82)$$

$$\nu_{12} \equiv -\frac{\varepsilon_2}{\varepsilon_1} = -\frac{S_{12}}{S_{11}}. \qquad (2.83)$$

- Apply a pure tensile load in direction 2 (Figure 2.18b), that is

$$\sigma_1 = 0,\ \sigma_2 \neq 0,\ \tau_{12} = 0. \qquad (2.84)$$

Then, from Equation (2.77),

$$\varepsilon_1 = S_{12}\sigma_2,$$
$$\varepsilon_2 = S_{22}\sigma_2, \qquad (2.85a\text{–}c)$$
$$\gamma_{12} = 0.$$

By definition, if the only nonzero stress is σ_2, as is the case here, then

$$E_2 \equiv \frac{\sigma_2}{\varepsilon_2} = \frac{1}{S_{22}}, \qquad (2.86)$$

$$\nu_{21} \equiv -\frac{\varepsilon_1}{\varepsilon_2} = -\frac{S_{12}}{S_{22}}. \qquad (2.87)$$

The ν_{21} term is called the minor Poisson's ratio. From Equation (2.82), Equation (2.83), Equation (2.86), and Equation (2.87), we have the reciprocal relationship

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}.$$

- Apply a pure shear stress in the plane 1–2 (Figure 2.18c) — that is,

$$\sigma_1 = 0,\ \sigma_2 = 0 \text{ and } \tau_{12} \neq 0. \tag{2.89}$$

Then, from Equation (2.77),

$$\varepsilon_1 = 0,$$

$$\varepsilon_2 = 0,$$

$$\gamma_{12} = S_{66}\tau_{12}. \tag{2.90a–c}$$

By definition, if τ_{12} is the only nonzero stress, as is the case here, then

$$G_{12} \equiv \frac{\tau_{12}}{\gamma_{12}} = \frac{1}{S_{66}}. \tag{2.91}$$

Thus, we have proved that

$$S_{11} = \frac{1}{E_1},$$

$$S_{12} = -\frac{\nu_{12}}{E_1},$$

$$S_{22} = \frac{1}{E_2},$$

$$S_{66} = \frac{1}{G_{12}}. \tag{2.92a–d}$$

Also, the stiffness coefficients Q_{ij} are related to the engineering constants through Equation (2.98) and Equation (2.92) as

$$Q_{11} = \frac{E_1}{1 - \nu_{21}\nu_{12}},$$

$$Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{21}\nu_{12}},$$

$$Q_{22} = \frac{E_2}{1 - \nu_{21}\nu_{12}}, \text{ and}$$

$$Q_{66} = G_{12}. \tag{2.93a–d}$$

Equation (2.77), Equation (2.78), Equation (2.92), and Equation (2.93) relate stresses and strains through any of the following combinations of four constants.

Q_{11}, Q_{12}, Q_{22}, Q_{66}, or
S_{11}, S_{12}, S_{22}, S_{66}, or
E_1, E_2, ν_{12}, G_{12}

The unidirectional lamina is a *specially orthotropic* lamina because normal stresses applied in the 1–2 direction do not result in any shearing strains in the 1–2 plane because $Q_{16} = Q_{26} = 0 = S_{16} = S_{26}$. Also, the shearing stresses applied in the 1–2 plane do not result in any normal strains in the 1 and 2 directions because $Q_{16} = Q_{26} = 0 = S_{16} = S_{26}$.

A woven composite with its weaves perpendicular to each other and short fiber composites with fibers arranged perpendicularly to each other or aligned in one direction also are *specially orthotropic*. Thus, any discussion in this chapter or in Chapter 4 ("Macromechanics of a Laminate") is valid for such a lamina as well. Mechanical properties of some typical unidirectional lamina are given in Table 2.1 and Table 2.2.

Example 2.6

For a graphite/epoxy unidirectional lamina, find the following

1. Compliance matrix
2. Minor Poisson's ratio
3. Reduced stiffness matrix
4. Strains in the 1–2 coordinate system if the applied stresses (Figure 2.19) are

$$\sigma_1 = 2MPa,\ \sigma_2 = -3MPa,\ \tau_{12} = 4MPa$$

s of unidirectional graphite/epoxy la

TABLE 2.1

Typical Mechanical Properties of a Unidirectional Lamina (SI System of Units)

Property	Symbol	Units	Glass/ epoxy	Boron/ epoxy	Graphite/ epoxy
Fiber volume fraction	V_f		0.45	0.50	0.70
Longitudinal elastic modulus	E_1	GPa	38.6	204	181
Transverse elastic modulus	E_2	GPa	8.27	18.50	10.30
Major Poisson's ratio	ν_{12}		0.26	0.23	0.28
Shear modulus	G_{12}	GPa	4.14	5.59	7.17
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	MPa	1062	1260	1500
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	MPa	610	2500	1500
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	MPa	31	61	40
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	MPa	118	202	246
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	MPa	72	67	68
Longitudinal coefficient of thermal expansion	α_1	μm/m/°C	8.6	6.1	0.02
Transverse coefficient of thermal expansion	α_2	μm/m/°C	22.1	30.3	22.5
Longitudinal coefficient of moisture expansion	β_1	m/m/kg/kg	0.00	0.00	0.00
Transverse coefficient of moisture expansion	β_2	m/m/kg/kg	0.60	0.60	0.60

Source: Tsai, S.W. and Hahn, H.T., *Introduction to Composite Materials*, CRC Press, Boca Raton, FL, Table 1.7, p. 19; Table 7.1, p. 292; Table 8.3, p. 344. Reprinted with permission.

Solution

From Table 2.1, the engineering elastic constants of the unidirectional graphite/epoxy lamina are

$$E_1 = 181\ GPa,\ E_2 = 10.3\ GPa,\ \nu_{12} = 0.28,\ G_{12} = 7.17\ GPa.$$

1. Using Equation (2.92), the compliance matrix elements are

$$S_{11} = \frac{1}{181 \times 10^9} = 0.5525 \times 10^{-11}\,Pa^{-1},$$

$$S_{12} = -\frac{0.28}{181 \times 10^9} = -0.1547 \times 10^{-11}\,Pa^{-}$$

TABLE 2.2

Typical Mechanical Properties of a Unidirectional Lamina (USCS System of Units)

Property	Symbol	Units	Glass/ epoxy	Boron/ epoxy	Graphite/ epoxy
Fiber volume fraction	V_f	—	0.45	0.50	0.70
Longitudinal elastic modulus	E_1	Msi	5.60	29.59	26.25
Transverse elastic modulus	E_2	Msi	1.20	2.683	1.49
Major Poisson's ratio	υ_{12}		0.26	0.23	0.28
Shear modulus	G_{12}	Msi	0.60	0.811	1.040
Ultimate longitudinal tensile strength	$(\sigma_1^T)_{ult}$	ksi	154.03	182.75	217.56
Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	ksi	88.47	362.6	217.56
Ultimate transverse tensile strength	$(\sigma_2^T)_{ult}$	ksi	4.496	8.847	5.802
Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	ksi	17.12	29.30	35.68
Ultimate in-plane shear strength	$(\tau_{12})_{ult}$	ksi	10.44	9.718	9.863
Longitudinal coefficient of thermal expansion	α_1	µin./in./°F	4.778	3.389	0.0111
Transverse coefficient of thermal expansion	α_2	µin./in./°F	12.278	16.83	12.5
Longitudinal coefficient of moisture expansion	β_1	in./in./lb/lb	0.00	0.00	0.00
Transverse coefficient of moisture expansion	β_2	in./in./lb/lb	0.60	0.60	0.60

Source: Tsai, S.W. and Hahn, H.T., *Introduction to Composite Materials*, CRC Press, Boca Raton, FL, Table 1.7, p. 19; Table 7.1, p. 292; Table 8.3, p. 344. USCS system used for tables reprinted with permission.

$$S_{22} = \frac{1}{10.3 \times 10^9} = 0.9709 \times 10^{-10}\,Pa^{-1},$$

$$S_{66} = \frac{1}{7.17 \times 10^9} = 0.1395 \times 10^{-9}\,Pa^{-1}.$$

2. Using the reciprocal relationship (2.88), the minor Poisson's ratio is

$$\nu_{21} = \frac{0.28}{181 \times 10^9} \times (10.3 \times 10^9) = 0.01593.$$

3. Using Equation (2.93), the reduced stiffness matri

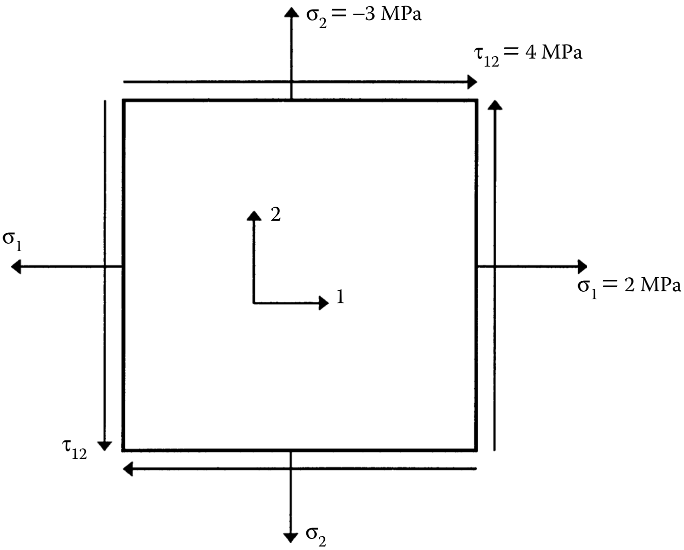


FIGURE 2.19
Applied stresses in a unidirectional lamina in Example 2.6.

$$Q_{11} = \frac{181 \times 10^9}{1-(0.28)(0.01593)} = 181.8 \times 10^9 \, Pa,$$

$$Q_{12} = \frac{(0.28)(10.3 \times 10^9)}{1-(0.28)(0.01593)} = 2.897 \times 10^9 \, Pa,$$

$$Q_{22} = \frac{10.3 \times 10^9}{1-(0.28)(0.01593)} = 10.35 \times 10^9 \, Pa,$$

$$Q_{66} = 7.17 \times 10^9 \, Pa \; .$$

The reduced stiffness matrix [Q] could also be obtained by inverting the compliance matrix [S] of part 1:

$$[Q] = [S]^{-1} = \begin{bmatrix} 0.5525 \times 10^{-11} & -0.1547 \times 10^{-11} & \\ -0.1547 \times 10^{-11} & 0.9709 \times 10^{-10} & \\ 0 & 0 & 0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 181.8\times10^9 & 2.897\times10^9 & 0 \\ 2.897\times10^9 & 10.35\times10^9 & 0 \\ 0 & 0 & 7.17\times10^9 \end{bmatrix} Pa\ .$$

4. Using Equation (2.77), the strains in the 1–2 coordinate system are

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.5525\times10^{-11} & -0.1547\times10^{-11} & 0 \\ -0.1547\times10^{-11} & 0.9709\times10^{-10} & 0 \\ 0 & 0 & 0.1395\times10^{-9} \end{bmatrix} \begin{bmatrix} 2\times10^6 \\ -3\times10^6 \\ 4\times10^6 \end{bmatrix}$$

$$= \begin{bmatrix} 15.69 \\ -294.4 \\ 557.9 \end{bmatrix} (10^{-6}).$$

Thus, the strains in the local axes are

$$\varepsilon_1 = 15.69 \frac{\mu m}{m},$$

$$\varepsilon_2 = 294.4 \frac{\mu m}{m},$$

$$\gamma_{12} = 557.9 \frac{\mu m}{m}.$$

2.5 Hooke's Law for a Two-Dimensional Angle Lamina

Generally, a laminate does not consist only of unidirectional laminae because of their low stiffness and strength properties in the transverse direction. Therefore, in most laminates, some laminae are placed at an angle. It is thus necessary to develop the stress–strain relationship for an angle lamina.

The coordinate system used for showing an angle l
Figure 2.20. The axes in the 1–2 coordinate system are
or the material axes. The direction 1 is parallel to the fib
r to the fibers. In some literature, dir

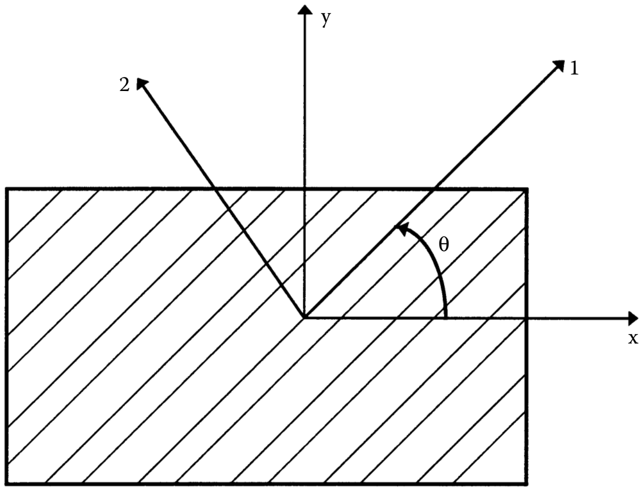


FIGURE 2.20
Local and global axes of an angle lamina.

the longitudinal direction *L* and the direction 2 is called the transverse direction *T*. The axes in the *x*–*y* coordinate system are called the global axes or the off-axes. The angle between the two axes is denoted by an angle θ. The stress–strain relationship in the 1–2 coordinate system has already been established in Section 2.4 and we are now going to develop the stress–strain equations for the *x*–*y* coordinate system.

The global and local stresses in an angle lamina are related to each other through the angle of the lamina, θ (Appendix B):

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}, \tag{2.94}$$

where [*T*] is called the transformation matrix and is defined as

$$[T]^{-1} = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix}, \tag{2.95}$$

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix}, \tag{2.96}$$

$$c = \text{Cos}\,(\theta),$$

$$s = \text{Sin}\,(\theta). \tag{2.97a,b}$$

Using the stress–strain Equation (2.78) in the local axes, Equation (2.94) can be written as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1}[Q] \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}. \tag{2.98}$$

The global and local strains are also related through the transformation matrix (Appendix B):

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{bmatrix} - [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}/2 \end{bmatrix}, \tag{2.99}$$

which can be rewritten as

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \tag{2.100}$$

where $[R]$ is the Reuter matrix[3] and is defined as

$$[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}. \tag{2.101}$$

ng Equation (2.100) in Equation (2.98

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1}[Q][R][T][R]^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}. \tag{2.102}$$

On carrying the multiplication of the first five matrices on the right-hand side of Equation (2.102),

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}, \tag{2.103}$$

where $\bar{Q}_{ij}$ are called the elements of the transformed reduced stiffness matrix $[\bar{Q}]$ and are given by

$$\bar{Q}_{11} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2,$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^2),$$

$$\bar{Q}_{22} = Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2,$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})c^3s - (Q_{22} - Q_{12} - 2Q_{66})s^3c,$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})cs^3 - (Q_{22} - Q_{12} - 2Q_{66})c^3s,$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4). \tag{2.104a–f}$$

Note that six elements are in the $[\bar{Q}]$ matrix. However, by looking at Equation (2.104), it can be seen that they are just functions of the four stiffness elements, Q_{11}, Q_{12}, Q_{22}, and Q_{66}, and the angle of the lamina, θ.

Inverting Equation (2.103) gives

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix},$$

where S_{ij} are the elements of the transformed reduced compliance matrix and are given by

$$\bar{S}_{11} = S_{11}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}s^4,$$

$$\bar{S}_{12} = S_{12}(s^4 + c^4) + (S_{11} + S_{22} - S_{66})s^2c^2,$$

$$\bar{S}_{22} = S_{11}s^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}c^4,$$

$$\bar{S}_{16} = (2S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c,$$

$$\bar{S}_{26} = (2S_{11} - 2S_{12} - S_{66})s^3c - (2S_{22} - 2S_{12} - S_{66})sc^3,$$

$$\bar{S}_{66} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})s^2c^2 + S_{66}(s^4 + c^4). \qquad (2.106a–f)$$

From Equation (2.77) and Equation (2.78), for a unidirectional lamina loaded in the material axes directions, no coupling occurs between the normal and shearing terms of strains and stresses. However, for an angle lamina, from Equation (2.103) and Equation (2.105), coupling takes place between the normal and shearing terms of strains and stresses. If only normal stresses are applied to an angle lamina, the shear strains are nonzero; if only shearing stresses are applied to an angle lamina, the normal strains are nonzero. Therefore, Equation (2.103) and Equation (2.105) are stress–strain equations for what is called a *generally orthotropic* lamina.

Example 2.7

Find the following for a 60° angle lamina (Figure 2.21) of graphite/epoxy. Use the properties of unidirectional graphite/epoxy lamina from Table 2.1.

1. Transformed compliance matrix
2. Transformed reduced stiffness matrix

If the applied stress is σ_x = 2 MPa, σ_y = –3 MPa, and τ_{xy} = 4 MPa, also find

3. Global strains
4. Local strains
5. Local stresses
6. Principal stresses

[illegible] hear stress

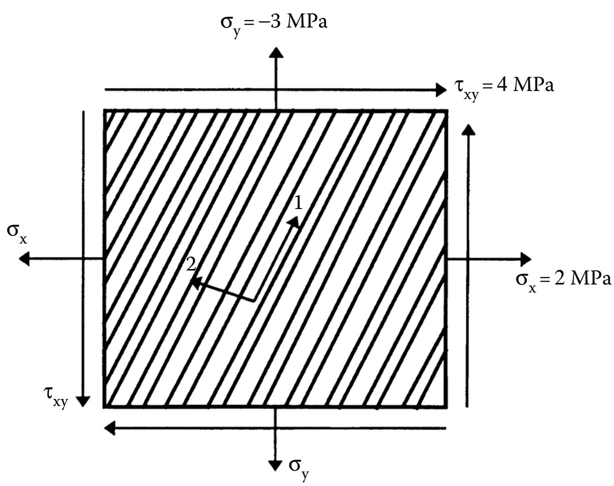


FIGURE 2.21
Applied stresses to an angle lamina in Example 2.7.

8. Principal strains
9. Maximum shear strain

Solution

c = Cos(60°) = 0.500
s = Sin(60°) = 0.866

1. From Example 2.6,

$$S_{11} = 0.5525 \times 10^{-11} \frac{1}{Pa},$$

$$S_{22} = 0.9709 \times 10^{-10} \frac{1}{Pa},$$

$$S_{12} = -0.1547 \times 10^{-11} \frac{1}{Pa},$$

$$S_{66} = 0.1395 \times 10^{-9} \frac{1}{Pa}.$$

ation (2.106a),

$$\bar{S}_{11} = 0.5525 \times 10^{-11}(0.500)^4 + [2(-0.1547 \times 10^{-11})$$

$$+0.1395 \times 10^{-9}](0.866)^2(0.5)^2 + 0.9709 \times 10^{-10}(0.866)^4$$

$$= 0.8053 \times 10^{-10}\,\frac{1}{Pa}$$

Similarly, using Equation (2.106b–f), one can evaluate

$$\bar{S}_{12} = -0.7878 \times 10^{-11}\,\frac{1}{Pa},$$

$$\bar{S}_{16} = -0.3234 \times 10^{-10}\,\frac{1}{Pa},$$

$$\bar{S}_{22} = 0.3475 \times 10^{-10}\,\frac{1}{Pa},$$

$$\bar{S}_{26} = -0.4696 \times 10^{-10}\,\frac{1}{Pa},$$

$$\bar{S}_{66} = 0.1141 \times 10^{-9}\,\frac{1}{Pa}.$$

2. Invert the transformed compliance matrix $[\bar{S}]$ to obtain the transformed reduced stiffness matrix $[\bar{Q}]$:

$$[\bar{Q}] = \begin{bmatrix} 0.8053 \times 10^{-10} & -0.7878 \times 10^{-11} & -0.3234 \times 10^{-10} \\ -0.7878 \times 10^{-11} & 0.3475 \times 10^{-10} & -0.4696 \times 10^{-10} \\ -0.3234 \times 10^{-10} & -0.4696 \times 10^{-10} & 0.1141 \times 10^{-9} \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0.2365 \times 10^{11} & 0.3246 \times 10^{11} & 0.2005 \times 10^{11} \\ 0.3246 \times 10^{11} & 0.1094 \times 10^{12} & 0.5419 \times \\ 0.2005 \times 10^{11} & 0.5419 \times 10^{11} & 0.3674 \times \end{bmatrix}$$

3. The global strains in the x–y plane are given by Equation (2.105) as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 0.8053 \times 10^{-10} & -0.7878 \times 10^{-11} & -0.3234 \times 10^{-10} \\ -0.7878 \times 10^{-11} & 0.3475 \times 10^{-10} & -0.4696 \times 10^{-10} \\ -0.3234 \times 10^{-10} & -0.4696 \times 10^{-10} & 0.1141 \times 10^{-9} \end{bmatrix} \begin{bmatrix} 2 \times 10^{6} \\ -3 \times 10^{6} \\ 4 \times 10^{6} \end{bmatrix}$$

$$= \begin{bmatrix} 0.5534 \times 10^{-4} \\ -0.3078 \times 10^{-3} \\ 0.5328 \times 10^{-3} \end{bmatrix}.$$

4. Using transformation Equation (2.99), the local strains in the lamina are

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12}/2 \end{bmatrix} = \begin{bmatrix} 0.2500 & 0.7500 & 0.8660 \\ 0.7500 & 0.2500 & -0.8660 \\ -0.4330 & 0.4330 & -0.500 \end{bmatrix} \begin{bmatrix} 0.5534 \times 10^{-4} \\ -0.3078 \times 10^{-3} \\ 0.5328 \times 10^{-3}/2 \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.1367 \times 10^{-4} \\ -0.2662 \times 10^{-3} \\ -0.5809 \times 10^{-3} \end{bmatrix}.$$

5. Using transformation Equation (2.94), the local stresses in the lamina are

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} 0.2500 & 0.7500 & 0.8660 \\ 0.7500 & 0.2500 & -0.8660 \\ -0.4330 & 0.4330 & -0.500 \end{bmatrix} \begin{bmatrix} 2 \times 10^{6} \\ -3 \times 10^{6} \\ 4 \times 10^{6} \end{bmatrix}$$

$$= \begin{bmatrix} 0.1714 \times 10^{7} \\ -0.2714 \times 10^{7} \\ -0.4165 \times 10^{7} \end{bmatrix} Pa.$$

6. The principal normal stresses are given by[4]

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_x^2}$$

$$= \frac{2\times10^6 - 3\times10^6}{2} \pm \sqrt{\left(\frac{2\times10^6 + 3\times10^6}{2}\right)^2 + (4\times10^6)^2}$$

$$= 4.217, -5.217 \ MPa.$$

The value of the angle at which the maximum normal stresses occur is[4]

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \tag{2.108}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{2(4\times10^6)}{2\times10^6 + 3\times10^6}\right)$$

$$= 29.00^0 \ .$$

Note that the principal normal stresses do not occur along the material axes. This should be also evident from the nonzero shear stresses in the local axes.

7. The maximum shear stress is given by[4]

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(2.109)

$$= \sqrt{\left(\frac{2\times10^6 - 3\times10^6}{2}\right)^2 + (4\times10^6)^2}$$

$$= 4.717 \ MPa.$$

The angle at which the maximum shear stress occurs is[4]

$$\theta_s = \frac{1}{2}\tan^{-1}\left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(-\frac{2\times10^{6}+3\times10^{6}}{2(4\times10^{6})}\right)$$

$$= 16.00^{0}$$

8. The principal strains are given by[4]

$$\varepsilon_{\max,\min} = \frac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{0.5534\times10^{-4}+0.3078\times10^{-3}}{2} \pm \sqrt{\left(\frac{0.5534\times10^{-4}+0.3078\times10^{-3}}{2}\right)^2+\left(\frac{0.5328\times10^{-3}}{2}\right)^2} \tag{2.111}$$

$$= 1.962\times10^{-4}, -4.486\times10^{-4}.$$

The value of the angle at which the maximum normal strains occur is[4]

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{xy}}{\varepsilon_x-\varepsilon_y}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{0.5328\times10^{-3}}{0.5534\times10^{-4}+0.3078\times10^{-3}}\right) \tag{2.112}$$

$$= 27.86^{0}.$$

Note that the principal normal strains do not occur along the material axes. This should also be clear from the nonzero shear strain in the local axes. In addition, the axes of principal normal stresses and principal normal strains do not match, unlike in i

9. The maximum shearing strain is given by[4]

$$\gamma_{max} = \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

$$= \sqrt{(0.5534 \times 10^{-4} + 0.3078 \times 10^{-3})^2 + (0.532 \times 10^{-3})^2} \quad (2.113)$$

$$= 6.448 \times 10^{-4}.$$

The value of the angle at which the maximum shearing strain occurs is[4]

$$\theta_s = \frac{1}{2}\tan^{-1}\left(-\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(-\frac{0.5534 \times 10^{-4} + 0.3078 \times 10^{-3}}{0.5328 \times 10^{-3}}\right) \quad (2.114)$$

$$= -17.14^0.$$

Example 2.8

As shown in Figure 2.22, a 60° angle graphite/epoxy lamina is subjected only to a shear stress τ_{xy} = 2 MPa in the global axes. What would be the value of the strains measured by the strain gage rosette — that is, what

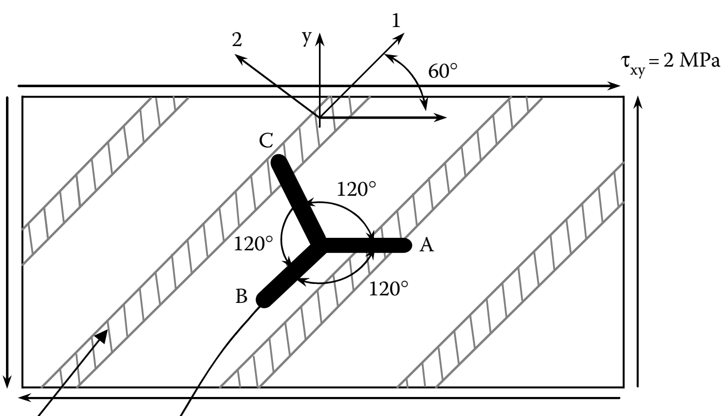


FIGURE 2.22
Strain gage rosette on an angle lamina.

would be the normal strains measured by strain gages A, B, and C? Use the properties of unidirectional graphite/epoxy lamina from Table 2.1.

Solution

Per Example 2.7, the reduced compliance matrix [$\bar{S}$] is

$$\begin{bmatrix} 0.8053\times10^{-10} & -0.7878\times10^{-11} & -0.3234\times10^{-10} \\ -0.7878\times10^{-11} & 0.3475\times10^{-10} & -0.4696\times10^{-10} \\ -0.3234\times10^{-10} & -0.4696\times10^{-10} & 0.1141\times10^{-9} \end{bmatrix} \frac{1}{Pa}.$$

The global strains in the x–y plane are given by Equation (2.105) as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 0.8053\times10^{-10} & -0.7878\times10^{-11} & -0.3234\times10^{-10} \\ -0.7878\times10^{-11} & 0.3475\times10^{-10} & -0.4696\times10^{-10} \\ -0.3234\times10^{-10} & -0.4696\times10^{-10} & 0.1141\times10^{-9} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2\times10^{6} \end{bmatrix}$$

$$= \begin{bmatrix} -6.468\times10^{-5} \\ -9.392\times10^{-5} \\ 2.283\times10^{-4} \end{bmatrix}.$$

For a strain gage placed at an angle, ϕ, to the x-axis, the normal strain recorded by the strain gage is given by Equation (B.15) in Appendix B.

$$\varepsilon_\phi = \varepsilon_x \text{Cos}^2\phi + \varepsilon_y \ \text{Sin}^2\phi + \gamma_{xy} \text{Sin}\,\phi \,\text{Cos}\,\phi .$$

For strain gage A, ϕ = 0°:

$$\varepsilon_A = -6.468\times10^{-5}\, Cos^2\, 0° + (-9.392\times10^{-5}) \text{Sin}^2\, 0° + 2.283\times10^{-4} \text{Sin}\, 0° \,\text{Cos}\, 0°$$

$$= -6.468\times10^{-5} .$$

For strain gage B, ϕ = 240°:

$$\varepsilon_B = -6.468\times10^{-5} \text{Cos}^2\, 240° + (-9.392\times10^{-5})$$

$$+ 2.283\times10^{-4} \text{Sin}\, 240° \,\text{Cos}\, 240°$$

$$= 1.724 \times 10^{-4} \, .$$

For strain gage C, ϕ = 120°:

$$\varepsilon_C = -6.468 \times 10^{-5} \, \text{Cos}^2 \, 120° + (-9.392 \times 10^{-5}) \, \text{Sin}^2 \, 120°$$

$$+ 2.283 \times 10^{-4} \, \text{Sin} \, 120° \, \text{Cos} \, 120°$$

$$= 1.083 \times 10^{-5} \, .$$

2.6 Engineering Constants of an Angle Lamina

The engineering constants for a unidirectional lamina were related to the compliance and stiffness matrices in Section 2.4.3. In this section, similar techniques are applied to relate the engineering constants of an angle ply to its transformed stiffness and compliance matrices.

1. For finding the engineering elastic moduli in direction x (Figure 2.23a), apply

$$\sigma_x \neq 0, \; \sigma_y = 0, \; \tau_{xy} = 0. \tag{2.115}$$

Then, from Equation (2.105),

$$\varepsilon_x = \bar{S}_{11} \sigma_x ,$$

$$\varepsilon_y = \bar{S}_{12} \sigma_x ,$$

$$\gamma_{xy} = \bar{S}_{16} \sigma_x \, . \tag{2.116a–c}$$

The elastic moduli in direction x is defined as

$$E_x \equiv \frac{\sigma_x}{\varepsilon_x} = \frac{1}{\bar{S}_{11}} .$$

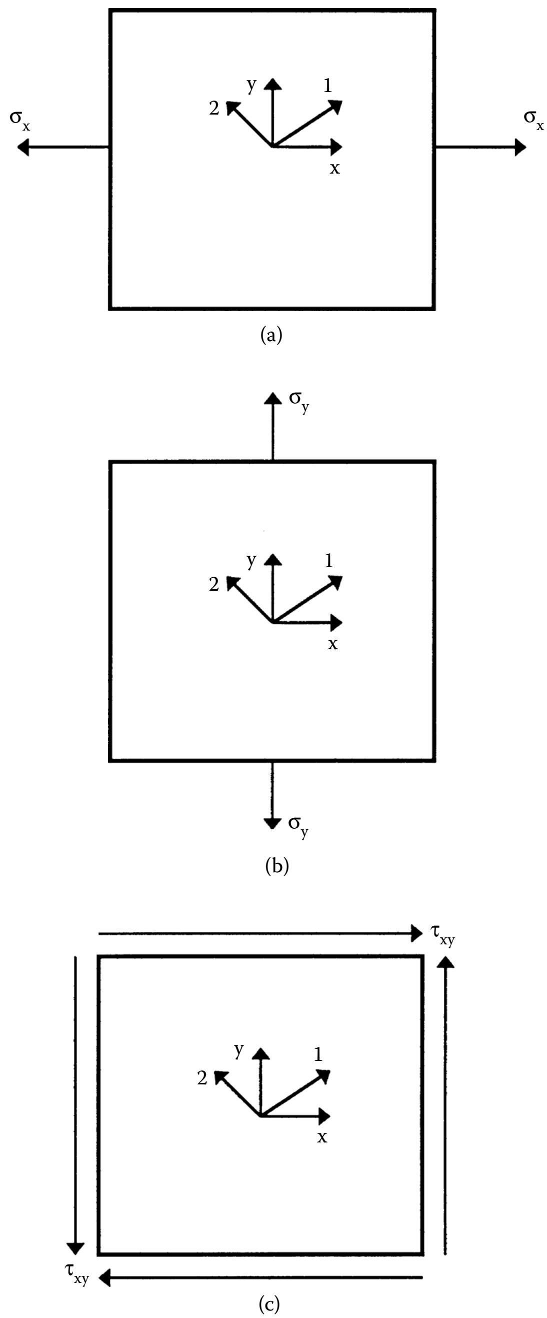


FIGURE 2.23
Application of stresses to find engineering constants of an angle lan

Also, the Poisson's ratio, ν_{xy}, is defined as

$$\nu_{xy} \equiv -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\bar{S}_{12}}{\bar{S}_{11}}. \tag{2.118}$$

In an angle lamina, unlike in a unidirectional lamina, interaction also occurs between the shear strain and the normal stresses. This is called shear coupling. The shear coupling term that relates the normal stress in the *x*-direction to the shear strain is denoted by m_x and is defined as

$$\frac{1}{m_x} \equiv -\frac{\sigma_x}{\gamma_{xy}E_1} = -\frac{1}{\bar{S}_{16}E_1}. \tag{2.119}$$

Note that m_x is a nondimensional parameter like the Poisson's ratio. Later, note that the same parameter, m_x, relates the shearing stress in the x–y plane to the normal strain in direction-*x*.

The shear coupling term is particularly important in tensile testing of angle plies. For example, if an angle lamina is clamped at the two ends, it will not allow shearing strain to occur. This will result in bending moments and shear forces at the clamped ends.[5]

2. Similarly, by applying stresses

$$\sigma_x = 0,\ \sigma_y \neq 0,\ \tau_{xy} = 0\ , \tag{2.120}$$

as shown in Figure 2.23b, it can be found

$$E_y = \frac{1}{\bar{S}_{22}}, \tag{2.121}$$

$$\nu_{yx} = -\frac{\bar{S}_{12}}{\bar{S}_{22}},\ \text{and} \tag{2.122}$$

$$\frac{1}{m_y} = -\frac{1}{\bar{S}_{26}E_1}. \tag{2.123}$$

The shear coupling term m_y relates the normal stre
strain γ_{xy}. In the following section (3), note that th
e shear stress τ_{xy} in the x–y plane to th

From Equation (2.117), Equation (2.118), Equation (2.121), and Equation (2.122), the reciprocal relationship is given by

$$\frac{\nu_{yx}}{E_y} = \frac{\nu_{xy}}{E_x} . \tag{2.124}$$

3. Also, by applying the stresses

$$\sigma_x = 0,\ \sigma_y = 0,\ \tau_{xy} \neq 0 , \tag{2.125}$$

as shown in Figure 2.23c, it is found that

$$\frac{1}{m_x} = -\frac{1}{\bar{S}_{16} E_1}, \tag{2.126}$$

$$\frac{1}{m_y} = -\frac{1}{\bar{S}_{26} E_1}, \text{ and} \tag{2.127}$$

$$G_{xy} = \frac{1}{\bar{S}_{66}}. \tag{2.128}$$

Thus, the strain–stress Equation (2.105) of an angle lamina can also be written in terms of the engineering constants of an angle lamina in matrix form as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{E_x} & -\dfrac{\nu_{xy}}{E_x} & -\dfrac{m_x}{E_1} \\ -\dfrac{\nu_{xy}}{E_x} & \dfrac{1}{E_y} & -\dfrac{m_y}{E_1} \\ -\dfrac{m_x}{E_1} & -\dfrac{m_y}{E_1} & \dfrac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}. \tag{2.129}$$

The preceding six engineering constants of an angle ply can also be written in terms of the engineering constants of a unidirectional ply using Equation (2.92) and Equation (2.106) in Equation (2.117) through Equation (2.119), Equation (2.121), Equation (2.123), and Equation (2.128):

$$\frac{1}{E_x} = \bar{S}_{11}$$

$$= S_{11}c^4 + (2S_{12} + S_{66})s^2c^2 + S_{22}s^4.$$

$$= \frac{1}{E_1}c^4 + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)s^2c^2 + \frac{1}{E_2}s^4, \tag{2.130}$$

$$\nu_{xy} = -E_x\bar{S}_{12}$$

$$= -E_x[S_{12}(s^4 + c^4) + (S_{11} + S_{22} - S_{66})s^2c^2]$$

$$= E_x\left[\frac{\nu_{12}}{E_1}\left(s^4 + c^4\right) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)s^2c^2\right], \tag{2.131}$$

$$\frac{1}{E_y} = \bar{S}_{22}$$

$$= S_{11}s^4 + (2S_{12} + S_{66})c^2s^2 + S_{22}c^4$$

$$= \frac{1}{E_1}s^4 + \left(-\frac{2\nu_{12}}{E_1} + \frac{1}{G_{12}}\right)c^2s^2 + \frac{1}{E_2}c^4, \tag{2.132}$$

$$\frac{1}{G_{xy}} = \bar{S}_{66}$$

$$= 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})s^2c^2 + S_{66}(s^4 + c^4)$$

$$= 2\left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)s^2c^2 + \frac{1}{G_{12}}(s^4 + c^4), \tag{2.133}$$

$$m_x = -\bar{S}_{16}E_1$$

$$= -E_1[(S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c]$$

$$= E_1\left[\left(-\frac{2}{E_1} - \frac{2\nu_{12}}{E_1} + \frac{1}{G_{12}}\right)sc^3 + \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \ldots\right.\right.$$

$$m_y = -\bar{S}_{26}E_1$$

$$= -E_1[(2S_{11} - 2S_{12} - S_{66})s^3c - (2S_{22} - 2S_{12} - S_{66})sc^3]$$

$$= E_1\left[\left(-\frac{2}{E_1} - \frac{2\nu_{12}}{E_1} + \frac{1}{G_{12}}\right)s^3c + \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)sc^3\right]. \tag{2.135}$$

Example 2.9

Find the engineering constants of a 60° graphite/epoxy lamina. Use the properties of a unidirectional graphite/epoxy lamina from Table 2.1.

Solution

From Example 2.7, we have

$$\bar{S}_{11} = 0.8053 \times 10^{-10}\,\frac{1}{Pa},$$

$$\bar{S}_{12} = -0.7878 \times 10^{-11}\,\frac{1}{Pa},$$

$$\bar{S}_{16} = -0.3234 \times 10^{-10}\,\frac{1}{Pa},$$

$$\bar{S}_{22} = 0.3475 \times 10^{-10}\,\frac{1}{Pa},$$

$$\bar{S}_{26} = -0.4696 \times 10^{-10}\,\frac{1}{Pa},\ \text{and}$$

$$\bar{S}_{66} = 0.1141 \times 10^{-9}\,\frac{1}{Pa}.$$

From Equation (2.117),

$$E_x = \frac{1}{0.8053 \times 10^{-10}}$$

$$= 12.42\,GPa.$$

From Equation (2.118),

$$\nu_{xy} = -\frac{-0.7878 \times 10^{-11}}{0.8053 \times 10^{-10}}$$

$$= 0.09783.$$

From Equation (2.119),

$$\frac{1}{m_x} = -\frac{1}{(-0.3234 \times 10^{-10})(181 \times 10^{9})}$$

$$m_x = 5.854 \, .$$

From Equation (2.121),

$$E_y = \frac{1}{0.3475 \times 10^{-10}}$$

$$= 28.78 \; GPa.$$

From Equation (2.123),

$$\frac{1}{m_y} = -\frac{1}{(-0.4696 \times 10^{-10})(181 \times 10^{9})}$$

$$m_y = 8.499.$$

From Equation (2.128),

$$G_{xy} = \frac{1}{0.1141 \times 10^{-9}}$$

$$= 8.761 \; GPa.$$

The variations of the six engineering elastic constants are shown as a function of the angle for the preceding graphite/epoxy
through Figure 2.29.

…f the Young's modulus, E_x and E_y are i…
…tation (angle of ply) varies from 0° t…

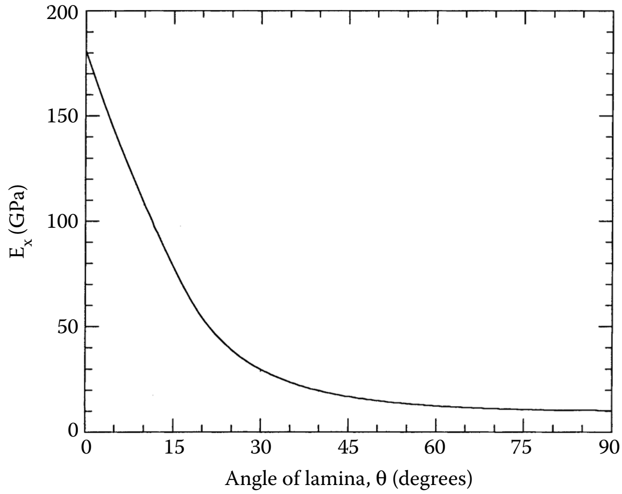


FIGURE 2.24
Elastic modulus in direction-x as a function of angle of lamina for a graphite/epoxy lamina.

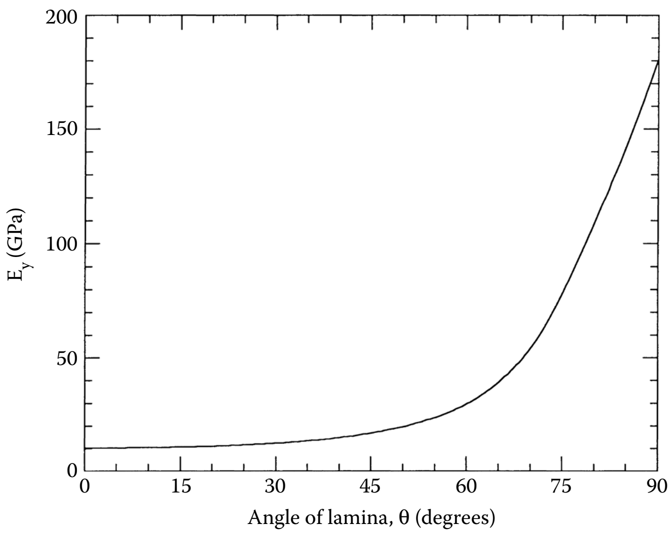


FIGURE 2.25
Elastic modulus in direction-y as a function of angle of lamina for a

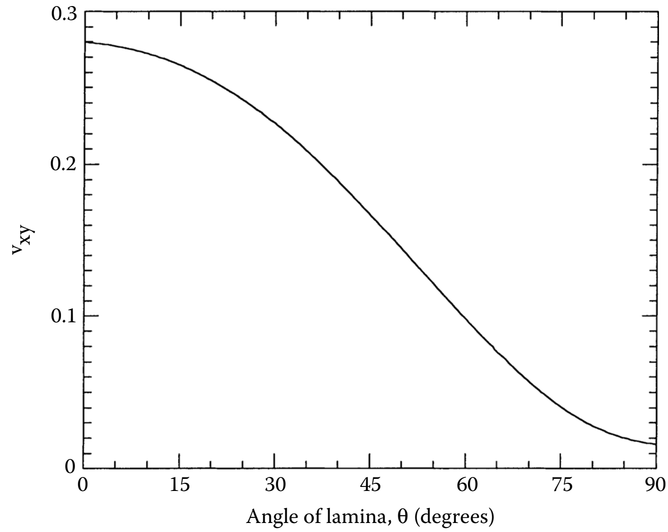


FIGURE 2.26
Poisson's ratio ν_{xy} as a function of angle of lamina for a graphite/epoxy lamina.

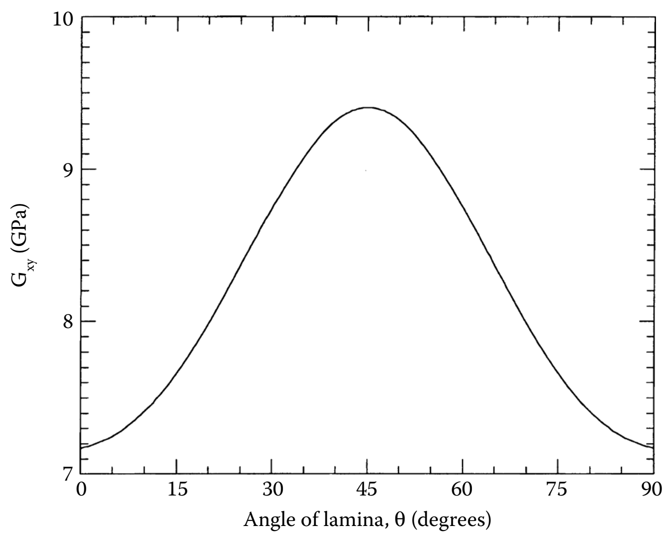


FIGURE 2.27
In-plane shear modulus in xy-plane as a function of angle of lamina fo

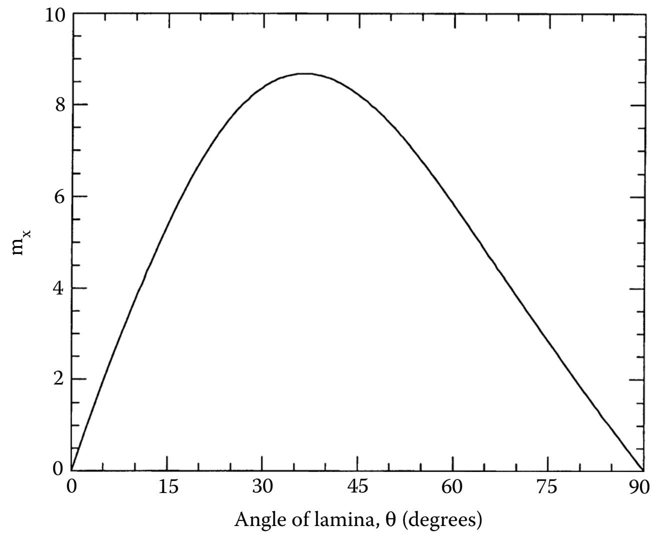


FIGURE 2.28
Shear coupling coefficient m_x as a function of angle of lamina for a graphite/epoxy lamina.

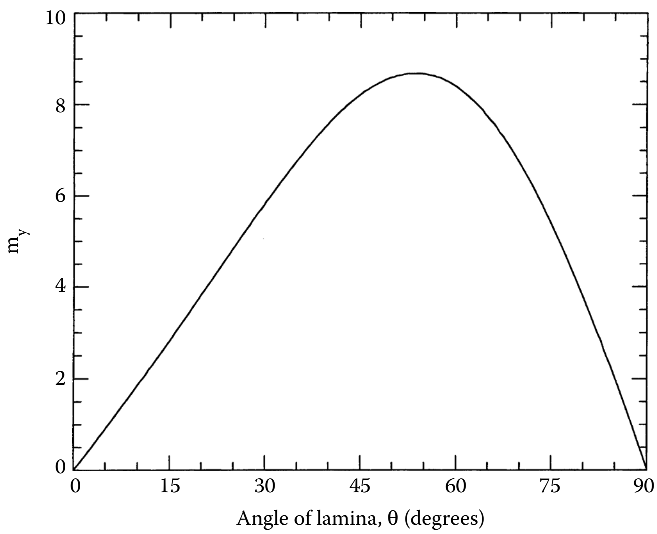


FIGURE 2.29
Shear coupling coefficient m_y as a function of angle of lamina for a

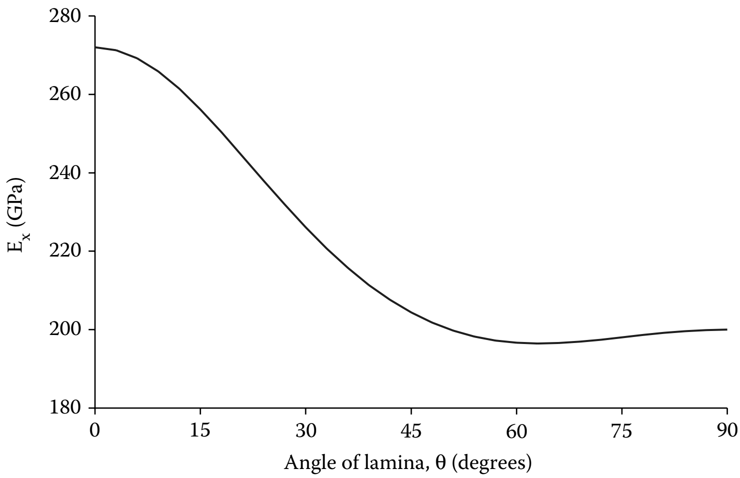


FIGURE 2.30
Variation of elastic modulus in direction-x as a function of angle of lamina for a typical $SCS - 6/Ti6 - Al - 4V$ lamina.

varies from the value of the longitudinal (E_1) to the transverse Young's modulus E_2. However, the maximum and minimum values of E_x do not necessarily exist for θ = 0° and θ = 90°, respectively, for every lamina.

Consider the case of a metal matrix composite such as a typical $SCS - 6/Ti6 - Al - 4V$ composite. The elastic moduli of such a lamina with a 55% fiber volume fraction is

$$E_1 = 272 \text{ GPa}$$

$$E_2 = 200 \text{ GPa}$$

$$\nu_{12} = 0.2770$$

$$G_{12} = 77.33 \text{ GPa}$$

In Figure 2.30, the lowest modulus value of E_x is found for θ = 63°. In fact, the angle of 63° at which E_x is minimum is independent of the fiber volume fraction, if one uses the "mechanics of materials approach" (Section 3.3.1) to evaluate the preceding four elastic moduli of a unidirectional lamina. See Exercise 3.13.

In Figure 2.27, the shear modulus G_{xy} is maximum for θ = 45° and is minimum for 0 and 90° plies. The shear modulus G_{xy} becomes maximum for 45° because the principal stresses for pure shear l
along the material axis.

(2.133), the expression for G_{xy} for a 4

$$G_{xy/45^\circ} = \frac{E_1}{\left(1 + 2\nu_{12} + \frac{E_1}{E_2}\right)}. \tag{2.136}$$

In Figure 2.28 and Figure 2.29, the shear coupling coefficients m_x and m_y are maximum at θ = 36.2° and θ = 53.78°, respectively. The values of these coefficients are quite extreme, showing that the normal-shear coupling terms have a stronger effect than the Poisson's effect. This phenomenon of shear coupling terms is missing in isotropic materials and unidirectional plies, but cannot be ignored in angle plies.

2.7 Invariant Form of Stiffness and Compliance Matrices for an Angle Lamina

Equation (2.104) and Equation (2.106) for the $[\bar{Q}]$ and $[\bar{S}]$ matrices are not analytically convenient because they do not allow a direct study of the effect of the angle of the lamina on the $[\bar{Q}]$ and $[\bar{S}]$ matrices. The stiffness elements can be written in invariant form as[6]

$$\bar{Q}_{11} = U_1 + U_2 \cos 2\theta + U_3 \cos 4\theta,$$

$$\bar{Q}_{12} = U_4 - U_3 \cos 4\theta,$$

$$\bar{Q}_{22} = U_1 - U_2 \cos 2\theta + U_3 \cos 4\theta$$

$$\bar{Q}_{16} = \frac{U_2}{2} \sin 2\theta + U_3 \sin 4\theta,$$

$$\bar{Q}_{26} = \frac{U_2}{2} \sin 2\theta - U_3 \sin 4\theta,$$

$$\bar{Q}_{66} = \frac{1}{2}(U_1 - U_4) - U_3 \cos 4\theta, \tag{2.137a–f}$$

where

$$U_1 = \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})$$

$$U_2 = \frac{1}{2}(Q_{11} - Q_{22}),$$

$$U_3 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}),$$

$$U_4 = \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}). \qquad (2.138a\text{–}d)$$

The terms U_1, U_2, U_3, and U_4 are the four invariants and are combinations of the Q_{ij}, which are invariants as well.

The transformed reduced compliance $[\bar{S}]$ matrix can similarly be written as

$$\bar{S}_{11} = V_1 + V_2 \text{Cos}\, 2\theta + V_3 \text{Cos}\, 4\theta,$$

$$\bar{S}_{12} = V_4 - V_3 \text{Cos}\, 4\theta,$$

$$\bar{S}_{22} = V_1 - V_2 \text{Cos}\, 2\theta + V_3 \text{Cos}\, 4\theta,$$

$$\bar{S}_{16} = V_2 \text{Sin}\, 2\theta + 2V_3 \text{Sin}\, 4\theta,$$

$$\bar{S}_{26} = V_2 \text{Sin}\, 2\theta - 2V_3 \text{Sin}\, 4\theta, \text{ and}$$

$$\bar{S}_{66} = 2(V_1 - V_4) - 4V_3 \text{Cos}\, 4\theta, \qquad (2.139a\text{–}f)$$

where

$$V_1 = \frac{1}{8}(3S_{11} + 3S_{22} + 2S_{12} + S_{66}),$$

$$V_2 = \frac{1}{2}(S_{11} - S_{22}),$$

$$V_3 = \frac{1}{8}(S_{11} + S_{22} - 2S_{12} - S_{66}),$$

$$V_4 = \frac{1}{8}(S_{11} + S_{22} + 6S_{12} - S_{66}). \qquad (2.140a–d)$$

The terms V_1, V_2, V_3, and V_4 are invariants and are combinations of S_{ij}, which are also invariants.

The main advantage of writing the equations in this form is that one can easily examine the effect of the lamina angle on the reduced stiffness matrix elements. Also, formulas given by Equation (2.137) and Equation (2.139) are easier to manipulate for integration, differentiation, etc. The concept is mainly important in deriving the laminate stiffness properties in Chapter 4.

The elastic moduli of quasi-isotropic laminates that behave like isotropic material are directly given in terms of these invariants. Because quasi-isotropic laminates have the minimum stiffness of any laminate, these can be used as a comparative measure of the stiffness of other types of laminates.[7]

Example 2.10

Starting with the expression for $\bar{Q}_{11}$ from Equation (2.104a), $\bar{Q}_{11} = Q_{11}\,\mathrm{Cos}^4\,\theta,$ $+ Q_{22}\,\mathrm{Sin}^4\,\theta + 2(Q_{12} + 2Q_{66})\mathrm{Sin}^2\,\theta\,\mathrm{Cos}^2\,\theta$, reduce it to the expression for $\bar{Q}_{11}$ of Equation (2.137a) — that is,

$$\bar{Q}_{11} = U_1 + U_2\,\mathrm{Cos}\,2\theta + U_3\,\mathrm{Cos}\,4\theta$$

Solution

Given

$$\bar{Q}_{11} = Q_{11}\,\mathrm{Cos}^4\,\theta + Q_{22}\,\mathrm{Sin}^4\,\theta + 2(Q_{12} + 2Q_{66})\mathrm{Sin}^2\,\theta\,\mathrm{Cos}^2\,\theta\,,$$

and substituting

$$\mathrm{Cos}^2\,\theta = \frac{1 + \mathrm{Cos}\,2\theta}{2},$$

$$\mathrm{Sin}^2\,\theta = \frac{1 - \mathrm{Cos}\,2\theta}{2},$$

$$\text{Cos}^2 2\theta = \frac{1+\text{Cos}\,4\theta}{2}, \text{ and}$$

$$2\,\text{Sin}\,\theta\,\text{Cos}\,\theta = \text{Sin}\,2\theta,$$

$$\text{Sin}^2 2\theta = \frac{1-\text{Cos}\,4\theta}{2},$$

we get

$$\bar{Q}_{11} = U_1 + U_2\,\text{Cos}\,2\theta + U_3\,\text{Cos}\,4\theta\ ,$$

where

$$U_1 = \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}),$$

$$U_2 = \frac{1}{2}(Q_{11} - Q_{22})$$

$$U_3 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})\ .$$

Example 2.11

Evaluate the four compliance and four stiffness invariants for a graphite/epoxy angle lamina. Use the properties for a unidirectional graphite/epoxy lamina from Table 2.1.

Solution

From Example 2.6, the compliance matrix [S] elements are

$$S_{11} = 0.5525 \times 10^{-11}\,\frac{1}{Pa},$$

$$S_{12} = -0.1547 \times 10^{-11}\,\frac{1}{Pa},$$

$$S_{22} = 0.9709 \times 10^{-10} \frac{1}{Pa},$$

$$S_{66} = 0.1395 \times 10^{-9} \frac{1}{Pa}.$$

The stiffness matrix $[Q]$ elements are

$$[Q] = [S]^{-1},$$

$$Q_{11} = 0.1818 \times 10^{12}\ Pa,$$

$$Q_{12} = 0.2897 \times 10^{10}\ Pa,$$

$$Q_{22} = 0.1035 \times 10^{11}\ Pa,$$

$$Q_{66} = 0.7170 \times 10^{10}\ Pa.$$

Using Equation (2.138),

$$U_1 = \frac{1}{8}[3(0.1818 \times 10^{12}) + 3(0.1035 \times 10^{11}) + 2(0.2897 \times 10^{10}) + 4(0.7171 \times 10^{10})]$$

$$= 0.7637 \times 10^{11}\ Pa,$$

$$U_2 = \frac{1}{2}(0.1818 \times 10^{12} - 0.1035 \times 10^{11})$$

$$= 0.8573 \times 10^{11}\ Pa,$$

$$U_3 = \frac{1}{8}[0.1818 \times 10^{12} + 0.1035 \times 10^{11} - 2(0.2897 \times 10^{10}) - 4(0.7171 \times 10^{10})]$$

$$= 0.1971 \times 10^{11}\ Pa,$$

$$U_4 = \frac{1}{8}[0.1818 \times 10^{12} + 0.1035 \times 10^{11} + 6(0.2897 \times 10^{10})$$

$$= 0.2261 \times 10^{11}\ Pa.$$

Using Equation (2.140),

$$V_1 = \frac{1}{8}[3(0.5525\times10^{-11}) + 3(-0.1547\times10^{-11}) + 2(0.9709\times10^{-10}) + 0.1395\times10^{-9}]$$

$$= 0.5553\times10^{-10}\,\frac{1}{Pa},$$

$$V_2 = \frac{1}{2}[(0.5525\times10^{-11} - (-0.1547\times10^{-11})]$$

$$= -0.4578\times10^{-10}\,\frac{1}{Pa},$$

$$V_3 = \frac{1}{8}[0.5525\times10^{-11} + 0.9709\times10^{-10} - 2(0.1547\times10^{-11}) - 0.1395\times10^{-9}]$$

$$= -0.4220\times10^{-11}\,\frac{1}{Pa},$$

$$V_4 = \frac{1}{8}[0.5525\times10^{-11} + 0.9709\times10^{-10} + 6(0.1547\times10^{-11}) - 0.1395\times10^{-9}]$$

$$= -0.5767\times10^{-11}\,\frac{1}{Pa}.$$

2.8 Strength Failure Theories of an Angle Lamina

A successful design of a structure requires efficient and safe use of materials. Theories need to be developed to compare the state of stress in a material to failure criteria. It should be noted that failure theories are only stated and their application is validated by experiments.

For a laminate, the strength is related to the strength of each individual lamina. This allows for a simple and economical method for finding the strength of a laminate. Various theories have been developed for studying the failure of an angle lamina. The theories are generally based on the normal and shear strengths of a unidirectional lamina.

An isotropic material, such as steel, generally has two strength parameters: normal strength and shear strength. In some cases, suc cast iron, the normal strengths are different in the tensi theory for an isotropic material is b stresses and the maximum shear stres stresses, if greater than any of the corresponding ultimate strengths, indicate failure in the material.

Example 2.12

A cylindrical rod made of gray cast iron is subjected to a uniaxial tensile load, *P*. Given:

Cross-sectional area of rod = 2 in.^{2}

Ultimate tensile strength = 25 ksi

Ultimate compressive strength = 95 ksi

Ultimate shear strength = 35 ksi

Modulus of elasticity = 10 Msi

Find the maximum load, *P*, that can be applied using maximum stress failure theory.

Solution

At any location, the stress state in the rod is $\sigma = P/2$. From a typical Mohr's circle analysis, the maximum principal normal stress is $P/2$. The maximum shear stress is $P/4$ and acts at a cross-section 45° to the plane of maximum normal stress. Comparing these maximum stresses to the corresponding ultimate strengths, we have

$$\frac{P}{2} < 25 \times 10^3 \ \text{or} \ P < 50,000 \text{ lb},$$

and

$$\frac{P}{4} < 35 \times 10^3 \ \text{or} \ P < 140,000 \text{ lb}.$$

Thus, the maximum load is 50,000 lb.

However, in a lamina, the failure theories are not based on principal normal stresses and maximum shear stresses. Rather, they are based on the stresses in the material or local axes because a lamina is orthotropic and its properties are different at different angles, unlike an isotropic material.

In the case of a unidirectional lamina, there are two material axes: one parallel to the fibers and one perpendicular to the fibers. Thus, there are four normal strength parameters for a unidirectional lamina one for compression, in each of the two material axes er is the shear strength of a unidirectio ositive or negative, does not have an e shear strengths of a unidirectional lamina. However, we will find later that the sign of the shear stress does affect the strength of an angle lamina. The five strength parameters of a unidirectional lamina are therefore

$(\sigma_1^T)_{ult}$ = Ultimate longitudinal tensile strength (in direction 1),
$(\sigma_1^C)_{ult}$ = Ultimate longitudinal compressive strength (in direction 1),
$(\sigma_2^T)_{ult}$ = Ultimate transverse tensile strength (in direction 2),
$(\sigma_2^C)_{ult}$ = Ultimate transverse compressive strength (in direction 2), and
$(\tau_{12})_{ult}$ = Ultimate in-plane shear strength (in plane 12).

Unlike the stiffness parameters, these strength parameters cannot be transformed directly for an angle lamina. Thus, the failure theories are based on first finding the stresses in the local axes and then using these five strength parameters of a unidirectional lamina to find whether a lamina has failed. Four common failure theories are discussed here. Related concepts of strength ratio and the development of failure envelopes are also discussed.

2.8.1 Maximum Stress Failure Theory

Related to the maximum normal stress theory by Rankine and the maximum shearing stress theory by Tresca, this theory is similar to those applied to isotropic materials. The stresses acting on a lamina are resolved into the normal and shear stresses in the local axes. Failure is predicted in a lamina, if any of the normal or shear stresses in the local axes of a lamina is equal to or exceeds the corresponding ultimate strengths of the unidirectional lamina.

Given the stresses or strains in the global axes of a lamina, one can find the stresses in the material axes by using Equation (2.94). The lamina is considered to be failed if

$$-(\sigma_1^C)_{ult} < \sigma_1 < (\sigma_1^T)_{ult}, \ or$$

$$-(\sigma_2^C)_{ult} < \sigma_2 < (\sigma_2^T)_{ult}, \ or$$

$$-(\tau_{12})_{ult} < \tau_{12} < (\tau_{12})_{ult} \qquad (2.141a–c)$$

is violated. Note that all five strength parameters ar
numbers, and the normal stresses are positive if ter
compressive

nt of stress is compared with the cor
nent of stress does not interact with

Example 2.13

Find the maximum value of $S > 0$ if a stress of $\sigma_x = 2S$, $\sigma_y = -3S$, and $\tau_{xy} = 4S$ is applied to the 60° lamina of graphite/epoxy. Use maximum stress failure theory and the properties of a unidirectional graphite/epoxy lamina given in Table 2.1.

Solution

Using Equation (2.94), the stresses in the local axes are

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} 0.2500 & 0.7500 & 0.8660 \\ 0.7500 & 0.2500 & -0.8660 \\ -0.4330 & 0.4330 & -0.5000 \end{bmatrix} \begin{bmatrix} 2S \\ -3S \\ 4S \end{bmatrix}$$

$$= \begin{bmatrix} 0.1714 \times 10^1 \\ -0.2714 \times 10^1 \\ -0.4165 \times 10^1 \end{bmatrix} S.$$

From Table 2.1, the ultimate strengths of a unidirectional graphite/epoxy lamina are

$$(\sigma_1^T)_{ult} = 1500 \text{ MPa}$$

$$(\sigma_1^C)_{ult} = 1500 \text{ MPa}$$

$$(\sigma_2^T)_{ult} = 40 \text{ MPa}$$

$$(\sigma_2^C)_{ult} = 246 \text{ MPa}$$

$$(\tau_{12})_{ult} = 68 \text{ MPa}$$

Then, using the inequalities (2.141) of the maximum stress failure theory,

$$-1500 \times 10^6 < 0.1714 \times 10^1 S < 1500 \times 10^6$$

$$-246 \times 10^6 < -0.2714 \times 10^1 S < 40 \times 10^6$$

$$-68 \times 10^6 < -0.4165 \times 10^1 S < 68 \times 1$$

$$-875.1 \times 10^6 < S < 875.1 \times 10^6$$

$$-14.73 \times 10^6 < S < 90.64 \times 10^6$$

$$-16.33 \times 10^6 < S < 16.33 \times 10^6.$$

All the inequality conditions (and $S > 0$) are satisfied if $0 < S < 16.33$ MPa. The preceding inequalities also show that the angle lamina will fail in shear. The maximum stress that can be applied before failure is

$$\sigma_x = 32.66\,MPa, \sigma_y = -48.99\,MPa, \tau_{xy} = 65.32\,MPa.$$

Example 2.14

Find the off-axis shear strength of a 60° graphite/epoxy lamina. Use the properties of unidirectional graphite/epoxy from Table 2.1 and apply the maximum stress failure theory.

Solution

The off-axis shear strength of a lamina is defined as the minimum of the magnitude of positive and negative shear stress (Figure 2.31) that can be applied to an angle lamina before failure.

Assume the following stress state

$$\sigma_x = 0,\ \sigma_y = 0,\ \tau_{xy} = \tau.$$

Then, using the transformation Equation (2.94),

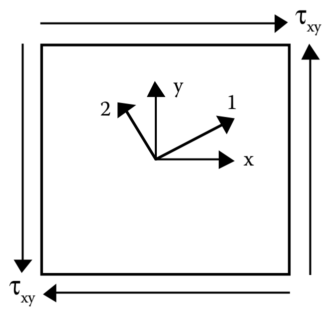
$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} 0.2500 & 0.7500 & 0.8660 \\ 0.7500 & 0.2500 & -0.8660 \\ -0.4330 & 0.4330 & -0.5000 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \tau \end{bmatrix}$$

$$\sigma_1 = 0.866\tau$$

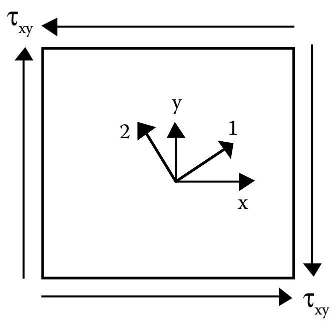
$$\sigma_2 - 0.866\tau$$

$$\tau_{12} = -0.500\tau\ .$$

ualities (2.141) of the maximum stre



(a) Positive shear stress



(b) Negative shear stress

FIGURE 2.31
Positive and negative shear stresses applied to an angle lamina.

$$-1500 < 0.866\tau < 1500 \text{ or } -1732 < \tau < 1732$$

$$-246 < -0.866\tau < 40 \text{ or } -46.19 < \tau < 284.1$$

$$-68 < -0.500\tau < 68 \text{ or } -136.0 < \tau < 136.0,$$

which shows that τ_{xy} = 46.19 MPa is the largest magnitude of shear stress that can be applied to the 60° graphite/epoxy lamina. However, the largest positive shear stress that could be applied is τ_{xy} = 136.0 MPa, and the largest negative shear stress is τ_{xy} = –46.19 MPa.

This shows that the maximum magnitude of allowable shear stress in other than the material axes' direction depends on the sign of the shear stress. This is mainly because the local axes' stresses in the direction perpendicular to the fibers are opposite in sign to each other for opposite signs of shear stress ($\sigma_2 = -0.866\tau$ for positive τ_{xy} and $\sigma_2 = 0.866\tau$ for negative τ_{xy}). Because the tensile strength perpendicular to the fiber direction is compressive strength perpendicular to the fiber direct lifferent.

TABLE 2.3

Effect of Sign of Shear Stress as a Function of Angle of Lamina

Angle, Degrees	Positive τ_{xy} MPa	Negative τ_{xy} MPa	Shear strength MPa
0	68.00 (S)	68.00 (S)	68.00
15	78.52 (S)	78.52 (S)	78.52
30	136.0 (S)	46.19 (2T)	46.19
45	246.0 (2C)	40.00 (2T)	40.00
60	136.0 (S)	46.19 (2T)	46.19
75	78.52 (S)	78.52 (S)	78.52
90	68.00 (S)	68.00 (S)	68.00

Note: The notation in the parentheses denotes the mode of failure of the angle lamina as follows:
(1T) — longitudinal tensile failure;
(1C) — longitudinal compressive failure;
(2T) — transverse tensile failure;
(2C) — transverse compressive failure;
(S) — shear failure.

Table 2.3 shows the maximum negative and positive values of shear stress that can be applied to different angle plies of graphite/epoxy of Table 2.1. The minimum magnitude of the two stresses is the shear strength of the angle lamina.

2.8.2 Strength Ratio

In a failure theory such as the maximum stress failure theory of Section 2.8.1, it can be determined whether a lamina has failed if any of the inequalities of Equation (2.141) are violated. However, this does not give the information about how much the load can be increased if the lamina is safe or how much the load should be decreased if the lamina has failed. The definition of strength ratio (SR) is helpful here. The strength ratio is defined as

$$SR = \frac{Maximum\ Load\ Which\ Can\ Be\ Applied}{Load\ Applied}. \tag{2.142}$$

The concept of strength ratio is applicable to any failure theory. If SR > 1, then the lamina is safe and the applied stress can be increased by a factor of SR. If SR < 1, the lamina is unsafe and the applied stress needs to be reduced by a factor of SR. A value of SR = 1 implies the failure load.

Example 2.15

is applying a load of

$$\sigma_x = 2\,MPa, \sigma_y = -3\,MPa, \tau_{xy} = 4\,MPa$$

to a 60° angle lamina of graphite/epoxy. Find the strength ratio using the maximum stress failure theory.

Solution

If the strength ratio is R, then the maximum stress that can be applied is

$$\sigma_x = 2R,\ \sigma_y = -3R,\ \tau_{xy} = 4R\ .$$

Following Example 2.13 for finding the local stresses gives

$$\sigma_1 0.1714 \times 10^1\, R$$

$$\sigma_2 = -0.2714 \times 10^1\, R$$

$$\tau_{12} = -0.4165 \times 10^1\, R\ .$$

Using the maximum stress failure theory as given by Equation (2.141) yields

$$R = 16.33.$$

Thus, the load that can be applied just before failure is

$$\sigma_x = 16.33 \times 2\ MPa,\ \sigma_y = 16.33 \times (-3)\ MPa,\ \tau_{xy} = 16.33 \times 4\ Mpa,$$

$$\sigma_x = 32.66\ MPa,\ \sigma_y = -48.99\ MPa,\ \tau_{xy} = 65.32\ MPa.$$

Note that all the components of the stress vector must be multiplied by the strength ratio.

2.8.3 Failure Envelopes

A failure envelope is a three-dimensional plot of the combinations of the normal and shear stresses that can be applied to an ang[...]
failure. Because drawing three dimensional graphs car[...]
one may develop failure envelopes for constant shear s[...]
[...]resses σ_x and σ_y as the two axes. Ther[...]
[...]re envelope, the lamina is safe; other[...]

Example 2.16

Develop a failure envelope for the 60° lamina of graphite/epoxy for a constant shear stress of τ_{xy} = 24 MPa. Use the properties for the unidirectional graphite/epoxy lamina from Table 2.1.

Solution

From Equation (2.94), the stresses in the local axes for a 60° lamina are given by

$$\sigma_1 = 0.2500\,\sigma_x + 0.7500\,\sigma_y + 20.78\,MPa,$$

$$\sigma_2 = 0.7500\,\sigma_x + 0.2500\,\sigma_y - 20.78\,MPa,$$

$$\tau_{12} = -0.4330\,\sigma_x + 0.4330\,\sigma_y - 12.00\,MPa,$$

where σ_x and σ_y are also in units of MPa.

Using the preceding inequalities,

$$-1500 < 0.2500\,\sigma_x + 0.7500\,\sigma_y + 20.78 < 1500$$

$$-246 < 0.7500\,\sigma_x + 0.2500\,\sigma_y - 20.78 < 40$$

$$-68 < -0.4330\,\sigma_x + 0.4330\,\sigma_y - 12.00 < 68\,.$$

Various combinations of (σ_x, σ_y) can be found to satisfy the preceding inequalities. However, the objective is to find the points on the failure envelope. These are combinations of σ_x and σ_y, where one of the three inequalities is just violated and the other two are satisfied. Some of the values of (σ_x, σ_y) obtained on the failure envelope are given in Table 2.4.

Several methods can be used to obtain the points on the failure envelope for a constant shear stress. One way is to fix the value of σ_x and find the maximum value of σ_y that can be applied without violating any of the conditions. For example, for σ_x = 100 MPa, from the inequalities we have

$$-2061 < \sigma_y < 1939,$$

$$-1201 < \sigma_y < -56.88,$$

$$-29.33 < \sigma_y < 284.80.$$

TABLE 2.4

Typical Values of (σ_x, σ_y) on the Failure Envelope for Example 2.16

σ_x (MPa)	σ_y (MPa)
50.0	93.1
50.0	–79.3
–50.0	179
–50.0	–135
25.0	168
25.0	–104
–25.0	160
–25.0	–154

The preceding three inequalities show no allowable value of σ_y for this value of σ_x = 100 MPa.

As another example, for σ_x = 50 MPa, we have from inequalities,

$$-2044 < \sigma_y < 1956,$$

$$-1051 < \sigma_y < 93.12,$$

$$-79.33 < \sigma_y < 234.80.$$

The preceding three inequalities show two maximum allowable values of the normal stress, σ_y. These are σ_y = 93.12 MPa and σ_y = –79.33 MPa. The failure envelope for τ_{xy} = 24 MPa is shown in Figure 2.32.

2.8.4 Maximum Strain Failure Theory

This theory is based on the maximum normal strain theory by St. Venant and the maximum shear stress theory by Tresca as applied to isotropic materials. The strains applied to a lamina are resolved to strains in the local axes. Failure is predicted in a lamina, if any of the normal or shearing strains in the local axes of a lamina equal or exceed the corresponding ultimate strains of the unidirectional lamina. Given the strains/stresses in an angle lamina, one can find the strains in the local axes. A lamina is considered to be failed if

$$-(\varepsilon_1^C)_{ult} < \varepsilon_1 < (\varepsilon_1^T)_{ult}, \; or$$

$$-(\varepsilon_2^C)_{ult} < \varepsilon_2 < (\varepsilon_2^T)_{ult}, \; or$$

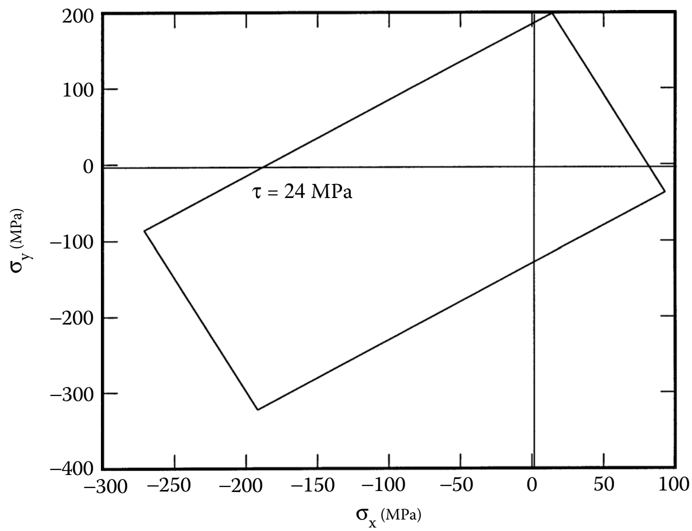


FIGURE 2.32
Failure envelopes for constant shear stress using maximum stress failure theory.

$$-(\gamma_{12})_{ult} < \gamma_{12} < (\gamma_{12})_{ult} \qquad (2.143a\text{–}c)$$

is violated, where

$(\varepsilon_1^T)_{ult}$ = ultimate longitudinal tensile strain (in direction 1)
$(\varepsilon_1^C)_{ult}$ = ultimate longitudinal compressive strain (in direction 1)
$(\varepsilon_2^T)_{ult}$ = ultimate transverse tensile strain (in direction 2)
$(\varepsilon_2^C)_{ult}$ = ultimate transverse compressive strain (in direction 2)
$(\gamma_{12})_{ult}$ = ultimate in-plane shear strain (in plane 1–2)

The ultimate strains can be found directly from the ultimate strength parameters and the elastic moduli, assuming the stress–strain response is linear until failure. The maximum strain failure theory is similar to the maximum stress failure theory in that no interaction occurs between various components of strain. However, the two failure theories give different results because the local strains in a lamina include the Poisson's ratio effect. In fact, if the Poisson's ratio is zero in the unidirectional lamina, the two failure theories will give identical results.

Example 2.17

Find the maximum value of $S > 0$ if a stress, $\sigma_x = 2S$, σ
0° graphite/epoxy lamina. Use ma

theory. Use the properties of the graphite/epoxy unidirectional lamina given in Table 2.1.

Solution

In Example 2.6, the compliance matrix [S] was obtained and, in Example 2.13, the local stresses for this problem were obtained. Then, from Equation (2.77),

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [S] \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

$$= \begin{bmatrix} 0.5525\times10^{-11} & -0.1547\times10^{-11} & 0 \\ -0.1547\times10^{-11} & 0.9709\times10^{-10} & 0 \\ 0 & 0 & 0.1395\times10^{-9} \end{bmatrix} \begin{bmatrix} 0.1714\times10^{1} \\ -0.2714\times10^{1} \\ -0.4165\times10^{1} \end{bmatrix} S$$

$$= \begin{bmatrix} 0.1367\times10^{-10} \\ -0.2662\times10^{-9} \\ -0.5809\times10^{-9} \end{bmatrix} S.$$

Assume a linear relationship between all the stresses and strains until failure; then the ultimate failure strains are

$$(\varepsilon_1^T)_{ult} = \frac{(\sigma_1^T)_{ult}}{E_1} = \frac{1500\times10^6}{181\times10^9} = 8.287\times10^{-3},$$

$$(\varepsilon_1^C)_{ult} = \frac{(\sigma_1^C)_{ult}}{E_1} = \frac{1500\times10^6}{181\times10^9} = 8.287\times10^{-3},$$

$$(\varepsilon_2^T)_{ult} = \frac{(\sigma_2^T)_{ult}}{E_2} = \frac{40\times10^6}{10.3\times10^9} = 3.883\times10^{-3},$$

$$(\varepsilon_2^C)_{ult} = \frac{(\sigma_2^C)_{ult}}{E_2} = \frac{246\times10^6}{10.3\times10^9} = 2.388\times1($$

$$(\gamma_{12})_{ult} = \frac{(\tau_{12})_{ult}}{G_{12}} = \frac{68 \times 10^6}{7.17 \times 10^6} = 9.483 \times 10^{-3}.$$

The preceding values for the ultimate strains also assume that the compressive and tensile stiffnesses are identical. Using the inequalities (2.143) and recognizing that $S > 0$,

$$-8.287 \times 10^{-3} < 0.1367 \times 10^{-10} S < 8.287 \times 10^{-3},$$

$$-2.388 \times 10^{-2} < -0.2662 \times 10^{-9} S < 3.883 \times 10^{-3},$$

$$-9.483 \times 10^{-3} < -0.5809 \times 10^{-9} S < 9.483 \times 10^{-3},$$

or

$$-606.2 \times 10^6 < S < 606.2 \times 10^6,$$

$$-14.58 \times 10^6 < S < 89.71 \times 10^6$$

$$-16.33 \times 10^6 < S < 16.33 \times 10^6,$$

which give

$$0 < S < 16.33 \, MPa.$$

The maximum value of S before failure is 16.33 MPa. The same maximum value of S = 16.33 MPa is also found using maximum stress failure theory. There is no difference between the two values because the mode of failure is shear. However, if the mode of failure were other than shear, a difference in the prediction of failure loads would have been present due to the Poisson's ratio effect, which couples the normal strains and stresses in the local axes.

Neither the maximum stress failure theory nor the maximum strain failure theory has any coupling among the five possible modes of failure. The following theories are based on the interaction failure theory.

2.8.5 Tsai–Hill Failure Theory

...sed on the distortion energy failure ...
...y yield criterion for isotropic material...

tropic materials. Distortion energy is actually a part of the total strain energy in a body. The strain energy in a body consists of two parts; one due to a change in volume and is called the dilation energy and the second is due to a change in shape and is called the distortion energy. It is assumed that failure in the material takes place only when the distortion energy is greater than the failure distortion energy of the material. Hill[8] adopted the Von-Mises' distortional energy yield criterion to anisotropic materials. Then, Tsai[7] adapted it to a unidirectional lamina. Based on the distortion energy theory, he proposed that a lamina has failed if

$$(G_2+G_3)\sigma_1^2+(G_1+G_3)\sigma_2^2+(G_1+G_2)\sigma_3^2-2G_3\sigma_1\sigma_2-2G_2\sigma_1\sigma_3 \tag{2.144}$$

$$-\,2G_1\sigma_2\sigma_3+2G_4\tau_{23}^2+2G_5\tau_{13}^2+2G_6\tau_{12}^2<1$$

is violated. The components G_1, G_2, G_3, G_4, G_5, and G_6 of the strength criterion depend on the failure strengths and are found as follows.

1. Apply $\sigma_1=(\sigma_1^T)_{ult}$ to a unidirectional lamina; then, the lamina will fail. Thus, Equation (2.144) reduces to

$$(G_2+G_3)(\sigma_1^T)_{ult}^2=1. \tag{2.145}$$

2. Apply $\sigma_2=(\sigma_2^T)_{ult}$ to a unidirectional lamina; then, the lamina will fail. Thus, Equation (2.144) reduces to

$$(G_1+G_3)(\sigma_2^T)_{ult}^2=1. \tag{2.146}$$

3. Apply $\sigma_3=(\sigma_2^T)_{ult}$ to a unidirectional lamina and, assuming that the normal tensile failure strength is same in directions (2) and (3), the lamina will fail. Thus, Equation (2.144) reduces to

$$(G_1+G_2)(\sigma_2^T)_{ult}^2=1. \tag{2.147}$$

4. Apply τ_{12} = $(\tau_{12})_{ult}$ to a unidirectional lamina; then, the lamina will fail. Thus, Equation (2.144) reduces to

$$2G_6(\tau_{12})_{ult}^2=1. \tag{2.148}$$

From Equation (2.145) to Equation (2.148),

$$G_1 = \frac{1}{2}\left(\frac{2}{[(\sigma_2^T)_{ult}]^2} - \frac{1}{[(\sigma_1^T)_{ult}]^2}\right),$$

$$G_2 = \frac{1}{2}\left(\frac{1}{[(\sigma_1^T)_{ult}]^2}\right),$$

$$G_3 = \frac{1}{2}\left(\frac{1}{[(\sigma_1^T)_{ult}]^2}\right),$$

$$G_6 = \frac{1}{2}\left(\frac{1}{[(\tau_{12})_{ult}]^2}\right). \tag{2.149a–d}$$

Because the unidirectional lamina is assumed to be under plane stress — that is, $\sigma_3 = \tau_{31} = \tau_{23} = 0$, then Equation (2.144) reduces through Equation (2.149) to

$$\left[\frac{\sigma_1}{(\sigma_1^T)_{ult}}\right]^2 - \left[\frac{\sigma_1\sigma_2}{(\sigma_1^T)_{ult}^2}\right] + \left[\frac{\sigma_2}{(\sigma_2^T)_{ult}}\right]^2 + \left[\frac{\tau_{12}}{(\tau_{12})_{ult}}\right]^2 < 1. \tag{2.150}$$

Given the global stresses in a lamina, one can find the local stresses in a lamina and apply the preceding failure theory to determine whether the lamina has failed.

Example 2.18

Find the maximum value of $S > 0$ if a stress of $\sigma_x = 2S$, $\sigma_y = -3S$, and $\tau_{xy} = 4S$ is applied to a 60° graphite/epoxy lamina. Use Tsai–Hill failure theory. Use the unidirectional graphite/epoxy lamina properties given in Table 2.1.

Solution

From Example 2.13,

$$\sigma_1 = 1.714\ S,$$

$$\sigma_2 = -2.714\ S,$$

$$\tau_{12} = -4.165\ S.$$

Using the Tsai–Hill failure theory from Equation (2.150),

$$\left(\frac{1.714S}{1500\times10^6}\right)^2 - \left(\frac{1.714S}{1500\times10^6}\right)\left(\frac{-2.714S}{1500\times10^6}\right) + \left(\frac{-2.714S}{40\times10^6}\right)^2 + \left(\frac{-4.165S}{68\times10^6}\right)^2 < 1$$

$$S < 10.94\ MPa$$

1. Unlike the maximum strain and maximum stress failure theories, the Tsai–Hill failure theory considers the interaction among the three unidirectional lamina strength parameters.
2. The Tsai–Hill failure theory does not distinguish between the compressive and tensile strengths in its equations. This can result in underestimation of the maximum loads that can be applied when compared to other failure theories. For the load of σ_x = 2 MPa, σ_y = –3 MPa, and τ_{xy} = 4 MPa, as found in Example 2.15, Example 2.17, and Example 2.18, the strength ratios are given by

   SR = 10.94 (Tsai–Hill failure theory)

   SR = 16.33 (maximum stress failure theory)

   SR = 16.33 (maximum strain failure theory)

Tsai–Hill failure theory underestimates the failure stress because the transverse tensile strength of a unidirectional lamina is generally much less than its transverse compressive strength. The compressive strengths are not used in the Tsai–Hill failure theory, but it can be modified to use corresponding tensile or compressive strengths in the failure theory as follows

$$\left[\frac{\sigma_1}{X_1}\right]^2 - \left[\left(\frac{\sigma_1}{X_2}\right)\left(\frac{\sigma_2}{X_2}\right)\right] + \left[\frac{\sigma_2}{Y}\right]^2 + \left[\frac{\tau_{12}}{S}\right]^2 < 1, \quad (2.151)$$

where

$$X_1 = (\sigma_1^T)_{ult} \text{ if } \sigma_1 > 0$$

$$= (\sigma_1^C)_{ult} \text{ if } \sigma_1 < 0;$$

$$X_2 = (\sigma_1^T)_{ult} \text{ if } \sigma_2 > 0$$

$$= (\sigma_1^C)_{ult} \text{ if } \sigma_2 < 0;$$

$$Y = (\sigma_2^T)_{ult} \text{ if } \sigma_2 > 0$$

$$= (\sigma_2^C)_{ult} \text{ if } \sigma_2 < 0$$

$$S = (\tau_{12})_{ult}.$$

For Example 2.18, the modified Tsai–Hill failure theory given by Equation (2.151) now gives

$$\left(\frac{1.714\sigma}{1500\times10^6}\right)^2 - \left(\frac{1.714\sigma}{1500\times10^6}\right)\left(\frac{-2.714\sigma}{1500\times10^6}\right) + \left(\frac{-2.714\sigma}{246\times10^6}\right)^2 + \left(\frac{-4.165\sigma}{68\times10^6}\right)^2 < 1$$

$$\sigma < 16.06 \; MPa,$$

which implies that the strength ratio is $SR = 16.06$ (modified Tsai–Hill failure theory). This value is closer to the values obtained using maximum stress and maximum strain failure theories.

3. The Tsai–Hill failure theory is a unified theory and thus does not give the mode of failure like the maximum stress and maximum strain failure theories do. However, one can make a reasonable guess of the failure mode by calculating $\left|\sigma_1/(\sigma_1^T)_{ult}\right|$, $\left|\sigma_2/(\sigma_2^T)_{ult}\right|$ and $|\tau_{12}/(\tau_{12})_{ult}|$. The maximum of these three values gives the associated mode of failure. In the modified Tsai–Hill failure theory, calculate the maximum of $|\sigma_1/X_1|$, $|\sigma_2/Y|$, and $|\tau_{12}/S|$ for the associated mode of failure.

2.8.6 Tsai–Wu Failure Theory

This failure theory is based on the total strain energy failure theory of Beltrami. Tsai-Wu[9] applied the failure theory to a lamina in plane stress. A lamina is considered to be failed if

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\,\sigma_1^2 + H_{22}\,\sigma_2^2 + H_{66}\,\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \quad (2.152)$$

is violated. This failure theory is more general than
theory because it distinguishes between the comp
ina.

The components H_1, H_2, H_6, H_{11}, H_{22}, and H_{66} of the failure theory are found using the five strength parameters of a unidirectional lamina as follows:

1. Apply $\sigma_1 = (\sigma_1^T)_{ult}$, $\sigma_2 = 0$, $\tau_{12} = 0$ to a unidirectional lamina; the lamina will fail. Equation (2.152) reduces to

$$H_1(\sigma_1^T)_{ult} + H_{11}(\sigma_1^T)_{ult}^2 = 1. \tag{2.153}$$

2. Apply $\sigma_1 = -(\sigma_1^C)_{ult}$, $\sigma_2 = 0$, $\tau_{12} = 0$ to a unidirectional lamina; the lamina will fail. Equation (2.152) reduces to

$$-H_1(\sigma_1^C)_{ult} + H_{11}(\sigma_1^C)_{ult}^2 = 1. \tag{2.154}$$

From Equation (2.153) and Equation (2.154),

$$H_1 = \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}, \tag{2.155}$$

$$H_{11} = \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}. \tag{2.156}$$

3. Apply $\sigma_1 = 0$, $\sigma_2 = (\sigma_2^T)_{ult}$, $\tau_{12} = 0$ to a unidirectional lamina; the lamina will fail. Equation (2.152) reduces to

$$H_2(\sigma_2^T)_{ult} + H_{22}(\sigma_2^T)_{ult}^2 = 1. \tag{2.157}$$

4. Apply $\sigma_1 = 0$, $\sigma_2 = -(\sigma_2^C)_{ult}$, $\tau_{12} = 0$ to a unidirectional lamina; the lamina will fail. Equation (2.152) reduces to

$$-H_2(\sigma_2^C)_{ult} + H_{22}(\sigma_2^C)_{ult}^2 = 1. \tag{2.158}$$

From Equation (2.157) and Equation (2.158),

$$H_2 = \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}}, \tag{2.159}$$

$$H_{22} = \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}.$$

5. Apply $\sigma_1 = 0$, $\sigma_2 = 0$, and $\tau_{12} = (\tau_{12})_{ult}$ to a unidirectional lamina; it will fail. Equation (2.152) reduces to

$$H_6(\tau_{12})_{ult} + H_{66}(\tau_{12})_{ult}^2 = 1. \tag{2.161}$$

6. Apply $\sigma_1 = 0$, $\sigma_2 = 0$, and $\tau_{12} = -(\tau_{12})_{ult}$ to a unidirectional lamina; the lamina will fail. Equation (2.152) reduces to

$$-H_6(\tau_{12})_{ult} + H_{66}(\tau_{12})_{ult}^2 = 1. \tag{2.162}$$

From Equation (2.161) and Equation (2.162),

$$H_6 = 0, \tag{2.163}$$

$$H_{66} = \frac{1}{(\tau_{12})_{ult}^2}. \tag{2.164}$$

The only component of the failure theory that cannot be found directly from the five strength parameters of the unidirectional lamina is H_{12}. This can be found experimentally by knowing a biaxial stress at which the lamina fails and then substituting the values of σ_1, σ_2, and τ_{12} in the Equation (2.152). Note that σ_1 and σ_2 need to be nonzero to find H_{12}. Experimental methods to find H_{12} include the following.

1. Apply equal tensile loads along the two material axes in a unidirectional composite. If $\sigma_x = \sigma_y = \sigma$, $\tau_{xy} = 0$ is the load at which the lamina fails, then

$$(H_1 + H_2)\sigma + (H_{11} + H_{22} + 2H_{12})\sigma^2 = 1. \tag{2.165}$$

The solution of Equation (2.165) gives

$$H_{12} = \frac{1}{2\sigma^2}[1 - (H_1 + H_2)\sigma - (H_{11} + H_{22})\sigma^2]. \tag{2.166}$$

It is not necessary to pick tensile loads in the preceding biaxial test, but one may apply any combination of

$$\sigma_1 = \sigma, \sigma_2 = \sigma,$$

$$\sigma_1 = -\sigma,\ \sigma_2 = -\sigma,$$

$$\sigma_1 = \sigma,\ \sigma_2 = -\sigma,$$

$$\sigma_1 = -\sigma,\ \sigma_2 = \sigma. \tag{2.167}$$

This will give four different values of H_{12}, each corresponding to the four tests.

2. Take a 45° lamina under uniaxial tension σ_x. The stress σ_x at failure is noted. If this stress is $\sigma_x = \sigma$, then, using Equation (2.94), the local stresses at failure are

$$\sigma_1 = \frac{\sigma}{2},$$

$$\sigma_2 = \frac{\sigma}{2}, \tag{2.168a–c}$$

$$\tau_{12} = -\frac{\sigma}{2}.$$

Substituting the preceding local stresses in Equation (2.152),

$$(H_1 + H_2)\frac{\sigma}{2} + \frac{\sigma^2}{4}(H_{11} + H_{22} + H_{66} + 2H_{12}) = 1. \tag{2.169}$$

$$H_{12} = \frac{2}{\sigma^2} - \frac{(H_1 + H_2)}{\sigma} - \frac{1}{2}(H_{11} + H_{22} + H_{66}). \tag{2.170}$$

Some empirical suggestions for finding the value of H_{12} include

$$H_{12} = -\frac{1}{2(\sigma_1^T)_{ult}^2}, \text{ per Tsai–Hill failure theory}^{8} \tag{2.171a–c}$$

$$H_{12} = -\frac{1}{2(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}, \text{ per Hoffman criterion}^{10}$$

$$H_{12} = -\frac{1}{2}\sqrt{\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}}, \text{ per Mises–He}$$

Example 2.19

Find the maximum value of $S > 0$ if a stress $\sigma_x = 2S$, $\sigma_y = -3S$, and $\tau_{xy} = 4S$ are applied to a 60° lamina of graphite/epoxy. Use Tsai–Wu failure theory. Use the properties of a unidirectional graphite/epoxy lamina from Table 2.1.

Solution

From Example 2.13,

$$\sigma_1 = 1.714S,$$

$$\sigma_2 = -2.714S,$$

$$\tau_{12} = -4.165S.$$

From Equations (2.155), (2.156), (2.159), (2.160), (2.163), and (2.164),

$$H_1 = \frac{1}{1500 \times 10^6} - \frac{1}{1500 \times 10^6} = 0 \ Pa^{-1},$$

$$H_2 = \frac{1}{40 \times 10^6} - \frac{1}{246 \times 10^6} = 2.093 \times 10^{-8} \ Pa^{-1},$$

$$H_6 = 0 \ Pa^{-1},$$

$$H_{11} = \frac{1}{(1500 \times 10^6)(1500 \times 10^6)} = 4.4444 \times 10^{-19} \ Pa^{-2},$$

$$H_{22} = \frac{1}{(40 \times 10^6)(246 \times 10^6)} = 1.0162 \times 10^{-16} \ Pa^{-2},$$

$$H_{66} = \frac{1}{(68 \times 10^6)^2} = 2.1626 \times 10^{-16} \ Pa^{-2} \ .$$

Using the Mises–Hencky criterion for evaluation of H_{12}, (Equation 2.165c),

$$\frac{1}{\ldots} \sqrt{\frac{1}{\ldots \times 10^6)(1500 \times 10^6)(40 \times 10^6)(246 \times 10^6)}}$$

Substituting these values in Equation (2.152), we obtain

$$(0)(1.714S) + (2.093 \times 10^{-8})(-2.714S)$$

$$+ (0)(-4.165S) + (4.444 \times 10^{-19})(1.714S)^2$$

$$+ (1.0162 \times 10^{-16})(-2.714S)^2 + (2.1626 \times 10^{-16})(-4.165S)^2$$

$$+ 2(-3.360 \times 10^{-18})(1.714S)(-2.714S) < 1,$$

or

$$S < 22.39 \ MPa \ .$$

If one uses the other two empirical criteria for H_{12}, per Equation (2.171), this yields

$$S < 22.49 \ MPa \ \text{for} \ H_{12} = -\frac{1}{2(\sigma_1^T)_{ult}^2},$$

$$S < 22.49 \ MPa \ \mathit{for} \ H_{12} = -\frac{1}{2}\frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}.$$

Summarizing the four failure theories for the same stress state, the value of S obtained is

S = 16.33 (maximum stress failure theory)
S = 16.33 (maximum strain failure theory)
S = 10.94 (Tsai–Hill failure theory)
S = 16.06 (modified Tsai–Hill failure theory)
S = 22.39 (Tsai–Wu failure theory)

2.8.7 Comparison of Experimental Results with Failure Theories

Tsai[7] compared the results from various failure theories to some experimental results. He considered an angle lamina subjected the x-direction, σ_x, as shown in Figure 2.33. The f ientally for tensile and compressive ina.

UNIT-VI

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3

Micromechanical Analysis of a Lamina

Chapter Objectives

- Develop concepts of volume and weight fraction (mass fraction) of fiber and matrix, density, and void fraction in composites.
- Find the nine mechanical and four hygrothermal constants: four elastic moduli, five strength parameters, two coefficients of thermal expansion, and two coefficients of moisture expansion of a unidirectional lamina from the individual properties of the fiber and the matrix, fiber volume fraction, and fiber packing.
- Discuss the experimental characterization of the nine mechanical and four hygrothermal constants.

3.1 Introduction

In Chapter 2, the stress–strain relationships, engineering constants, and failure theories for an angle lamina were developed using four elastic moduli, five strength parameters, two coefficients of thermal expansion (CTE), and two coefficients of moisture expansion (CME) for a unidirectional lamina. These 13 parameters can be found experimentally by conducting several tension, compression, shear, and hygrothermal tests on unidirectional lamina (laminates). However, unlike in isotropic materials, experimental evaluation of these parameters is quite costly and time consuming because they are functions of several variables: the individual constituents of the composite material, fiber volume fraction, packing geometry, processing, etc. Thus, the need and motivation for developing analytical models to find these parameters are very important. In this chapter, we will develop simple relationships for the these parameters in terms of the stiffnesses, strengths, coefficients of thermal and moisture expansion of the individual cons
ite, fiber volume fraction, packing geometry, etc. An u

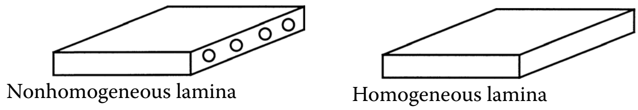

FIGURE 3.1
A nonhomogeneous lamina with fibers and matrix approximated as a homogeneous lamina.

relationship, called micromechanics of lamina, helps the designer to select the constituents of a composite material for use in a laminated structure.

Because this text is for a first course in composite materials, details will be explained only for the simple models based on the mechanics of materials approach and the semi-empirical approach. Results from other methods based on advanced topics such as elasticity are also explained for completeness.

As mentioned in Chapter 2, a unidirectional lamina is not homogeneous. However, one can assume the lamina to be homogeneous by focusing on the average response of the lamina to mechanical and hygrothermal loads (Figure 3.1). The lamina is simply looked at as a material whose properties are different in various directions, but not different from one location to another.

Also, the chapter focuses on a unidirectional continuous fiber-reinforced lamina. This is because it forms the basic building block of a composite structure, which is generally made of several unidirectional laminae placed at various angles. The modeling in the evaluation of the parameters is discussed first. This is followed by examples and experimental methods for finding these parameters.

3.2 Volume and Mass Fractions, Density, and Void Content

Before modeling the 13 parameters of a unidirectional composite, we introduce the concept of relative fraction of fibers by volume. This concept is critical because theoretical formulas for finding the stiffness, strength, and hygrothermal properties of a unidirectional lamina are a function of fiber volume fraction. Measurements of the constituents are generally based on their mass, so fiber mass fractions must also be defined. Moreover, defining the density of a composite also becomes necessary because its value is used in the experimental determination of fiber volume and void fractions of a composite. Also, the value of density is used in the definition of specific modulus and specific strength in Chapter 1.

3.2.1 Volume Fractions

…osite consisting of fiber and matrix …:

…LLC

$v_{c,f,m}$ = volume of composite, fiber, and matrix, respectively
$\rho_{c,f,m}$ = density of composite, fiber, and matrix, respectively.

Now define the fiber volume fraction V_f and the matrix volume fraction V_m as

$$V_f = \frac{v_f}{v_c},$$

and

$$V_m = \frac{v_m}{v_c}. \tag{3.1a, b}$$

Note that the sum of volume fractions is

$$V_f + V_m = 1,$$

from Equation (3.1) as

$$v_f + v_m = v_c.$$

3.2.2 Mass Fractions

Consider a composite consisting of fiber and matrix and take the following symbol notation: $w_{c,f,m}$ = mass of composite, fiber, and matrix, respectively. The mass fraction (weight fraction) of the fibers (W_f) and the matrix (W_m) are defined as

$$W_f = \frac{w_f}{w_c}, \text{ and}$$

$$W_m = \frac{w_m}{w_c}. \tag{3.2a, b}$$

Note that the sum of mass fractions is

$$W_f + W_m = 1,$$

from Equation (3.2) as

$$w_f + w_m = w_c \ .$$

From the definition of the density of a single material,

$$w_c = r_c v_c ,$$

$$w_f = r_f v_f , \text{ and} \tag{3.3a–c}$$

$$w_m = r_m v_m .$$

Substituting Equation (3.3) in Equation (3.2), the mass fractions and volume fractions are related as

$$W_f = \frac{\rho_f}{\rho_c} V_f , \text{ and}$$

$$W_m = \frac{\rho_m}{\rho_c} V_m , \tag{3.4a, b}$$

in terms of the fiber and matrix volume fractions. In terms of individual constituent properties, the mass fractions and volume fractions are related by

$$W_f = \frac{\frac{\rho_f}{\rho_m}}{\frac{\rho_f}{\rho_m} V_f + V_m} V_f ,$$

$$W_m = \frac{1}{\frac{\rho_f}{\rho_m}(1 - V_m) + V_m} V_m \ . \tag{3.5a, b}$$

One should always state the basis of calculating the fiber content of a composite. It is given in terms of mass or volume. Bas
it is evident that volume and mass fractions are not
...n the mass and volume fractions i...
...ity of fiber and matrix differs from o...

3.2.3 Density

The derivation of the density of the composite in terms of volume fractions is found as follows. The mass of composite w_c is the sum of the mass of the fibers w_f and the mass of the matrix w_m as

$$w_c = w_f + w_m. \tag{3.6}$$

Substituting Equation (3.3) in Equation (3.6) yields

$$\rho_c v_c = \rho_f v_f + \rho_m v_m,$$

and

$$\rho_c = \rho_f \frac{v_f}{v_c} + \rho_m \frac{v_m}{v_c}. \tag{3.7}$$

Using the definitions of fiber and matrix volume fractions from Equation (3.1),

$$\rho_c = \rho_f V_f + \rho_m V_m. \tag{3.8}$$

Now, consider that the volume of a composite v_c is the sum of the volumes of the fiber v_f and matrix (v_m):

$$v_c = v_f + v_m. \tag{3.9}$$

The density of the composite in terms of mass fractions can be found as

$$\frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}. \tag{3.10}$$

Example 3.1

A glass/epoxy lamina consists of a 70% fiber volume fraction. Use properties of glass and epoxy from Table 3.1* and Table 3.2, respectively, to determine the

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* Table 3.1 and Table 3.2 give the typical properties of common fibers ... ely. Note that fibers such as graphite and aram... ;enerally isotropic. The typical properties of co... le 3.3 and Table 3.4, respectively, in the USCS sy...

... LLC

TABLE 3.1

Typical Properties of Fibers (SI System of Units)

Property	Units	Graphite	Glass	Aramid
Axial modulus	GPa	230	85	124
Transverse modulus	GPa	22	85	8
Axial Poisson's ratio	—	0.30	0.20	0.36
Transverse Poisson's ratio	—	0.35	0.20	0.37
Axial shear modulus	GPa	22	35.42	3
Axial coefficient of thermal expansion	μm/m/°C	–1.3	5	–5.0
Transverse coefficient of thermal expansion	μm/m/°C	7.0	5	4.1
Axial tensile strength	MPa	2067	1550	1379
Axial compressive strength	MPa	1999	1550	276
Transverse tensile strength	MPa	77	1550	7
Transverse compressive strength	MPa	42	1550	7
Shear strength	MPa	36	35	21
Specific gravity	—	1.8	2.5	1.4

TABLE 3.2

Typical Properties of Matrices (SI System of Units)

Property	Units	Epoxy	Aluminum	Polyamide
Axial modulus	GPa	3.4	71	3.5
Transverse modulus	GPa	3.4	71	3.5
Axial Poisson's ratio	—	0.30	0.30	0.35
Transverse Poisson's ratio	—	0.30	0.30	0.35
Axial shear modulus	GPa	1.308	27	1.3
Coefficient of thermal expansion	μm/m/°C	63	23	90
Coefficient of moisture expansion	m/m/kg/kg	0.33	0.00	0.33
Axial tensile strength	MPa	72	276	54
Axial compressive strength	MPa	102	276	108
Transverse tensile strength	MPa	72	276	54
Transverse compressive strength	MPa	102	276	108
Shear strength	MPa	34	138	54
Specific gravity	—	1.2	2.7	1.2

1. Density of lamina
2. Mass fractions of the glass and epoxy
3. Volume of composite lamina if the mass of the lamina is 4 kg
4. Volume and mass of glass and epoxy in part (3)

Solution

1. From Table 3.1, the density of the fiber is

$$\rho_f = 2500 \ kg/m^3.$$

TABLE 3.3

Typical Properties of Fibers (USCS System of Units)

Property	Units	Graphite	Glass	Aramid
Axial modulus	Msi	33.35	12.33	17.98
Transverse modulus	Msi	3.19	12.33	1.16
Axial Poisson's ratio	—	0.30	0.20	0.36
Transverse Poisson's ratio	—	0.35	0.20	0.37
Axial shear modulus	Msi	3.19	5.136	0.435
Axial coefficient of thermal expansion	μin./in./°F	–0.7222	2.778	–2.778
Transverse coefficient of thermal expansion	μin./in./°F	3.889	2.778	2.278
Axial tensile strength	ksi	299.7	224.8	200.0
Axial compressive strength	ksi	289.8	224.8	40.02
Transverse tensile strength	ksi	11.16	224.8	1.015
Transverse compressive strength	ksi	6.09	224.8	1.015
Shear strength	ksi	5.22	5.08	3.045
Specific gravity	—	1.8	2.5	1.4

TABLE 3.4

Typical Properties of Matrices (USCS System of Units)

Property	Units	Epoxy	Aluminum	Polyamide
Axial modulus	Msi	0.493	10.30	0.5075
Transverse modulus	Msi	0.493	10.30	0.5075
Axial Poisson's ratio	—	0.30	0.30	0.35
Transverse Poisson's ratio	—	0.30	0.30	0.35
Axial shear modulus	Msi	0.1897	3.915	0.1885
Coefficient of thermal expansion	μin./in./°F	35	12.78	50
Coefficient of moisture expansion	in./in./lb/lb	0.33	0.00	0.33
Axial tensile strength	ksi	10.44	40.02	7.83
Axial compressive strength	ksi	14.79	40.02	15.66
Transverse tensile strength	ksi	10.44	40.02	7.83
Transverse compressive strength	ksi	14.79	40.02	15.66
Shear strength	ksi	4.93	20.01	7.83
Specific gravity	—	1.2	2.7	1.2

From Table 3.2, the density of the matrix is

$$\rho_m = 1200\ kg/m^3.$$

Using Equation (3.8), the density of the composite is

$$\rho_c = (2500)(0.7) + (1200)(0.3)$$

$$= 2110\ kg/m^3.$$

3. Using Equation (3.4), the fiber and matrix mass f

$$W_f = \frac{2500}{2110} \times 0.3$$

$$= 0.8294$$

$$W_m = \frac{1200}{2110} \times 0.3 .$$

$$= 0.1706$$

Note that the sum of the mass fractions,

$$W_f + W_m = 0.8294 + 0.1706$$

$$= 1.000.$$

3. The volume of composite is

$$v_c = \frac{w_c}{\rho_c}$$

$$= \frac{4}{2110}$$

$$= 1.896 \times 10^{-3} m^3 .$$

4. The volume of the fiber is

$$v_f = V_f v_c$$

$$= (0.7)(1.896 \times 10^{-3})$$

$$= 1.327 \times 10^{-3} m^3 .$$

The volume of the matrix is

$$v_m = V_m v_c$$

$$=(0.3)(0.1896 \times 10^{-3})$$

$$= 0.5688 \times 10^{-3}\ m^3\ .$$

The mass of the fiber is

$$w_f = \rho_f v_f$$

$$= (2500)(1.327 \times 10^{-3})$$

$$= 3.318\ kg\ .$$

The mass of the matrix is

$$w_m = \rho_m v_m$$

$$= (1200)(0.5688 \times 10^{-3})$$

$$= 0.6826\ kg\ .$$

3.2.4 Void Content

During the manufacture of a composite, voids are introduced in the composite as shown in Figure 3.2. This causes the theoretical density of the composite to be higher than the actual density. Also, the void content of a

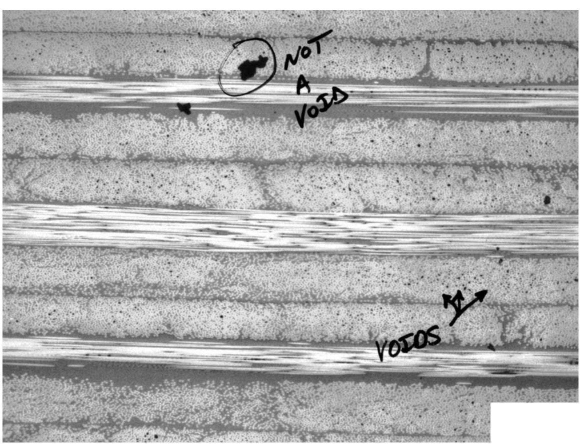

FIGURE 3.2
cross-section of a lamina with voids.

composite is detrimental to its mechanical properties. These detriments include lower

- Shear stiffness and strength
- Compressive strengths
- Transverse tensile strengths
- Fatigue resistance
- Moisture resistance

A decrease of 2 to 10% in the preceding matrix-dominated properties generally takes place with every 1% increase in the void content.[1]

For composites with a certain volume of voids V_v the volume fraction of voids V_v is defined as

$$V_v = \frac{v_v}{v_c}. \tag{3.11}$$

Then, the total volume of a composite (v_c) with voids is given by

$$v_c = v_f + v_m + v_v. \tag{3.12}$$

By definition of the experimental density ρ_{ce} of a composite, the actual volume of the composite is

$$v_c = \frac{w_c}{\rho_{ce}}, \tag{3.13}$$

and, by the definition of the theoretical density ρ_{ct} of the composite, the theoretical volume of the composite is

$$v_f + v_m = \frac{w_c}{\rho_{ct}}. \tag{3.14}$$

Then, substituting the preceding expressions (3.13) and (3.14) in Equation (3.12),

$$\frac{w_c}{\rho_{ce}} = \frac{w_c}{\rho_{ct}} + v_v .$$

…oid is given by

$$v_v = \frac{w_c}{\rho_{ce}}\left(\frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}\right). \tag{3.15}$$

Substituting Equation (3.13) and Equation (3.15) in Equation (3.11), the volume fraction of the voids is

$$\begin{aligned} V_v &= \frac{v_v}{v_c} \\ &= \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}. \end{aligned} \tag{3.16}$$

Example 3.2

A graphite/epoxy cuboid specimen with voids has dimensions of $a \times b \times c$ and its mass is M_c. After it is put it into a mixture of sulfuric acid and hydrogen peroxide, the remaining graphite fibers have a mass M_f. From independent tests, the densities of graphite and epoxy are ρ_f and ρ_m, respectively. Find the volume fraction of the voids in terms of a, b, c, M_f, M_c, ρ_f, and ρ_m.

Solution

The total volume of the composite v_c is the sum total of the volume of fiber v_f, matrix v_m, and voids v_v:

$$v_c = v_f + v_m + v_v. \tag{3.17}$$

From the definition of density,

$$v_f = \frac{M_f}{\rho_f}, \tag{3.18a}$$

$$v_m = \frac{M_c - M_f}{\rho_m}. \tag{3.18b}$$

The specimen is a cuboid, so the volume of the composite is

$$v_c = abc.$$

uation (3.18) and Equation (3.19) in E

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$$abc = \frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m} + v_v \,,$$

and the volume fraction of voids then is

$$V_v = \frac{v_v}{abc} = 1 - \frac{1}{abc}\left[\frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m}\right] \tag{3.20}$$

Alternative Solution

The preceding problem can also be solved by using Equation (3.16). The theoretical density of the composite is

$$\rho_{ct} = \rho_f V_f' + \rho_m (1 - V_f') \,, \tag{3.21}$$

where V_f' is the theoretical fiber volume fraction given as

$$V_f' = \frac{volume\ of\ \ fibers}{volume\ of\ \ fibers + volume\ of\ \ matrix}$$

$$V_f' = \frac{\dfrac{M_f}{\rho_f}}{\dfrac{M_f}{\rho_f} + \dfrac{M_c - M_f}{\rho_m}}. \tag{3.22}$$

The experimental density of the composite is

$$\rho_{ce} = \frac{M_c}{abc}. \tag{3.23}$$

Substituting Equation (3.21) through Equation (3.23) in the definition of void volume fractions given by Equation (3.16),

$$V_v = 1 - \frac{1}{abc}\left[\frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m}\right]. \tag{3.24}$$

Experimental determination: the fiber volume fractions
...und generally by the burn or the acid
...g a sample of composite and weighin

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of the specimen is found by the liquid displacement method in which the sample is weighed in air and then in water. The density of the composite is given by

$$\rho_c = \frac{w_c}{w_c - w_i}\rho_w , \tag{3.25}$$

where

w_c = weight of composite
w_i = weight of composite when immersed in water
ρ_w = density of water (1000 kg/m^3 or 62.4 lb/ft^3)

For specimens that float in water, a sinker is attached. The density of the composite is then found by

$$\rho_c = \frac{w_c}{w_c + w_s - w_w}\rho_w , \tag{3.26}$$

where

w_c = weight of composite
w_s = weight of sinker when immersed in water
w_w = weight of sinker and specimen when immersed in water

The sample is then dissolved in an acid solution or burned.[2] Glass-based composites are burned, and carbon and aramid-based composites are digested in solutions. Carbon and aramid-based composites cannot be burned because carbon oxidizes in air above 300°C (572°F) and the aramid fiber can decompose at high temperatures. Epoxy-based composites can be digested by nitric acid or a hot mixture of ethylene glycol and potassium hydroxide; polyamide- and phenolic resin-based composites use mixtures of sulfuric acid and hydrogen peroxide. When digestion or burning is complete, the remaining fibers are washed and dried several times and then weighed. The fiber and matrix weight fractions can be found using Equation (3.2). The densities of the fiber and the matrix are known; thus, one can use Equation (3.4) to determine the volume fraction of the constituents of the composite and Equation (3.8) to calculate the theoretical density of the composite.

3.3 Evaluation of the Four Elastic Moduli

As shown in Section 2.4.3, there are four elastic modu

- Longitudinal Young's modulus, E_1
- Transverse Young's modulus, E_2
- Major Poisson's ratio, ν_{12}
- In-plane shear modulus, G_{12}

Three approaches for determining the four elastic moduli are discussed next.

3.3.1 Strength of Materials Approach

From a unidirectional lamina, take a representative volume element* that consists of the fiber surrounded by the matrix (Figure 3.3). This representative volume element (RVE) can be further represented as rectangular blocks. The fiber, matrix, and the composite are assumed to be of the same width, h, but of thicknesses t_f, t_m, and t_c, respectively. The area of the fiber is given by

$$A_f = t_f h \ . \tag{3.27a}$$

The area of the matrix is given by

$$A_m = t_m h, \tag{3.27b}$$

and the area of the composite is given by

$$A_c = t_c h. \tag{3.27c}$$

The two areas are chosen in the proportion of their volume fractions so that the fiber volume fraction is defined as

$$V_f = \frac{A_f}{A_c} = \frac{t_f}{t_c}, \tag{3.28a}$$

and the matrix fiber volume fraction V_m is

* ... ume element (RVE) of a material is the smalle...
... l as a whole. It could be otherwise intractable t...
... s of the material.

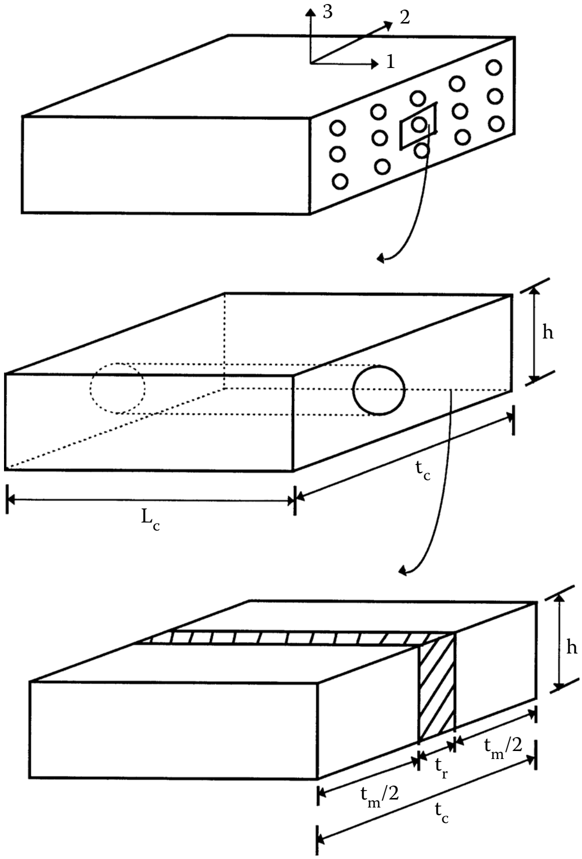

FIGURE 3.3
Representative volume element of a unidirectional lamina.

$$V_m = \frac{A_m}{A_c}$$

$$= \frac{t_m}{t_c} \qquad (3.28b)$$

$$= 1 - V_f.$$

The following assumptions are made in the strength of materials approach model:

- The bond between fibers and matrix is perfect.
- The elastic moduli, diameters, and space between
  re continuous and parallel.

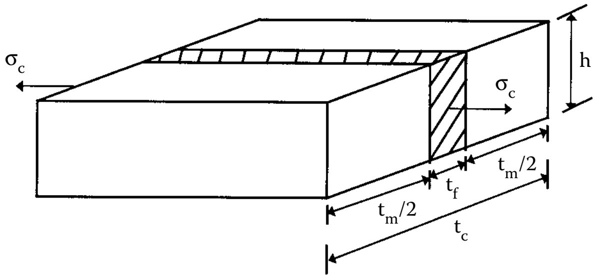

FIGURE 3.4
A longitudinal stress applied to the representative volume element to calculate the longitudinal Young's modulus for a unidirectional lamina.

- The fibers and matrix follow Hooke's law (linearly elastic).
- The fibers possess uniform strength.
- The composite is free of voids.

3.3.1.1 *Longitudinal Young's Modulus*

From Figure 3.4, under a uniaxial load F_c on the composite RVE, the load is shared by the fiber F_f and the matrix F_m so that

$$F_c = F_f + F_m. \tag{3.29}$$

The loads taken by the fiber, the matrix, and the composite can be written in terms of the stresses in these components and cross-sectional areas of these components as

$$F_c = \sigma_c A_c, \tag{3.30a}$$

$$F_f = \sigma_f A_f, \tag{3.30b}$$

$$F_m = \sigma_m A_m, \tag{3.30c}$$

where

$\sigma_{c,f,m}$ = stress in composite, fiber, and matrix, respectively
$A_{c,f,m}$ = area of composite, fiber, and matrix, respectively

Assuming that the fibers, matrix, and composite foll
that the fibers and the matrix are isotropic, the stress–s
and the composite is

$$\sigma_c = E_1 \varepsilon_c , \tag{3.31a}$$

$$\sigma_f = E_f \varepsilon_f , \tag{3.31b}$$

and

$$\sigma_m = E_m \varepsilon_m , \tag{3.31c}$$

where

$\varepsilon_{c,f,m}$ = strains in composite, fiber, and matrix, respectively
$E_{1,f,m}$ = elastic moduli of composite, fiber, and matrix, respectively

Substituting Equation (3.30) and Equation (3.31) in Equation (3.29) yields

$$E_1 \varepsilon_c A_c = E_f \varepsilon_f A_f + E_m \varepsilon_m A_m . \tag{3.32}$$

The strains in the composite, fiber, and matrix are equal ($\varepsilon_c = \varepsilon_f = \varepsilon_m$); then, from Equation (3.32),

$$E_1 = E_f \frac{A_f}{A_c} + E_m \frac{A_m}{A_c} . \tag{3.33}$$

Using Equation (3.28), for definitions of volume fractions,

$$E_1 = E_f V_f + E_m V_m . \tag{3.34}$$

Equation 3.34 gives the longitudinal Young's modulus as a weighted mean of the fiber and matrix modulus. It is also called the rule of mixtures.

The ratio of the load taken by the fibers F_f to the load taken by the composite F_c is a measure of the load shared by the fibers. From Equation (3.30) and Equation (3.31),

$$\frac{F_f}{F_c} = \frac{E_f}{E_1} V_f . \tag{3.35}$$

In Figure 3.5, the ratio of the load carried by the fibers to the load taken by the composite is plotted as a function of fiber-to-ma
ratio E_f/E_m for the constant fiber volume fraction V_f. It sl
ratio increases, the load taken by th

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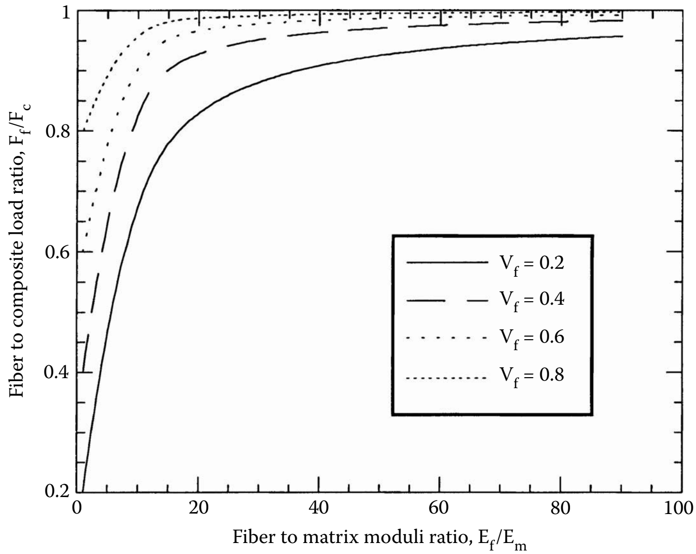


FIGURE 3.5
Fraction of load of composite carried by fibers as a function of fiber volume fraction for constant fiber to matrix moduli ratio.

Example 3.3

Find the longitudinal elastic modulus of a unidirectional glass/epoxy lamina with a 70% fiber volume fraction. Use the properties of glass and epoxy from Table 3.1 and Table 3.2, respectively. Also, find the ratio of the load taken by the fibers to that of the composite.

Solution

From Table 3.1, the Young's modulus of the fiber is

$$E_f = 85 \text{ GPa}.$$

From Table 3.2, the Young's modulus of the matrix is

$$E_m = 3.4 \text{ GPa}.$$

Using Equation (3.34), the longitudinal elastic modulus of the unidirectional lamina is

$$E_1 = (85)(0.7) + (3.4)(0.3)$$

$$= 60.52 \; GPa.$$

3.35), the ratio of the load taken by th

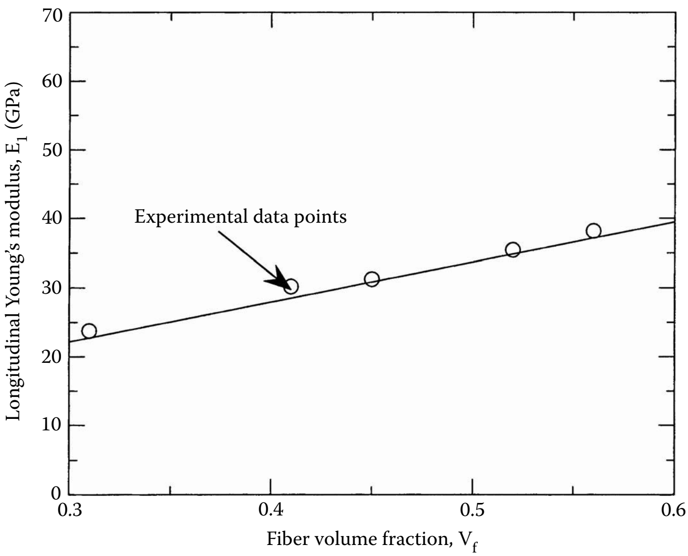


FIGURE 3.6
Longitudinal Young's modulus as function of fiber volume fraction and comparison with experimental data points for a typical glass/polyester lamina. (Experimental data points reproduced with permission of ASM International.)

$$\frac{F_f}{F_c} = \frac{85}{60.52}(0.7)$$

$$= 0.9831.$$

Figure 3.6 shows the linear relationship between the longitudinal Young's modulus of a unidirectional lamina and fiber volume fraction for a typical graphite/epoxy composite per Equation (3.34). It also shows that Equation (3.34) predicts results that are close to the experimental data points.[3]

3.3.1.2 *Transverse Young's Modulus*

Assume now that, as shown in Figure 3.7, the composite is stressed in the transverse direction. The fibers and matrix are again represented by rectangular blocks as shown. The fiber, the matrix, and composite stresses are equal. Thus,

$$\sigma_c = \sigma_f = \sigma_m, \tag{3.36}$$

where $\sigma_{c,f,m}$ = stress in composite, fiber, and matrix, res
verse extension in the composite Δ_c is
n the fiber Δ_f, and that is the matrix,

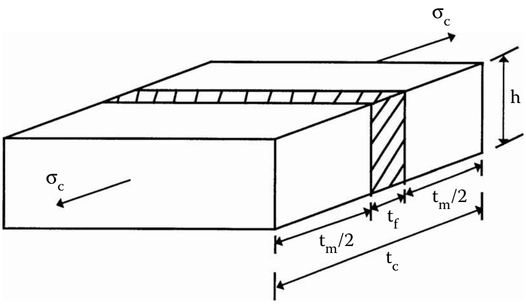


FIGURE 3.7
A transverse stress applied to a representative volume element used to calculate transverse Young's modulus of a unidirectional lamina.

$$\Delta_c = \Delta_f + \Delta_m. \tag{3.37}$$

Now, by the definition of normal strain,

$$\Delta_c = t_c \varepsilon_c, \tag{3.38a}$$

$$\Delta_f = t_f \varepsilon_f, \tag{3.38b}$$

and

$$\Delta_m = t_m \varepsilon_m, \tag{3.38c}$$

where

$t_{c,f,m}$ = thickness of the composite, fiber and matrix, respectively
$\varepsilon_{c,f,m}$ = normal transverse strain in the composite, fiber, and matrix, respectively

Also, by using Hooke's law for the fiber, matrix, and composite, the normal strains in the composite, fiber, and matrix are

$$\varepsilon_c = \frac{\sigma_c}{E_2}, \tag{3.39a}$$

$$\varepsilon_f = \frac{\sigma_f}{E_f}, \tag{3.39b}$$

and

$$\varepsilon_m = \frac{\sigma_m}{E_m}.$$

Substituting Equation (3.38) and Equation (3.39) in Equation (3.37) and using Equation (3.36) gives

$$\frac{1}{E_2} = \frac{1}{E_f}\frac{t_f}{t_c} + \frac{1}{E_m}\frac{t_m}{t_c}. \tag{3.40}$$

Because the thickness fractions are the same as the volume fractions as the other two dimensions are equal for the fiber and the matrix (see Equation 3.28):

$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}. \tag{3.41}$$

Equation (3.41) is based on the weighted mean of the compliance of the fiber and the matrix.

Example 3.4

Find the transverse Young's modulus of a glass/epoxy lamina with a fiber volume fraction of 70%. Use the properties of glass and epoxy from Table 3.1 and Table 3.2, respectively.

Solution

From Table 3.1, the Young's modulus of the fiber is

$$E_f = 85 \text{ GPa}.$$

From Table 3.2, the Young's modulus of the matrix is

$$E_m = 3.4 \text{ GPa}.$$

Using Equation (3.41), the transverse Young's modulus, E_2, is

$$\frac{1}{E_2} = \frac{0.7}{85} + \frac{0.3}{3.4},$$

$$E_2 = 10.37\ GPa.$$

Figure 3.8 plots the transverse Young's modulus as a function of fiber volume fraction for constant fiber-to-matrix elastic moduli ratio, E_f/E_m. For metal and ceramic matrix composites, the fiber and matrix elastic moduli are of the same order. (For example, for a SiC/alur
composite, $E_f/E_m = 4$ and for a SiC/CAS ceramic matri
: Young's modulus of the composite i
; a function of the fiber volume fracti

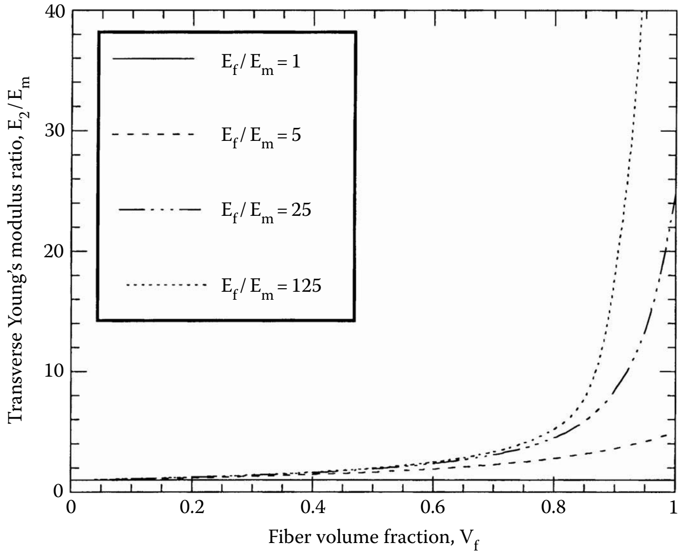


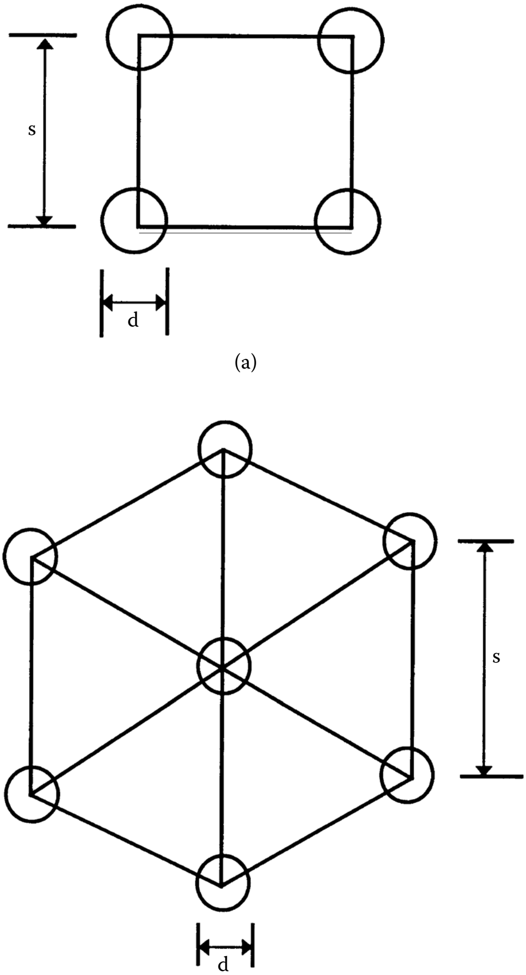
FIGURE 3.8
Transverse Young's modulus as a function of fiber volume fraction for constant fiber to matrix moduli ratio.

For polymeric composites, the fiber-to-matrix moduli ratio is very high. (For example, for a glass/epoxy polymer matrix composite, $E_f/E_m = 25$). The transverse Young's modulus of the composite in such cases changes appreciably only for large fiber volume fractions. Figure 3.8 shows that, for high E_f/E_m ratios, the contribution of the fiber modulus only increases substantially for a fiber volume fraction greater than 80%. These fiber volume fractions are not practical and in many cases are physically impossible due to the geometry of fiber packing. Figure 3.9 shows various possibilities of fiber packing. Note that the ratio of the diameter, d, to fiber spacing, s, d/s varies with geometrical packing. For circular fibers with square array packing (Figure 3.9a),

$$\frac{d}{s} = \left(\frac{4V_f}{\pi}\right)^{1/2}. \tag{3.42a}$$

This gives a maximum fiber volume fraction of 78.54% as $s \geq d$. For circular fibers with hexagonal array packing (Figure 3.9b),

$$\frac{d}{s} = \left(\frac{2\sqrt{3}V_f}{\pi}\right)^{1/2}.$$



(b)

FIGURE 3.9
Fiber to fiber spacing in (a) square packing geometry and (b) hexagonal packing geometry.

This gives a maximum fiber volume fraction of 90.69% because $s \geq d$. These maximum fiber volume fractions are not practical to use because the fibers touch each other and thus have surfaces where the matrix cannot wet out the fibers.

In Figure 3.10, the transverse Young's modulus is plotted as a function of fiber volume fraction using Equation (3.41) for a typical boron/epoxy lamina. Also given are the experimental data points.[4] In Figure 3 and analytical results are not as close to each other ng's modulus in Figure 3.6.

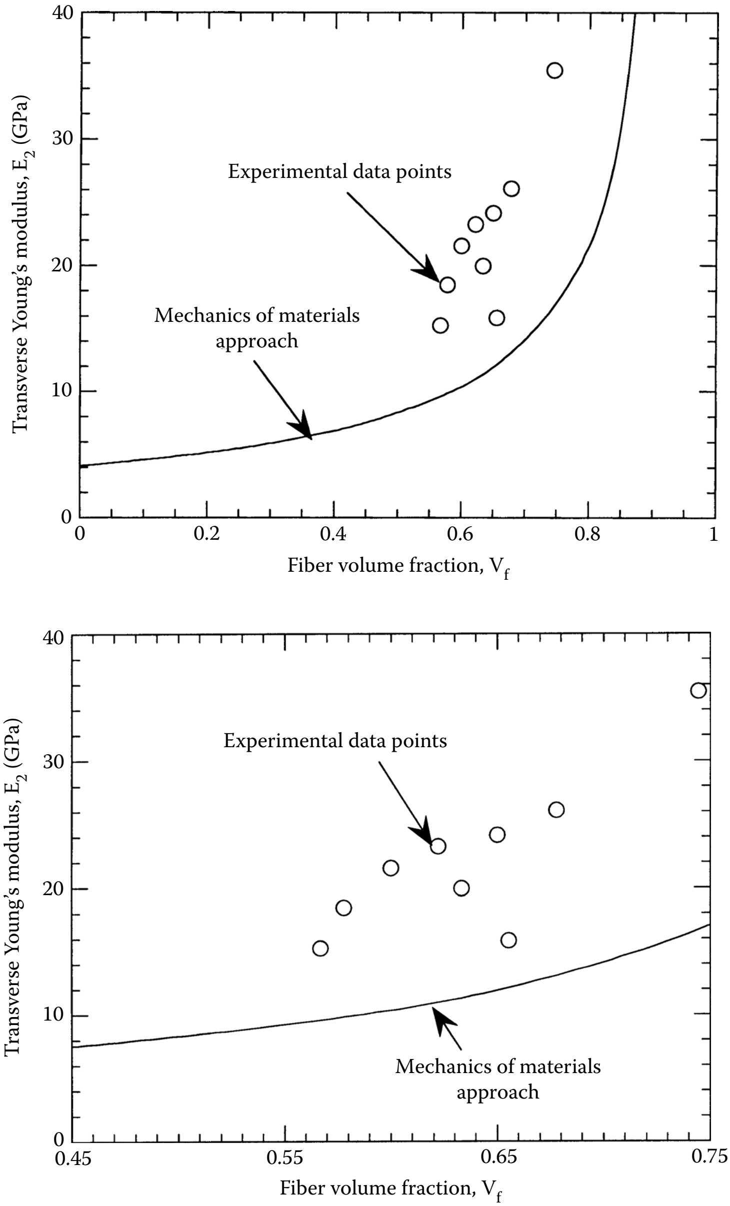


FIGURE 3.10
Theoretical values of transverse Young's modulus as a function of fiber volume fraction for a Boron/Epoxy unidirectional lamina (E_f = 414 GPa, ν_f = 0.2, E_m = 4.14 GPa, ν_m = 0.35) and comparison with experimental values. Figure (b) zooms figure (a) between 0.45 and 0.75. (Experimental data from Hashin, Z., NASA te[...] 8818, November 1970.)

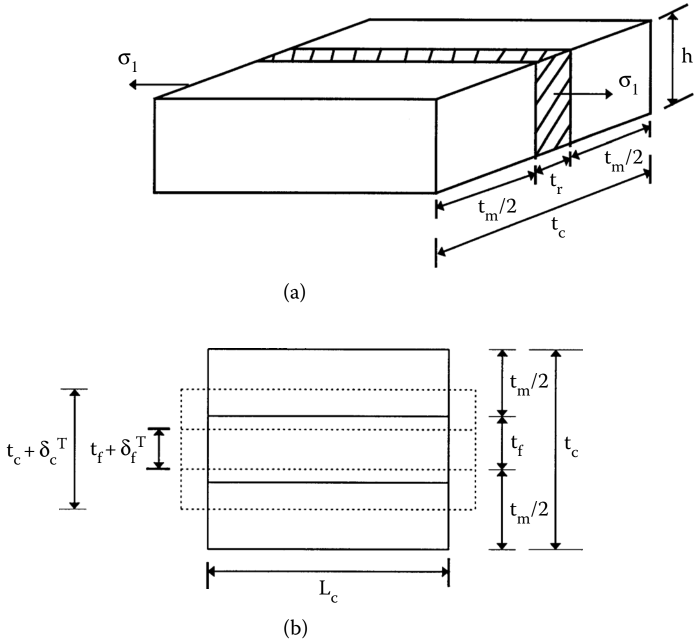


FIGURE 3.11
A longitudinal stress applied to a representative volume element to calculate Poisson's ratio of unidirectional lamina.

3.3.1.3 Major Poisson's Ratio

The major Poisson's ratio is defined as the negative of the ratio of the normal strain in the transverse direction to the normal strain in the longitudinal direction, when a normal load is applied in the longitudinal direction. Assume a composite is loaded in the direction parallel to the fibers, as shown in Figure 3.11. The fibers and matrix are again represented by rectangular blocks. The deformations in the transverse direction of the composite (δ_c^T) is the sum of the transverse deformations of the fiber (δ_f^T) and the matrix (δ_m^T) as

$$\delta_c^T = \delta_f^T + \delta_m^T. \tag{3.43}$$

Using the definition of normal strains,

$$\varepsilon_f^T = \frac{\delta_f^T}{t_f}, \tag{3.44a}$$

$$\varepsilon_m^T = \frac{\delta_m^T}{t_m},$$

and

$$\varepsilon_c^T = \frac{\delta_c^T}{t_c}, \tag{3.44c}$$

where $\varepsilon_{c,f,m}$ = transverse strains in composite, fiber, and matrix, respectively.

Substituting Equation (3.44) in Equation (3.43),

$$t_c\varepsilon_c^T = t_f\varepsilon_f^T + t_m\varepsilon_m^T. \tag{3.45}$$

The Poisson's ratios for the fiber, matrix, and composite, respectively, are

$$\nu_f = -\frac{\varepsilon_f^T}{\varepsilon_f^L}, \tag{3.46a}$$

$$\nu_m = -\frac{\varepsilon_m^T}{\varepsilon_m^L}, \tag{3.46b}$$

and

$$\nu_{12} = -\frac{\varepsilon_c^T}{\varepsilon_c^L}. \tag{3.46c}$$

Substituting in Equation (3.45),

$$-t_c\nu_{12}\varepsilon_c^L = -t_f\nu_f\varepsilon_f^L - t_m\nu_m\varepsilon_m^L, \tag{3.47}$$

where

$\nu_{12,f,m} v_{12,f,m}$ = Poisson's ratio of composite, fiber, and matrix, respectively

$\varepsilon_{c,f,m}^L$ = longitudinal strains of composite, fiber and matrix, respectively

However, the strains in the composite, fiber, and matrix are assumed to be the equal in the longitudinal direction ($\varepsilon_c^L = \varepsilon_f^L = \varepsilon_m^L$), which, from Equation (3.47), gives

$$t_c\nu_{12} = t_f\nu_f + t_m\nu_m,$$

$$\nu_{12} = \nu_f\frac{t_f}{t_c} + \nu_m\frac{t_m}{t_c}.$$

Because the thickness fractions are the same as the volume fractions, per Equation (3.28),

$$\nu_{12} = \nu_f V_f + \nu_m V_m. \tag{3.49}$$

Example 3.5

Find the major and minor Poisson's ratio of a glass/epoxy lamina with a 70% fiber volume fraction. Use the properties of glass and epoxy from Table 3.1 and Table 3.2, respectively.

Solution

From Table 3.1, the Poisson's ratio of the fiber is

$$\nu_f = 0.2.$$

From Table 3.2, the Poisson's ratio of the matrix is

$$\nu_m = 0.3.$$

Using Equation (3.49), the major Poisson's ratio is

$$\nu_{12} = (0.2)(0.7) + (0.3)(0.3)$$

$$= 0.230.$$

From Example 3.3, the longitudinal Young's modulus is

$$E_1 = 60.52 \text{ GPa}$$

and, from Example 3.4, the transverse Young's modulus is

$$E_2 = 10.37 \text{ GPa}.$$

Then, the minor Poisson's ratio from Equation (2.83) is

$$\nu_{21} = \nu_{12}\frac{E_2}{E_1}$$

$$= 0.230\left(\frac{10.37}{60.52}\right)$$

$$= 0.03941.$$

3.3.1.4 In-Plane Shear Modulus

ar stress τ_c to a lamina as shown in F

epresented by rectangular blocks as s

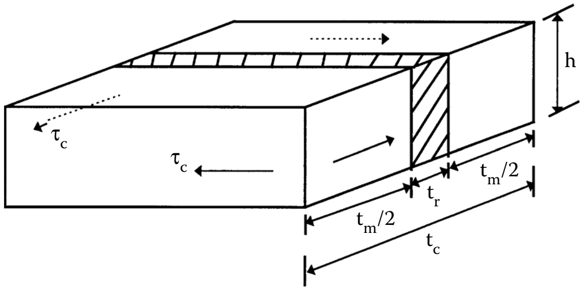


FIGURE 3.12
An in-plane shear stress applied to a representative volume element for finding in-plane shear modulus of a unidirectional lamina.

shear deformations of the composite δ_c the fiber δ_f, and the matrix δ_m are related by

$$\delta_c = \delta_f + \delta_m \, . \tag{3.50}$$

From the definition of shear strains,

$$\delta_c = \gamma_c t_c \, , \tag{3.51a}$$

$$\delta_f = \gamma_f t_f \, , \tag{3.51b}$$

and

$$\delta_m = \gamma_m t_m \, , \tag{3.51c}$$

where

$\gamma_{c,f,m}$ = shearing strains in the composite, fiber, and matrix, respectively

$t_{c,f,m}$ = thickness of the composite, fiber, and matrix, respectively.

From Hooke's law for the fiber, the matrix, and the composite,

$$\gamma_c = \frac{\tau_c}{G_{12}}, \tag{3.52a}$$

$$\gamma_f = \frac{\tau_f}{G_f},$$

$$\gamma_m = \frac{\tau_m}{G_m}, \tag{3.52c}$$

where $G_{12,f,m}$ = shear moduli of composite, fiber, and matrix, respectively.
From Equation (3.50) through Equation (3.52),

$$\frac{\tau_c}{G_{12}} t_c = \frac{\tau_f}{G_f} t_f + \frac{\tau_m}{G_m} t_m. \tag{3.53}$$

The shear stresses in the fiber, matrix, and composite are assumed to be equal ($\tau_c = \tau_f = \tau_m$), giving

$$\frac{1}{G_{12}} = \frac{1}{G_f}\frac{t_f}{t_c} + \frac{1}{G_m}\frac{t_m}{t_c}. \tag{3.54}$$

Because the thickness fractions are equal to the volume fractions, per Equation (3.28),

$$\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}. \tag{3.55}$$

Example 3.6

Find the in-plane shear modulus of a glass/epoxy lamina with a 70% fiber volume fraction. Use properties of glass and epoxy from Table 3.1 and Table 3.2, respectively.

Solution

The glass fibers and the epoxy matrix have isotropic properties. From Table 3.1, the Young's modulus of the fiber is

$$E_f = 85 \text{ GPa}$$

and the Poisson's ratio of the fiber is

$$\nu_f = 0.2.$$

The shear modulus of the fiber

$$G_f = \frac{E_f}{2(1+\nu_f)}$$

$$= \frac{85}{2(1+0.2)}$$

$$= 35.42 \; GPa.$$

From Table 3.2, the Young's modulus of the matrix is

$$E_m = 3.4 \text{ GPa}$$

and the Poisson's ratio of the fiber is

$$\nu_m = 0.3.$$

The shear modulus of the matrix is

$$G_m = \frac{E_m}{2(1+\nu_m)}$$

$$= \frac{3.40}{2(1+0.3)}$$

$$= 1.308 \; GPa.$$

From Equation (3.55), the in-plane shear modulus of the unidirectional lamina is

$$\frac{1}{G_{12}} = \frac{0.70}{35.42} + \frac{0.30}{1.308}$$

$$G_{12} = 4.014 \; GPa.$$

Figure 3.13a and Figure 3.13b show the analytical values from Equation (3.55) of the in-plane shear modulus as a function of fiber volume fraction for a typical glass/epoxy lamina. Experimental values[4] are also plotted in the same figure.

3.3.2 Semi-Empirical Models

The values obtained for transverse Young's modulus and in-plane shear modulus through Equation (3.41) and Equation (3.55), respectively, do not agree well with the experimental results shown in Figure 3.10 and Figure 3.13. This establishes a need for better modeling techniques. These techniques include numerical methods, such as finite element and finite difference, and boundary element methods, elasticity solution, and variational principal models.[5] Unfortunately, these models are available only as complicated equations or in graphical form. Due to these difficulties, semi-empirical models have been developed for design purposes. The most useful of these models include those of Halphin and Tsai[6] because the[illegible] wide range of elastic properties and fiber volume fract[illegible]

Halphin and Tsai[6] developed their models as simple equ[illegible] [illegible]based on elasticity. The equations are se[illegible] [illegible] parameters in the curve fitting carry p[illegible]

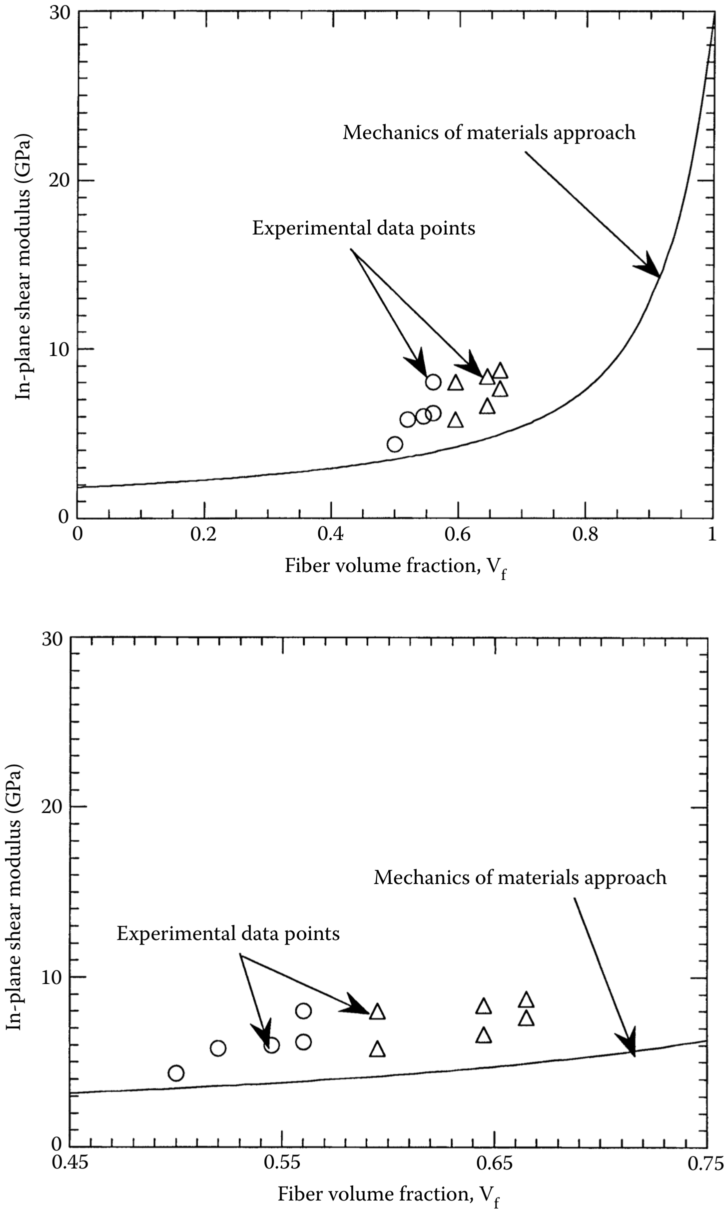


FIGURE 3.13
Theoretical values of in-plane shear modulus as a function of fiber volume fraction and comparison with experimental values for a unidirectional glass/epoxy lamina (G_f = 30.19 GPa, G_m = 1.83 GPa). Figure (b) zooms figure (a) for fiber volume fractio
(Experimental data from Hashin, Z., NASA tech. rep. contract No. NA

3.3.2.1 Longitudinal Young's Modulus

The Halphin–Tsai equation for the longitudinal Young's modulus, E_1, is the same as that obtained through the strength of materials approach — that is,

$$E_1 = E_f V_f + E_m V_m. \tag{3.56}$$

3.3.2.2 Transverse Young's Modulus

The transverse Young's modulus, E_2, is given by[6]

$$\frac{E_2}{E_m} = \frac{1 + \xi \eta V_f}{1 - \eta V_f}, \tag{3.57}$$

where

$$\eta = \frac{(E_f / E_m) - 1}{(E_f / E_m) + \xi}. \tag{3.58}$$

The term ξ is called the reinforcing factor and depends on the following:

- Fiber geometry
- Packing geometry
- Loading conditions

Halphin and Tsai[6] obtained the value of the reinforcing factor ξ by comparing Equation (3.57) and Equation (3.58) to the solutions obtained from the elasticity solutions. For example, for a fiber geometry of circular fibers in a packing geometry of a square array, $\xi = 2$. For a rectangular fiber cross-section of length a and width b in a hexagonal array, $\xi = 2(a/b)$, where b is in the direction of loading.[6] The concept of direction of loading is illustrated in Figure 3.14.

Example 3.7

Find the transverse Young's modulus for a glass/epoxy lamina with a 70% fiber volume fraction. Use the properties for glass and epoxy from Table 3.1 and Table 3.2, respectively. Use Halphin–Tsai equations for a circular fiber in a square array packing geometry.

Solution

; are circular and packed in a square
Table 3.1, the Young's modulus of th

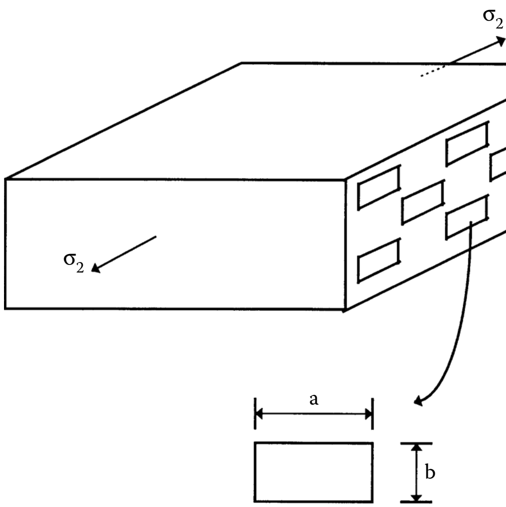


FIGURE 3.14
Concept of direction of loading for calculation of transverse Young's modulus by Halphin–Tsai equations.

From Table 3.2, the Young's modulus of the matrix is E_m = 3.4 GPa.
From Equation (3.58),

$$\eta = \frac{(85/3.4)-1}{(85/3.4)+2}$$

$$= 0.8889.$$

From Equation (3.57), the transverse Young's modulus of the unidirectional lamina is

$$\frac{E_2}{3.4} = \frac{1+2(0.8889)(0.7)}{1-(0.8889)(0.7)}$$

$$E_2 = 20.20\ GPa.$$

For the same problem, from Example 3.4, this value of E_2 was found to be 10.37 GPa by the mechanics of materials approach.

Figure 3.15a and Figure 3.15b show the transverse Young's modulus as a function of fiber volume fraction for a typical boron/epoxy composite. The Halphin–Tsai equations (3.57) and the mechanics of Equation (3.41) curves are shown and compared to expe reviously, the parameters ξ and η hav

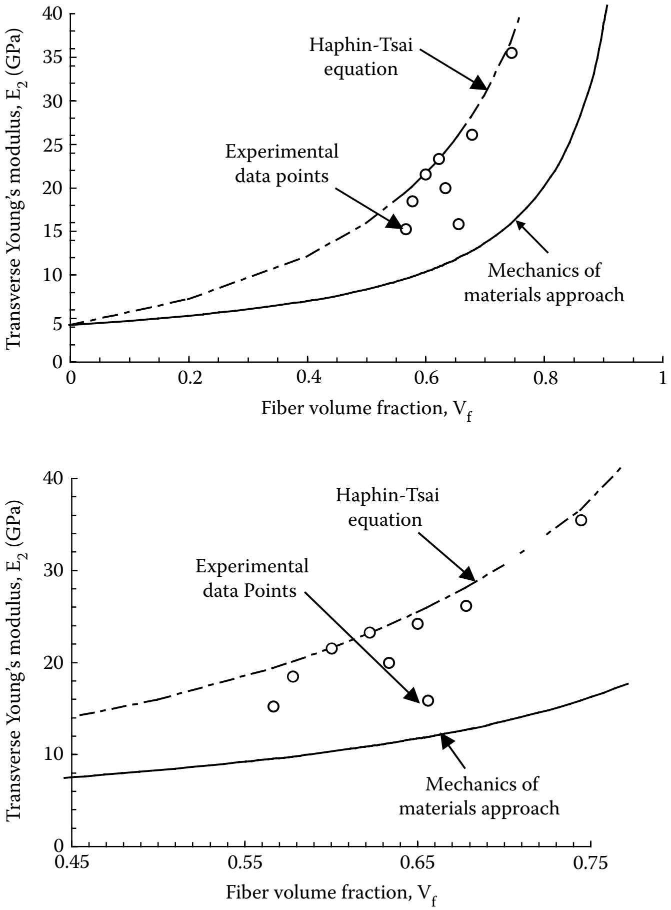


FIGURE 3.15
Theoretical values of transverse Young's modulus as a function of fiber volume fraction and comparison with experimental values for boron/epoxy unidirectional lamina (E_f = 414 GPa, ν_f = 0.2, E_m = 4.14 GPa, ν_m = 0.35). Figure (b) zooms figure (a) for fiber volume fraction between 0.45 and 0.75. (Experimental data from Hashin, Z., NASA tech. rep. contract no. NAS1-8818, November 1970.)

$$E_f/E_m = 1 \text{ implies } \eta = 0, \text{ (homogeneous medium)}$$
$$E_f/E_m \to \infty \text{ implies } \eta = 1 \text{ (rigid inclusions)}$$
$$E_f/E_m \to 0 \text{ implies } \eta = -\frac{1}{\xi} \text{ (voids)}$$

3.3.2.3 *Major Poisson's Ratio*

equation for the major Poisson's rat
ng the strength of materials approach

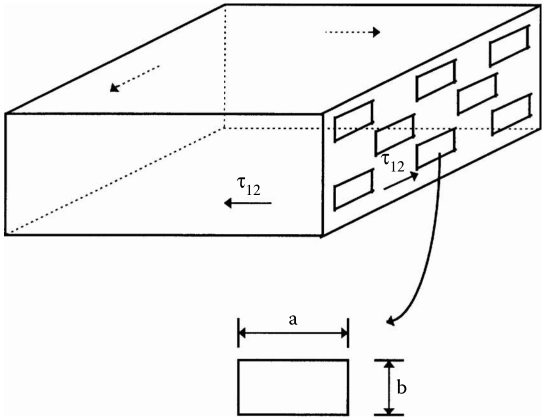


FIGURE 3.16
Concept of direction of loading to calculate in-plane shear modulus by Halphin–Tsai equations.

$$\nu_{12} = \nu_f V_f + \nu_m V_m. \tag{3.59}$$

3.3.2.4 In-Plane Shear Modulus

The Halphin–Tsai[6] equation for the in-plane shear modulus, G_{12}, is

$$\frac{G_{12}}{G_m} = \frac{1+\xi\eta V_f}{1-\eta V_f}, \tag{3.60}$$

where

$$\eta = \frac{(G_f / G_m) - 1}{(G_f / G_m) + \xi}. \tag{3.61}$$

The value of the reinforcing factor, ξ, depends on fiber geometry, packing geometry, and loading conditions. For example, for circular fibers in a square array, $\xi = 1$. For a rectangular fiber cross-sectional area of length a and width b in a hexagonal array, $\xi = \sqrt{3}\log_e(a/b)$, where a is the direction of loading. The concept of the direction of loading[7] is given in Figure 3.16.

The value of $\xi = 1$ for circular fibers in a square array gives reasonable results only for fiber volume fractions of up to 0.5. For example, for a typical glass/epoxy lamina with a fiber volume fraction of 0.75, the value of in-plane shear modulus using the Halphin–Tsai equation with $\xi = 1$ is 30% lower than that given by elasticity solutions. Hewitt
gested choosing a function,

$$\xi = 1 + 40V_f^{10}.$$

Example 3.8

Using Halphin–Tsai equations, find the shear modulus of a glass/epoxy composite with a 70% fiber volume fraction. Use the properties of glass and epoxy from Table 3.1 and Table 3.2, respectively. Assume that the fibers are circular and are packed in a square array. Also, get the value of the shear modulus by using Hewitt and Malherbe's[8] formula for the reinforcing factor.

Solution

For Halphin–Tsai's equations with circular fibers in a square array, the reinforcing factor $\xi = 1$. From Example 3.6, the shear modulus of the fiber is

$$G_f = 35.42 \text{ GPa}$$

and the shear modulus of the matrix is

$$G_m = 1.308 \text{ GPa}.$$

From Equation (3.61),

$$\eta = \frac{(35.42/1.308) - 1}{(35.42/1.308) + 1}$$

$$= 0.9288.$$

From Equation (3.60), the in-plane shear modulus is

$$\frac{G_{12}}{1.308} = \frac{1 + (1)(0.9288)(0.7)}{1 - (0.9288)(0.7)}$$

$$G_{12} = 6.169 \; GPa.$$

For the same problem, the value of G_{12} = 4.013 GPa was found by the mechanics of materials approach in Example 3.5.

Because the volume fraction is greater than 50%, Hewitt and Mahelbre[8] suggested a reinforcing factor (Equation 3.62):

$$\xi = 1 + 40V_f^{10}$$

$$= 1 + 40(0.7)^{10} \; .$$

$$= 2.130$$

ation (3.61),

$$\eta = \frac{(35.42/1.308) - 1}{(35.42/1.308) + 2.130}.$$

$$= 0.8928$$

From Equation (3.60), the in-plane shear modulus is

$$\frac{G_{12}}{1.308} = \frac{1 + (2.130)(0.8928)(0.7)}{1 - (0.8928)(0.7)}.$$

$$G = 8.130\ GPa$$

Figure 3.17a and Figure 3.17b show the in-plane shear modulus as a function of fiber volume fraction for a typical glass/epoxy composite. The Halphin–Tsai equation (3.60) and the mechanics of materials approach, Equation (3.55) are shown and compared to the experimental[4] data points.

3.3.3 Elasticity Approach

In addition to the strength of materials and semi-empirical equation approaches, expressions for the elastic moduli based on elasticity are also available. Elasticity accounts for equilibrium of forces, compatibility, and Hooke's law relationships in three dimensions; the strength of materials approach may not satisfy compatibility and/or account for Hooke's law in three dimensions, and semi-empirical approaches are just as the name implies — partly empirical.

The elasticity models described here are called composite cylinder assemblage (CCA) models.[4,9–12] In a CCA model, one assumes the fibers are circular in cross-section, spread in a periodic arrangement, and continuous, as shown in Figure 3.18. Then the composite can be considered to be made of repeating elements called the representative volume elements (RVEs). The RVE is considered to represent the composite and respond the same as the whole composite does.

The RVE consists of a composite cylinder made of a single inner solid cylinder (fiber) bonded to an outer hollow cylinder (matrix) as shown in Figure 3.19. The radius of the fiber, a, and the outer radius of the matrix, b, are related to the fiber volume fraction, V_f, as

$$V_f = \frac{a^2}{b^2}. \tag{3.63}$$

Appropriate boundary conditions are applied to thi... ...tic moduli being evaluated.

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UNIT-VI

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3

Micromechanical Analysis of a Lamina

Chapter Objectives

- Develop concepts of volume and weight fraction (mass fraction) of fiber and matrix, density, and void fraction in composites.
- Find the nine mechanical and four hygrothermal constants: four elastic moduli, five strength parameters, two coefficients of thermal expansion, and two coefficients of moisture expansion of a unidirectional lamina from the individual properties of the fiber and the matrix, fiber volume fraction, and fiber packing.
- Discuss the experimental characterization of the nine mechanical and four hygrothermal constants.

3.1 Introduction

In Chapter 2, the stress–strain relationships, engineering constants, and failure theories for an angle lamina were developed using four elastic moduli, five strength parameters, two coefficients of thermal expansion (CTE), and two coefficients of moisture expansion (CME) for a unidirectional lamina. These 13 parameters can be found experimentally by conducting several tension, compression, shear, and hygrothermal tests on unidirectional lamina (laminates). However, unlike in isotropic materials, experimental evaluation of these parameters is quite costly and time consuming because they are functions of several variables: the individual constituents of the composite material, fiber volume fraction, packing geometry, processing, etc. Thus, the need and motivation for developing analytical models to find these parameters are very important. In this chapter, we will develop simple relationships for the these parameters in terms of the stiffnesses, strengths, coefficients of thermal and moisture expansion of the individual constituents of a composite, fiber volume fraction, packing geometry, etc. An understand

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FIGURE 3.1
A nonhomogeneous lamina with fibers and matrix approximated as a homogeneous lamina.

relationship, called micromechanics of lamina, helps the designer to select the constituents of a composite material for use in a laminated structure.

Because this text is for a first course in composite materials, details will be explained only for the simple models based on the mechanics of materials approach and the semi-empirical approach. Results from other methods based on advanced topics such as elasticity are also explained for completeness.

As mentioned in Chapter 2, a unidirectional lamina is not homogeneous. However, one can assume the lamina to be homogeneous by focusing on the average response of the lamina to mechanical and hygrothermal loads (Figure 3.1). The lamina is simply looked at as a material whose properties are different in various directions, but not different from one location to another.

Also, the chapter focuses on a unidirectional continuous fiber-reinforced lamina. This is because it forms the basic building block of a composite structure, which is generally made of several unidirectional laminae placed at various angles. The modeling in the evaluation of the parameters is discussed first. This is followed by examples and experimental methods for finding these parameters.

3.2 Volume and Mass Fractions, Density, and Void Content

Before modeling the 13 parameters of a unidirectional composite, we introduce the concept of relative fraction of fibers by volume. This concept is critical because theoretical formulas for finding the stiffness, strength, and hygrothermal properties of a unidirectional lamina are a function of fiber volume fraction. Measurements of the constituents are generally based on their mass, so fiber mass fractions must also be defined. Moreover, defining the density of a composite also becomes necessary because its value is used in the experimental determination of fiber volume and void fractions of a composite. Also, the value of density is used in the definition of specific modulus and specific strength in Chapter 1.

3.2.1 Volume Fractions

Consider a composite consisting of fiber and matrix. Take th
l notations:

$v_{c,f,m}$ = volume of composite, fiber, and matrix, respectively
$\rho_{c,f,m}$ = density of composite, fiber, and matrix, respectively.

Now define the fiber volume fraction V_f and the matrix volume fraction V_m as

$$V_f = \frac{v_f}{v_c},$$

and

$$V_m = \frac{v_m}{v_c}. \tag{3.1a, b}$$

Note that the sum of volume fractions is

$$V_f + V_m = 1,$$

from Equation (3.1) as

$$v_f + v_m = v_c.$$

3.2.2 Mass Fractions

Consider a composite consisting of fiber and matrix and take the following symbol notation: $w_{c,f,m}$ = mass of composite, fiber, and matrix, respectively. The mass fraction (weight fraction) of the fibers (W_f) and the matrix (W_m) are defined as

$$W_f = \frac{w_f}{w_c}, \text{ and}$$

$$W_m = \frac{w_m}{w_c}. \tag{3.2a, b}$$

Note that the sum of mass fractions is

$$W_f + W_m = 1,$$

from Equation (3.2) as

$$w_f + w_m = w_c \,.$$

From the definition of the density of a single material,

$$w_c = r_c v_c ,$$

$$w_f = r_f v_f , \text{ and} \qquad (3.3a–c)$$

$$w_m = r_m v_m .$$

Substituting Equation (3.3) in Equation (3.2), the mass fractions and volume fractions are related as

$$W_f = \frac{\rho_f}{\rho_c} V_f \text{ , and}$$

$$W_m = \frac{\rho_m}{\rho_c} V_m , \qquad (3.4a, b)$$

in terms of the fiber and matrix volume fractions. In terms of individual constituent properties, the mass fractions and volume fractions are related by

$$W_f = \frac{\frac{\rho_f}{\rho_m}}{\frac{\rho_f}{\rho_m} V_f + V_m} V_f ,$$

$$W_m = \frac{1}{\frac{\rho_f}{\rho_m}(1 - V_m) + V_m} V_m \,. \qquad (3.5a, b)$$

One should always state the basis of calculating the fiber content of a composite. It is given in terms of mass or volume. Based on Equation (3.4), it is evident that volume and mass fractions are not equal a[...] mismatch between the mass and volume fractions increases a[...] [...]n the density of fiber and matrix differs from one.

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3.2.3 Density

The derivation of the density of the composite in terms of volume fractions is found as follows. The mass of composite w_c is the sum of the mass of the fibers w_f and the mass of the matrix w_m as

$$w_c = w_f + w_m. \tag{3.6}$$

Substituting Equation (3.3) in Equation (3.6) yields

$$\rho_c v_c = \rho_f v_f + \rho_m v_m,$$

and

$$\rho_c = \rho_f \frac{v_f}{v_c} + \rho_m \frac{v_m}{v_c}. \tag{3.7}$$

Using the definitions of fiber and matrix volume fractions from Equation (3.1),

$$\rho_c = \rho_f V_f + \rho_m V_m. \tag{3.8}$$

Now, consider that the volume of a composite v_c is the sum of the volumes of the fiber v_f and matrix (v_m):

$$v_c = v_f + v_m. \tag{3.9}$$

The density of the composite in terms of mass fractions can be found as

$$\frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}. \tag{3.10}$$

Example 3.1

A glass/epoxy lamina consists of a 70% fiber volume fraction. Use properties of glass and epoxy from Table 3.1* and Table 3.2, respectively, to determine the

* Table 3.1 and Table 3.2 give the typical properties of common fibers and matrices tem of units, respectively. Note that fibers such as graphite and aramids are trans natrices are generally isotropic. The typical properties of common fibers given in Table 3.3 and Table 3.4, respectively, in the USCS system of unit

TABLE 3.1

Typical Properties of Fibers (SI System of Units)

Property	Units	Graphite	Glass	Aramid
Axial modulus	GPa	230	85	124
Transverse modulus	GPa	22	85	8
Axial Poisson's ratio	—	0.30	0.20	0.36
Transverse Poisson's ratio	—	0.35	0.20	0.37
Axial shear modulus	GPa	22	35.42	3
Axial coefficient of thermal expansion	μm/m/°C	–1.3	5	–5.0
Transverse coefficient of thermal expansion	μm/m/°C	7.0	5	4.1
Axial tensile strength	MPa	2067	1550	1379
Axial compressive strength	MPa	1999	1550	276
Transverse tensile strength	MPa	77	1550	7
Transverse compressive strength	MPa	42	1550	7
Shear strength	MPa	36	35	21
Specific gravity	—	1.8	2.5	1.4

TABLE 3.2

Typical Properties of Matrices (SI System of Units)

Property	Units	Epoxy	Aluminum	Polyamide
Axial modulus	GPa	3.4	71	3.5
Transverse modulus	GPa	3.4	71	3.5
Axial Poisson's ratio	—	0.30	0.30	0.35
Transverse Poisson's ratio	—	0.30	0.30	0.35
Axial shear modulus	GPa	1.308	27	1.3
Coefficient of thermal expansion	μm/m/°C	63	23	90
Coefficient of moisture expansion	m/m/kg/kg	0.33	0.00	0.33
Axial tensile strength	MPa	72	276	54
Axial compressive strength	MPa	102	276	108
Transverse tensile strength	MPa	72	276	54
Transverse compressive strength	MPa	102	276	108
Shear strength	MPa	34	138	54
Specific gravity	—	1.2	2.7	1.2

1. Density of lamina
2. Mass fractions of the glass and epoxy
3. Volume of composite lamina if the mass of the lamina is 4 kg
4. Volume and mass of glass and epoxy in part (3)

Solution

1. From Table 3.1, the density of the fiber is

$$\rho_f = 2500\ kg/m^3.$$

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TABLE 3.3

Typical Properties of Fibers (USCS System of Units)

Property	Units	Graphite	Glass	Aramid
Axial modulus	Msi	33.35	12.33	17.98
Transverse modulus	Msi	3.19	12.33	1.16
Axial Poisson's ratio	—	0.30	0.20	0.36
Transverse Poisson's ratio	—	0.35	0.20	0.37
Axial shear modulus	Msi	3.19	5.136	0.435
Axial coefficient of thermal expansion	μin./in./°F	−0.7222	2.778	−2.778
Transverse coefficient of thermal expansion	μin./in./°F	3.889	2.778	2.278
Axial tensile strength	ksi	299.7	224.8	200.0
Axial compressive strength	ksi	289.8	224.8	40.02
Transverse tensile strength	ksi	11.16	224.8	1.015
Transverse compressive strength	ksi	6.09	224.8	1.015
Shear strength	ksi	5.22	5.08	3.045
Specific gravity	—	1.8	2.5	1.4

TABLE 3.4

Typical Properties of Matrices (USCS System of Units)

Property	Units	Epoxy	Aluminum	Polyamide
Axial modulus	Msi	0.493	10.30	0.5075
Transverse modulus	Msi	0.493	10.30	0.5075
Axial Poisson's ratio	—	0.30	0.30	0.35
Transverse Poisson's ratio	—	0.30	0.30	0.35
Axial shear modulus	Msi	0.1897	3.915	0.1885
Coefficient of thermal expansion	μin./in./°F	35	12.78	50
Coefficient of moisture expansion	in./in./lb/lb	0.33	0.00	0.33
Axial tensile strength	ksi	10.44	40.02	7.83
Axial compressive strength	ksi	14.79	40.02	15.66
Transverse tensile strength	ksi	10.44	40.02	7.83
Transverse compressive strength	ksi	14.79	40.02	15.66
Shear strength	ksi	4.93	20.01	7.83
Specific gravity	—	1.2	2.7	1.2

From Table 3.2, the density of the matrix is

$$\rho_m = 1200\ kg/m^3.$$

Using Equation (3.8), the density of the composite is

$$\rho_c = (2500)(0.7) + (1200)(0.3)$$
$$= 2110\ kg/m^3.$$

2. Using Equation (3.4), the fiber and matrix mass fractions a

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$$W_f = \frac{2500}{2110} \times 0.3$$

$$= 0.8294$$

$$W_m = \frac{1200}{2110} \times 0.3 .$$

$$= 0.1706$$

Note that the sum of the mass fractions,

$$W_f + W_m = 0.8294 + 0.1706$$

$$= 1.000.$$

3. The volume of composite is

$$v_c = \frac{w_c}{\rho_c}$$

$$= \frac{4}{2110}$$

$$= 1.896 \times 10^{-3} m^3 .$$

4. The volume of the fiber is

$$v_f = V_f v_c$$

$$= (0.7)(1.896 \times 10^{-3})$$

$$= 1.327 \times 10^{-3} m^3 .$$

The volume of the matrix is

$$v_m = V_m v_c$$

$$=(0.3)(0.1896 \times 10^{-3})$$

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$$= 0.5688 \times 10^{-3}\ m^3 .$$

The mass of the fiber is

$$w_f = \rho_f v_f$$

$$= (2500)(1.327 \times 10^{-3})$$

$$= 3.318\ kg .$$

The mass of the matrix is

$$w_m = \rho_m v_m$$

$$= (1200)(0.5688 \times 10^{-3})$$

$$= 0.6826\ kg .$$

3.2.4 Void Content

During the manufacture of a composite, voids are introduced in the composite as shown in Figure 3.2. This causes the theoretical density of the composite to be higher than the actual density. Also, the void content of a

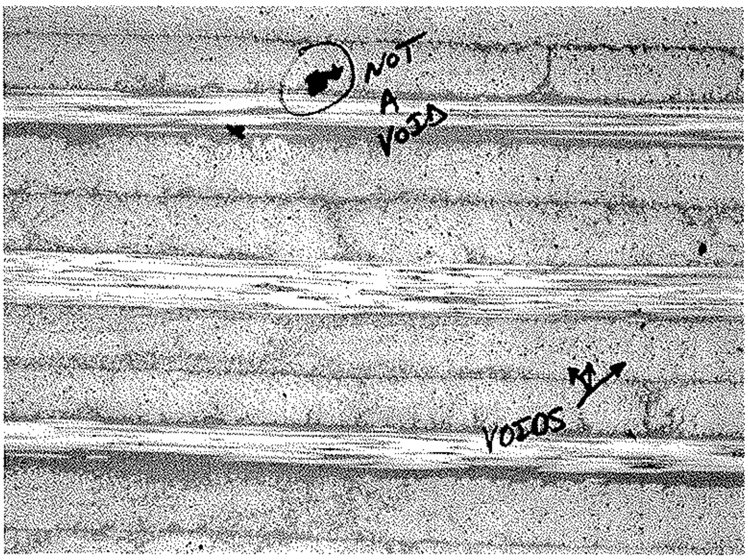

FIGURE 3.2
Photomicrographs of cross-section of a lamina with voids.

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composite is detrimental to its mechanical properties. These detriments include lower

- Shear stiffness and strength
- Compressive strengths
- Transverse tensile strengths
- Fatigue resistance
- Moisture resistance

A decrease of 2 to 10% in the preceding matrix-dominated properties generally takes place with every 1% increase in the void content.[1]

For composites with a certain volume of voids V_v the volume fraction of voids V_v is defined as

$$V_v = \frac{v_v}{v_c}. \tag{3.11}$$

Then, the total volume of a composite (v_c) with voids is given by

$$v_c = v_f + v_m + v_v. \tag{3.12}$$

By definition of the experimental density ρ_{ce} of a composite, the actual volume of the composite is

$$v_c = \frac{w_c}{\rho_{ce}}, \tag{3.13}$$

and, by the definition of the theoretical density ρ_{ct} of the composite, the theoretical volume of the composite is

$$v_f + v_m = \frac{w_c}{\rho_{ct}}. \tag{3.14}$$

Then, substituting the preceding expressions (3.13) and (3.14) in Equation (3.12),

$$\frac{w_c}{\rho_{ce}} = \frac{w_c}{\rho_{ct}} + v_v .$$

The volume of void is given by

$$v_v = \frac{w_c}{\rho_{ce}}\left(\frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}\right). \tag{3.15}$$

Substituting Equation (3.13) and Equation (3.15) in Equation (3.11), the volume fraction of the voids is

$$V_v = \frac{v_v}{v_c} = \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}. \tag{3.16}$$

Example 3.2

A graphite/epoxy cuboid specimen with voids has dimensions of $a \times b \times c$ and its mass is M_c. After it is put it into a mixture of sulfuric acid and hydrogen peroxide, the remaining graphite fibers have a mass M_f. From independent tests, the densities of graphite and epoxy are ρ_f and ρ_m, respectively. Find the volume fraction of the voids in terms of a, b, c, M_f, M_c, ρ_f, and ρ_m.

Solution

The total volume of the composite v_c is the sum total of the volume of fiber v_f, matrix v_m, and voids v_v:

$$v_c = v_f + v_m + v_v. \tag{3.17}$$

From the definition of density,

$$v_f = \frac{M_f}{\rho_f}, \tag{3.18a}$$

$$v_m = \frac{M_c - M_f}{\rho_m}. \tag{3.18b}$$

The specimen is a cuboid, so the volume of the composite is

$$v_c = abc. \tag{3.19}$$

...tituting Equation (3.18) and Equation (3.19) in Equation (

$$abc = \frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m} + v_v \, ,$$

and the volume fraction of voids then is

$$V_v = \frac{v_v}{abc} = 1 - \frac{1}{abc}\left[\frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m}\right] \tag{3.20}$$

Alternative Solution

The preceding problem can also be solved by using Equation (3.16). The theoretical density of the composite is

$$\rho_{ct} = \rho_f V_f' + \rho_m (1 - V_f') \, , \tag{3.21}$$

where V_f' is the theoretical fiber volume fraction given as

$$V_f' = \frac{volume \ of \ fibers}{volume \ of \ fibers + volume \ of \ matrix}$$

$$V_f' = \frac{\frac{M_f}{\rho_f}}{\frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m}} . \tag{3.22}$$

The experimental density of the composite is

$$\rho_{ce} = \frac{M_c}{abc} . \tag{3.23}$$

Substituting Equation (3.21) through Equation (3.23) in the definition of void volume fractions given by Equation (3.16),

$$V_v = 1 - \frac{1}{abc}\left[\frac{M_f}{\rho_f} + \frac{M_c - M_f}{\rho_m}\right] . \tag{3.24}$$

Experimental determination: the fiber volume fractions of the con
a composite are found generally by the burn or the acid digestion
volve taking a sample of composite and weighing it. Then

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of the specimen is found by the liquid displacement method in which the sample is weighed in air and then in water. The density of the composite is given by

$$\rho_c = \frac{w_c}{w_c - w_i}\rho_w , \quad (3.25)$$

where

w_c = weight of composite
w_i = weight of composite when immersed in water
ρ_w = density of water (1000 kg/m^3 or 62.4 lb/ft^3)

For specimens that float in water, a sinker is attached. The density of the composite is then found by

$$\rho_c = \frac{w_c}{w_c + w_s - w_w}\rho_w , \quad (3.26)$$

where

w_c = weight of composite
w_s = weight of sinker when immersed in water
w_w = weight of sinker and specimen when immersed in water

The sample is then dissolved in an acid solution or burned.[2] Glass-based composites are burned, and carbon and aramid-based composites are digested in solutions. Carbon and aramid-based composites cannot be burned because carbon oxidizes in air above 300°C (572°F) and the aramid fiber can decompose at high temperatures. Epoxy-based composites can be digested by nitric acid or a hot mixture of ethylene glycol and potassium hydroxide; polyamide- and phenolic resin-based composites use mixtures of sulfuric acid and hydrogen peroxide. When digestion or burning is complete, the remaining fibers are washed and dried several times and then weighed. The fiber and matrix weight fractions can be found using Equation (3.2). The densities of the fiber and the matrix are known; thus, one can use Equation (3.4) to determine the volume fraction of the constituents of the composite and Equation (3.8) to calculate the theoretical density of the composite.

3.3 Evaluation of the Four Elastic Moduli

As shown in Section 2.4.3, there are four elastic moduli of a un
lamina:

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- Longitudinal Young's modulus, E_1
- Transverse Young's modulus, E_2
- Major Poisson's ratio, ν_{12}
- In-plane shear modulus, G_{12}

Three approaches for determining the four elastic moduli are discussed next.

3.3.1 Strength of Materials Approach

From a unidirectional lamina, take a representative volume element* that consists of the fiber surrounded by the matrix (Figure 3.3). This representative volume element (RVE) can be further represented as rectangular blocks. The fiber, matrix, and the composite are assumed to be of the same width, h, but of thicknesses t_f, t_m, and t_c, respectively. The area of the fiber is given by

$$A_f = t_f h \,. \tag{3.27a}$$

The area of the matrix is given by

$$A_m = t_m h, \tag{3.27b}$$

and the area of the composite is given by

$$A_c = t_c h. \tag{3.27c}$$

The two areas are chosen in the proportion of their volume fractions so that the fiber volume fraction is defined as

$$\begin{aligned} V_f &= \frac{A_f}{A_c} \\ &= \frac{t_f}{t_c}, \end{aligned} \tag{3.28a}$$

and the matrix fiber volume fraction V_m is

* A representative volume element (RVE) of a material is the smallest part of th… …ts the material as a whole. It could be otherwise intractable to account fc… …ie constituents of the material.

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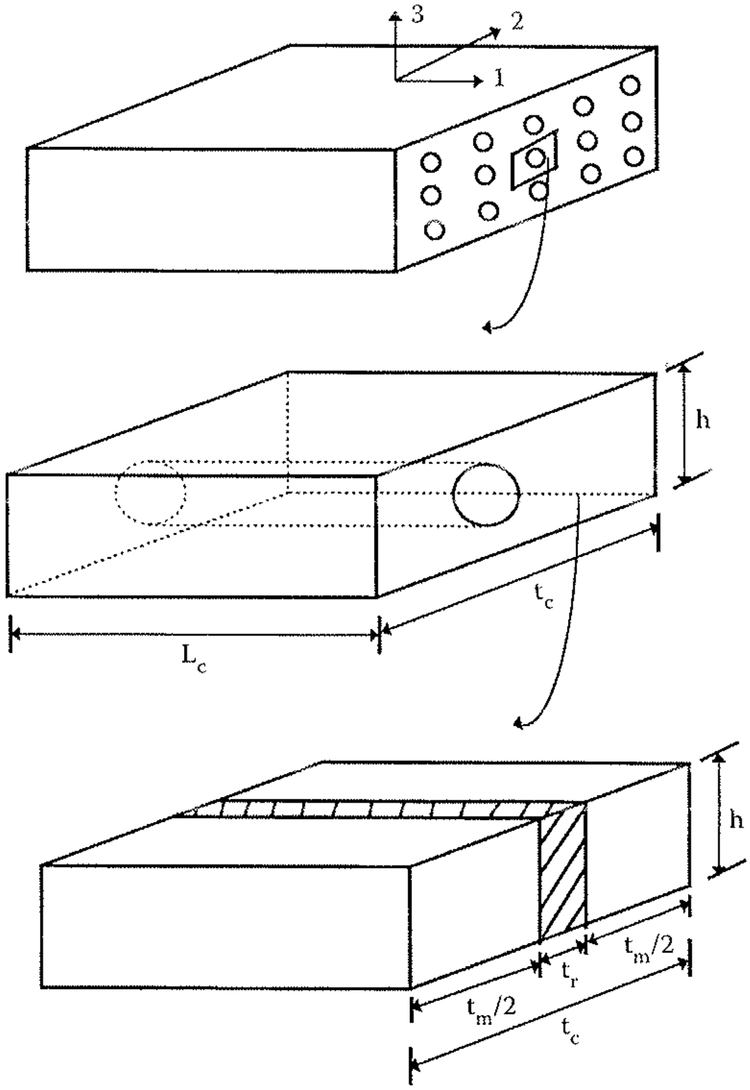

FIGURE 3.3
Representative volume element of a unidirectional lamina.

$$V_m = \frac{A_m}{A_c}$$

$$= \frac{t_m}{t_c} \quad (3.28b)$$

$$= 1 - V_f.$$

The following assumptions are made in the strength of materials approach model:

- The bond between fibers and matrix is perfect.
- The elastic moduli, diameters, and space between fibers are
  he fibers are continuous and parallel.

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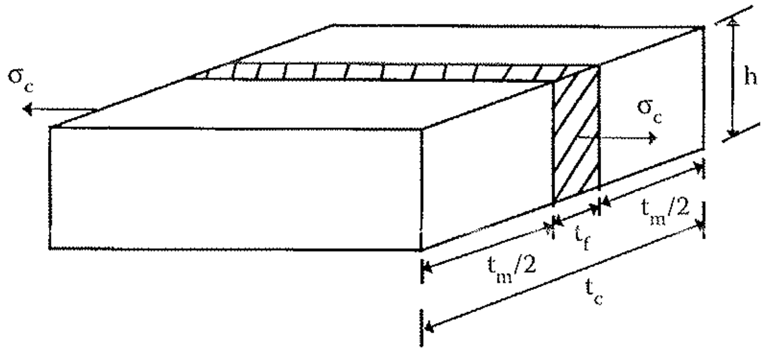

FIGURE 3.4
A longitudinal stress applied to the representative volume element to calculate the longitudinal Young's modulus for a unidirectional lamina.

- The fibers and matrix follow Hooke's law (linearly elastic).
- The fibers possess uniform strength.
- The composite is free of voids.

3.3.1.1 *Longitudinal Young's Modulus*

From Figure 3.4, under a uniaxial load F_c on the composite RVE, the load is shared by the fiber F_f and the matrix F_m so that

$$F_c = F_f + F_m. \tag{3.29}$$

The loads taken by the fiber, the matrix, and the composite can be written in terms of the stresses in these components and cross-sectional areas of these components as

$$F_c = \sigma_c A_c, \tag{3.30a}$$

$$F_f = \sigma_f A_f, \tag{3.30b}$$

$$F_m = \sigma_m A_m, \tag{3.30c}$$

where

$\sigma_{c,f,m}$ = stress in composite, fiber, and matrix, respectively
$A_{c,f,m}$ = area of composite, fiber, and matrix, respectively

Assuming that the fibers, matrix, and composite follow Hooke's law and that the fibers and the matrix are isotropic, the stress–strain relat each component and the composite is

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$$\sigma_c = E_1\varepsilon_c, \tag{3.31a}$$

$$\sigma_f = E_f\varepsilon_f, \tag{3.31b}$$

and

$$\sigma_m = E_m\varepsilon_m, \tag{3.31c}$$

where

$\varepsilon_{c,f,m}$ = strains in composite, fiber, and matrix, respectively
$E_{1,f,m}$ = elastic moduli of composite, fiber, and matrix, respectively

Substituting Equation (3.30) and Equation (3.31) in Equation (3.29) yields

$$E_1\varepsilon_c A_c = E_f\varepsilon_f A_f + E_m\varepsilon_m A_m. \tag{3.32}$$

The strains in the composite, fiber, and matrix are equal ($\varepsilon_c = \varepsilon_f = \varepsilon_m$); then, from Equation (3.32),

$$E_1 = E_f\frac{A_f}{A_c} + E_m\frac{A_m}{A_c}. \tag{3.33}$$

Using Equation (3.28), for definitions of volume fractions,

$$E_1 = E_f V_f + E_m V_m. \tag{3.34}$$

Equation 3.34 gives the longitudinal Young's modulus as a weighted mean of the fiber and matrix modulus. It is also called the rule of mixtures.

The ratio of the load taken by the fibers F_f to the load taken by the composite F_c is a measure of the load shared by the fibers. From Equation (3.30) and Equation (3.31),

$$\frac{F_f}{F_c} = \frac{E_f}{E_1}V_f. \tag{3.35}$$

In Figure 3.5, the ratio of the load carried by the fibers to the load taken by the composite is plotted as a function of fiber-to-matrix Young's moduli ratio E_f/E_m for the constant fiber volume fraction V_f. It shows that to matrix moduli ratio increases, the load taken by the fiber in usly.

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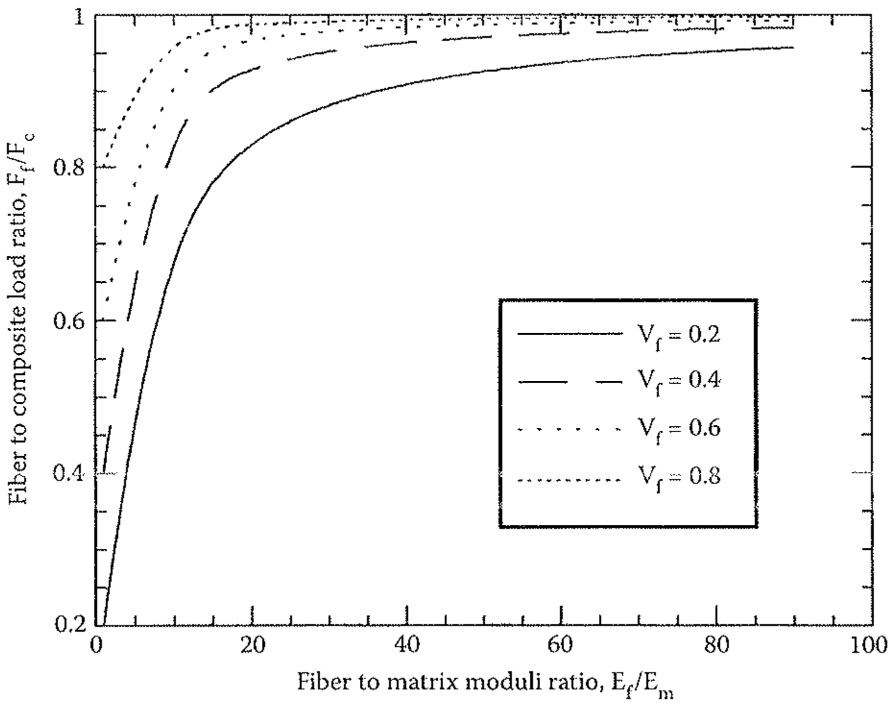

FIGURE 3.5
Fraction of load of composite carried by fibers as a function of fiber volume fraction for constant fiber to matrix moduli ratio.

Example 3.3

Find the longitudinal elastic modulus of a unidirectional glass/epoxy lamina with a 70% fiber volume fraction. Use the properties of glass and epoxy from Table 3.1 and Table 3.2, respectively. Also, find the ratio of the load taken by the fibers to that of the composite.

Solution

From Table 3.1, the Young's modulus of the fiber is

$$E_f = 85 \text{ GPa}.$$

From Table 3.2, the Young's modulus of the matrix is

$$E_m = 3.4 \text{ GPa}.$$

Using Equation (3.34), the longitudinal elastic modulus of the unidirectional lamina is

$$E_1 = (85)(0.7) + (3.4)(0.3)$$

$$= 60.52 \; GPa.$$

Using Equation (3.35), the ratio of the load taken by the fibers tc
site is

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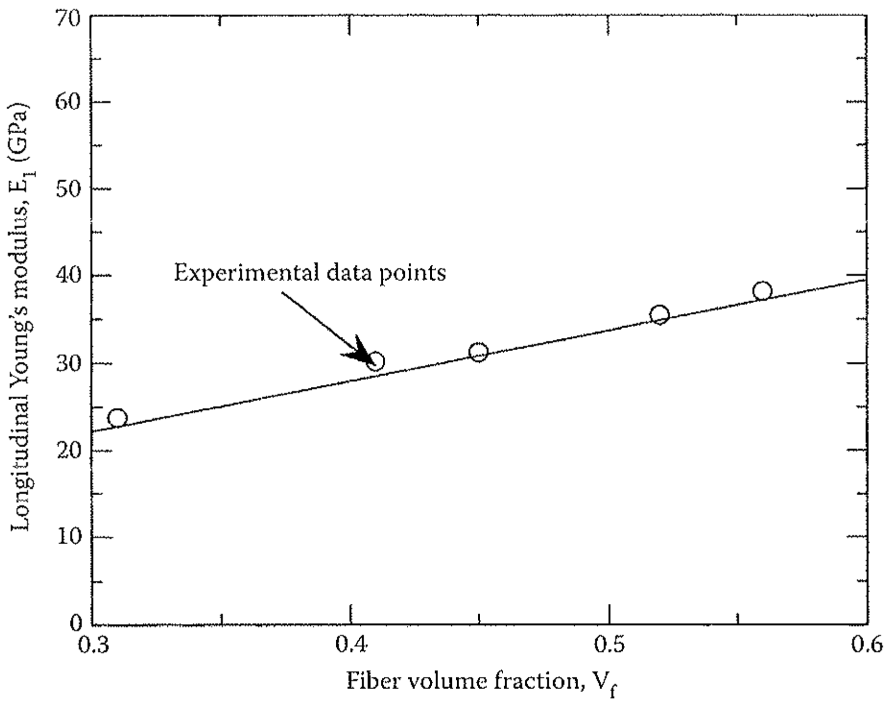


FIGURE 3.6
Longitudinal Young's modulus as function of fiber volume fraction and comparison with experimental data points for a typical glass/polyester lamina. (Experimental data points reproduced with permission of ASM International.)

$$\frac{F_f}{F_c} = \frac{85}{60.52}(0.7)$$

$$= 0.9831.$$

Figure 3.6 shows the linear relationship between the longitudinal Young's modulus of a unidirectional lamina and fiber volume fraction for a typical graphite/epoxy composite per Equation (3.34). It also shows that Equation (3.34) predicts results that are close to the experimental data points.[3]

3.3.1.2 Transverse Young's Modulus

Assume now that, as shown in Figure 3.7, the composite is stressed in the transverse direction. The fibers and matrix are again represented by rectangular blocks as shown. The fiber, the matrix, and composite stresses are equal. Thus,

$$\sigma_c = \sigma_f = \sigma_m, \tag{3.36}$$

where $\sigma_{c,f,m}$ = stress in composite, fiber, and matrix, respectively.

Now, the transverse extension in the composite Δ_c is the sum c

:xtension in the fiber Δ_f, and that is the matrix, Δ_m.

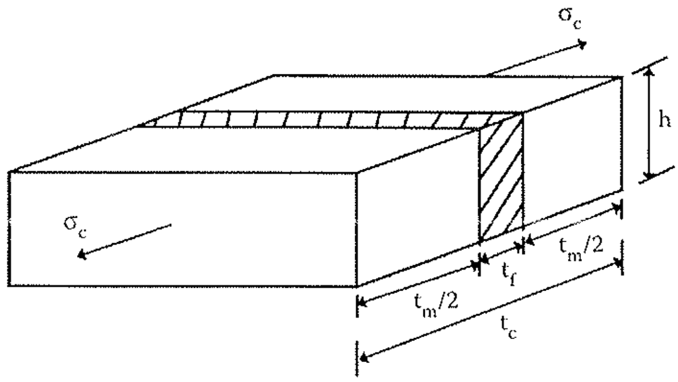


FIGURE 3.7
A transverse stress applied to a representative volume element used to calculate transverse Young's modulus of a unidirectional lamina.

$$\Delta_c = \Delta_f + \Delta_m. \tag{3.37}$$

Now, by the definition of normal strain,

$$\Delta_c = t_c \varepsilon_c, \tag{3.38a}$$

$$\Delta_f = t_f \varepsilon_f, \tag{3.38b}$$

and

$$\Delta_m = t_m \varepsilon_m, \tag{3.38c}$$

where

$t_{c,f,m}$ = thickness of the composite, fiber and matrix, respectively
$\varepsilon_{c,f,m}$ = normal transverse strain in the composite, fiber, and matrix, respectively

Also, by using Hooke's law for the fiber, matrix, and composite, the normal strains in the composite, fiber, and matrix are

$$\varepsilon_c = \frac{\sigma_c}{E_2}, \tag{3.39a}$$

$$\varepsilon_f = \frac{\sigma_f}{E_f}, \tag{3.39b}$$

and

$$\varepsilon_m = \frac{\sigma_m}{E_m}.$$

Substituting Equation (3.38) and Equation (3.39) in Equation (3.37) and using Equation (3.36) gives

$$\frac{1}{E_2} = \frac{1}{E_f}\frac{t_f}{t_c} + \frac{1}{E_m}\frac{t_m}{t_c}. \tag{3.40}$$

Because the thickness fractions are the same as the volume fractions as the other two dimensions are equal for the fiber and the matrix (see Equation 3.28):

$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}. \tag{3.41}$$

Equation (3.41) is based on the weighted mean of the compliance of the fiber and the matrix.

Example 3.4

Find the transverse Young's modulus of a glass/epoxy lamina with a fiber volume fraction of 70%. Use the properties of glass and epoxy from Table 3.1 and Table 3.2, respectively.

Solution

From Table 3.1, the Young's modulus of the fiber is

$$E_f = 85 \text{ GPa}.$$

From Table 3.2, the Young's modulus of the matrix is

$$E_m = 3.4 \text{ GPa}.$$

Using Equation (3.41), the transverse Young's modulus, E_2, is

$$\frac{1}{E_2} = \frac{0.7}{85} + \frac{0.3}{3.4},$$

$$E_2 = 10.37 \; GPa.$$

Figure 3.8 plots the transverse Young's modulus as a function of fiber volume fraction for constant fiber-to-matrix elastic moduli ratio, E_f/E_m. For metal and ceramic matrix composites, the fiber and matrix elastic moduli are of the same order. (For example, for a SiC/aluminum metal matrix composite, $E_f/E_m = 4$ and for a SiC/CAS ceramic matrix compos
2) The transverse Young's modulus of the composite in such cas
moothly as a function of the fiber volume fraction.

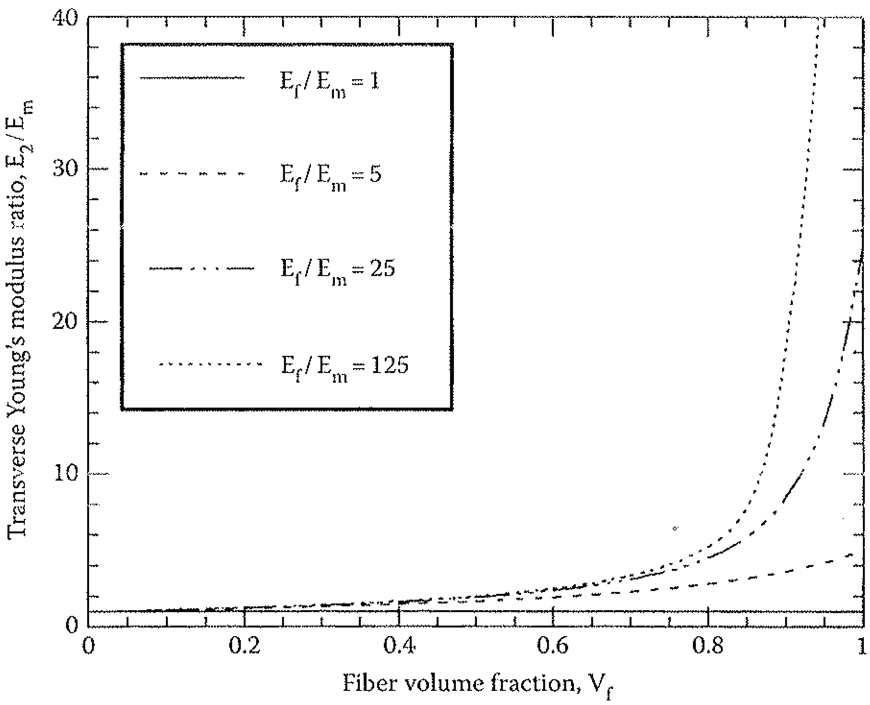


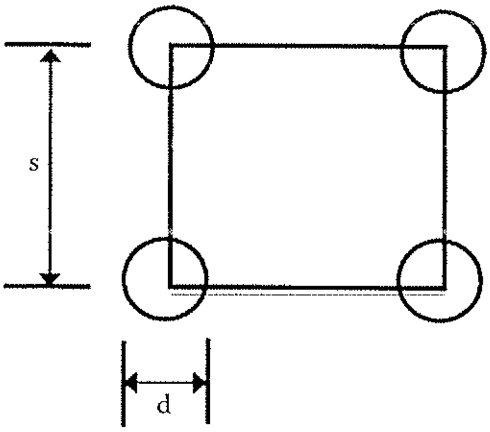
FIGURE 3.8
Transverse Young's modulus as a function of fiber volume fraction for constant fiber to matrix moduli ratio.

For polymeric composites, the fiber-to-matrix moduli ratio is very high. (For example, for a glass/epoxy polymer matrix composite, $E_f/E_m = 25$). The transverse Young's modulus of the composite in such cases changes appreciably only for large fiber volume fractions. Figure 3.8 shows that, for high E_f/E_m ratios, the contribution of the fiber modulus only increases substantially for a fiber volume fraction greater than 80%. These fiber volume fractions are not practical and in many cases are physically impossible due to the geometry of fiber packing. Figure 3.9 shows various possibilities of fiber packing. Note that the ratio of the diameter, d, to fiber spacing, s, d/s varies with geometrical packing. For circular fibers with square array packing (Figure 3.9a),

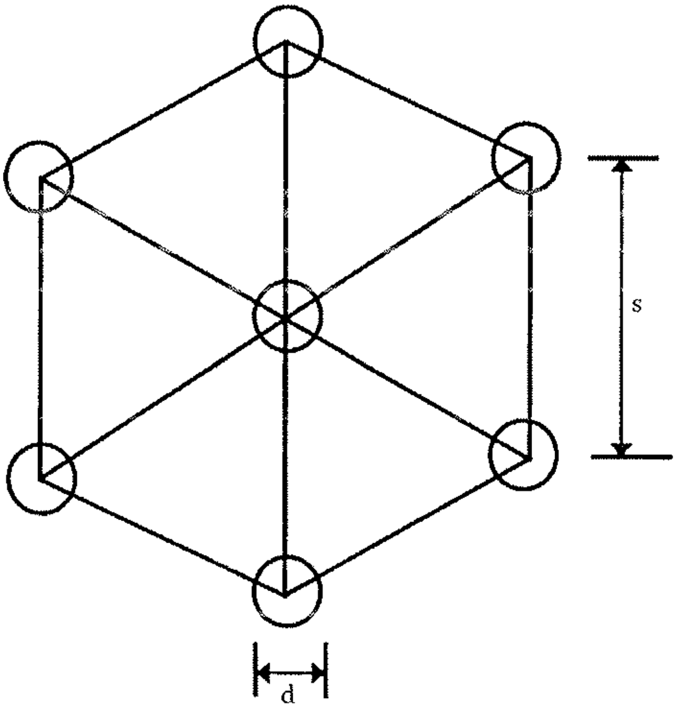
$$\frac{d}{s} = \left(\frac{4V_f}{\pi}\right)^{1/2}. \tag{3.42a}$$

This gives a maximum fiber volume fraction of 78.54% as $s \geq d$. For circular fibers with hexagonal array packing (Figure 3.9b),

$$\frac{d}{s} = \left(\frac{2\sqrt{3}V_f}{\pi}\right)^{1/2}. \tag{3.42b}$$



(a)



(b)

FIGURE 3.9
Fiber to fiber spacing in (a) square packing geometry and (b) hexagonal packing geometry.

This gives a maximum fiber volume fraction of 90.69% because $s \geq d$. These maximum fiber volume fractions are not practical to use because the fibers touch each other and thus have surfaces where the matrix cannot wet out the fibers.

In Figure 3.10, the transverse Young's modulus is plotted as a function of fiber volume fraction using Equation (3.41) for a typical boron/epoxy lamina. Also given are the experimental data points.[4] In Figure 3.10, the experimental and analytical results are not as close to each other as they longitudinal Young's modulus in Figure 3.6.

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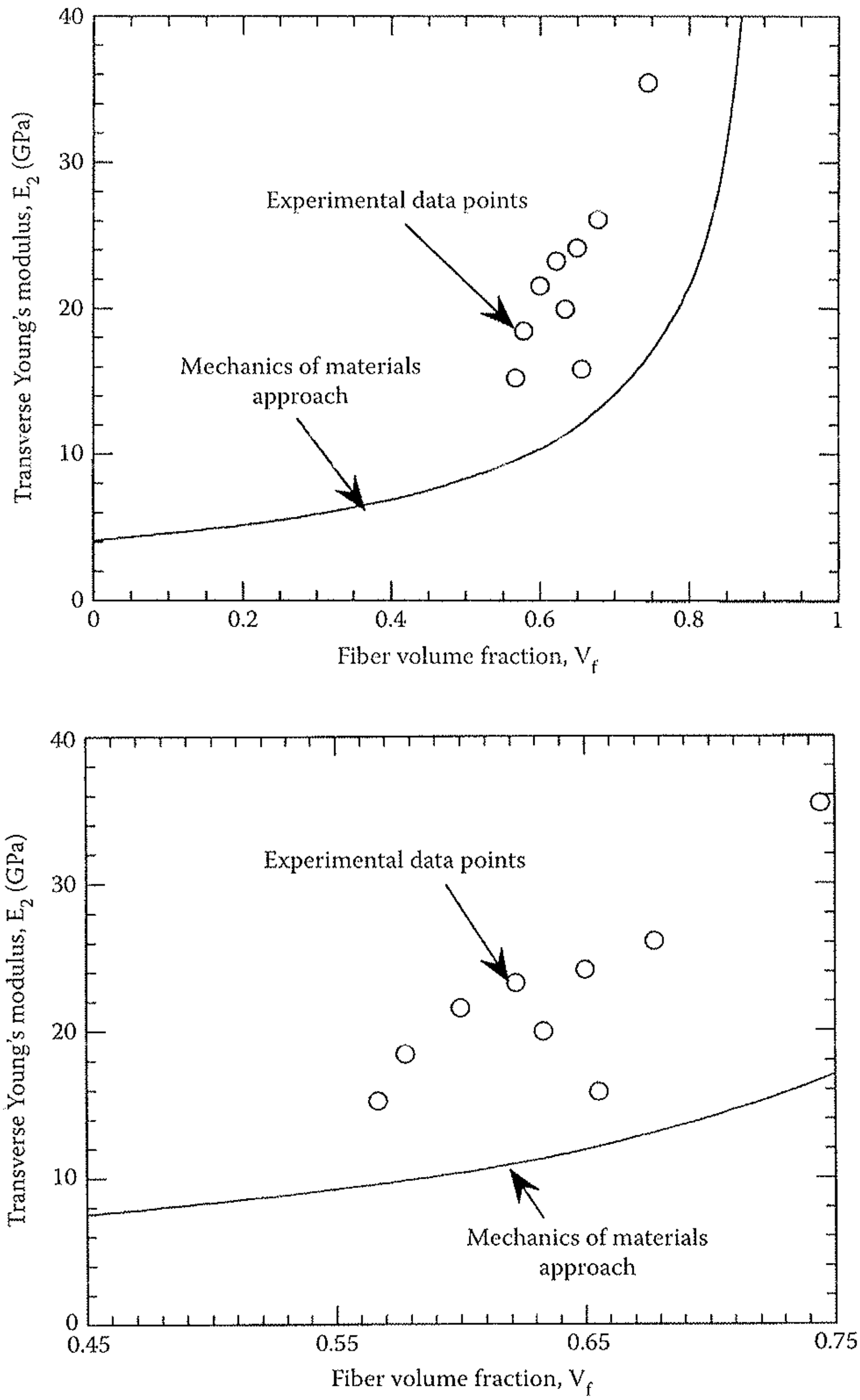

FIGURE 3.10
Theoretical values of transverse Young's modulus as a function of fiber volume fraction for a Boron/Epoxy unidirectional lamina (E_f = 414 GPa, ν_f = 0.2, E_m = 4.14 GPa, ν_m = 0.35) and comparison with experimental values. Figure (b) zooms figure (a) for fiber volume fraction between 0.45 and 0.75. (Experimental data from Hashin, Z., NASA tech. rep. contract no. NAS1-8818, November 1970.)

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Because the thickness fractions are the same as the volume fractions, per Equation (3.28),

$$\nu_{12} = \nu_f V_f + \nu_m V_m. \tag{3.49}$$

Example 3.5

Find the major and minor Poisson's ratio of a glass/epoxy lamina with a 70% fiber volume fraction. Use the properties of glass and epoxy from Table 3.1 and Table 3.2, respectively.

Solution

From Table 3.1, the Poisson's ratio of the fiber is

$$\nu_f = 0.2.$$

From Table 3.2, the Poisson's ratio of the matrix is

$$\nu_m = 0.3.$$

Using Equation (3.49), the major Poisson's ratio is

$$\nu_{12} = (0.2)(0.7) + (0.3)(0.3)$$

$$= 0.230.$$

From Example 3.3, the longitudinal Young's modulus is

$$E_1 = 60.52 \text{ GPa}$$

and, from Example 3.4, the transverse Young's modulus is

$$E_2 = 10.37 \text{ GPa}.$$

Then, the minor Poisson's ratio from Equation (2.83) is

$$\nu_{21} = \nu_{12}\frac{E_2}{E_1}$$

$$= 0.230\left(\frac{10.37}{60.52}\right)$$

$$= 0.03941.$$

3.3.1.4 In-Plane Shear Modulus

Apply a pure shear stress τ_c to a lamina as shown in Figure 3.12. ...atrix are represented by rectangular blocks as shown. Th

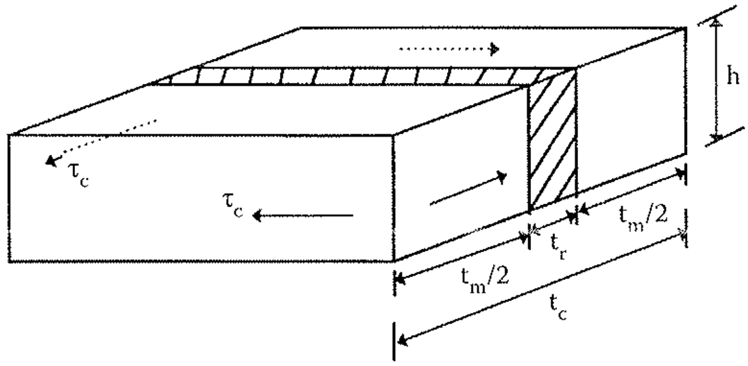

FIGURE 3.12
An in-plane shear stress applied to a representative volume element for finding in-plane shear modulus of a unidirectional lamina.

shear deformations of the composite δ_c the fiber δ_f, and the matrix δ_m are related by

$$\delta_c = \delta_f + \delta_m \,. \tag{3.50}$$

From the definition of shear strains,

$$\delta_c = \gamma_c t_c \,, \tag{3.51a}$$

$$\delta_f = \gamma_f t_f \,, \tag{3.51b}$$

and

$$\delta_m = \gamma_m t_m \,, \tag{3.51c}$$

where

$\gamma_{c,f,m}$ = shearing strains in the composite, fiber, and matrix, respectively
$t_{c,f,m}$ = thickness of the composite, fiber, and matrix, respectively.

From Hooke's law for the fiber, the matrix, and the composite,

$$\gamma_c = \frac{\tau_c}{G_{12}} \,, \tag{3.52a}$$

$$\gamma_f = \frac{\tau_f}{G_f} \,, \tag{3.52b}$$

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(208)

$$\gamma_m = \frac{\tau_m}{G_m}, \quad (3.52c)$$

where $G_{12,f,m}$ = shear moduli of composite, fiber, and matrix, respectively.
From Equation (3.50) through Equation (3.52),

$$\frac{\tau_c}{G_{12}} t_c = \frac{\tau_f}{G_f} t_f + \frac{\tau_m}{G_m} t_m. \quad (3.53)$$

The shear stresses in the fiber, matrix, and composite are assumed to be equal ($\tau_c = \tau_f = \tau_m$), giving

$$\frac{1}{G_{12}} = \frac{1}{G_f}\frac{t_f}{t_c} + \frac{1}{G_m}\frac{t_m}{t_c}. \quad (3.54)$$

Because the thickness fractions are equal to the volume fractions, per Equation (3.28),

$$\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}. \quad (3.55)$$

Example 3.6

Find the in-plane shear modulus of a glass/epoxy lamina with a 70% fiber volume fraction. Use properties of glass and epoxy from Table 3.1 and Table 3.2, respectively.

Solution

The glass fibers and the epoxy matrix have isotropic properties. From Table 3.1, the Young's modulus of the fiber is

$$E_f = 85 \text{ GPa}$$

and the Poisson's ratio of the fiber is

$$\nu_f = 0.2.$$

The shear modulus of the fiber

$$G_f = \frac{E_f}{2(1+\nu_f)}$$

$$= \frac{85}{2(1+0.2)}$$

$$= 35.42 \; GPa.$$

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From Table 3.2, the Young's modulus of the matrix is

$$E_m = 3.4 \text{ GPa}$$

and the Poisson's ratio of the fiber is

$$\nu_m = 0.3.$$

The shear modulus of the matrix is

$$G_m = \frac{E_m}{2(1+\nu_m)}$$

$$= \frac{3.40}{2(1+0.3)}$$

$$= 1.308 \; GPa.$$

From Equation (3.55), the in-plane shear modulus of the unidirectional lamina is

$$\frac{1}{G_{12}} = \frac{0.70}{35.42} + \frac{0.30}{1.308}$$

$$G_{12} = 4.014 \; GPa.$$

Figure 3.13a and Figure 3.13b show the analytical values from Equation (3.55) of the in-plane shear modulus as a function of fiber volume fraction for a typical glass/epoxy lamina. Experimental values[4] are also plotted in the same figure.

3.3.2 Semi-Empirical Models

The values obtained for transverse Young's modulus and in-plane shear modulus through Equation (3.41) and Equation (3.55), respectively, do not agree well with the experimental results shown in Figure 3.10 and Figure 3.13. This establishes a need for better modeling techniques. These techniques include numerical methods, such as finite element and finite difference, and boundary element methods, elasticity solution, and variational principal models.[5] Unfortunately, these models are available only as complicated equations or in graphical form. Due to these difficulties, semi-empirical models have been developed for design purposes. The most useful of these models include those of Halphin and Tsai[6] because they can be used over a wide range of elastic properties and fiber volume fractions.

Halphin and Tsai[6] developed their models as simple equations by
to results that are based on elasticity. The equations are semi-empiri
e involved parameters in the curve fitting carry physical me

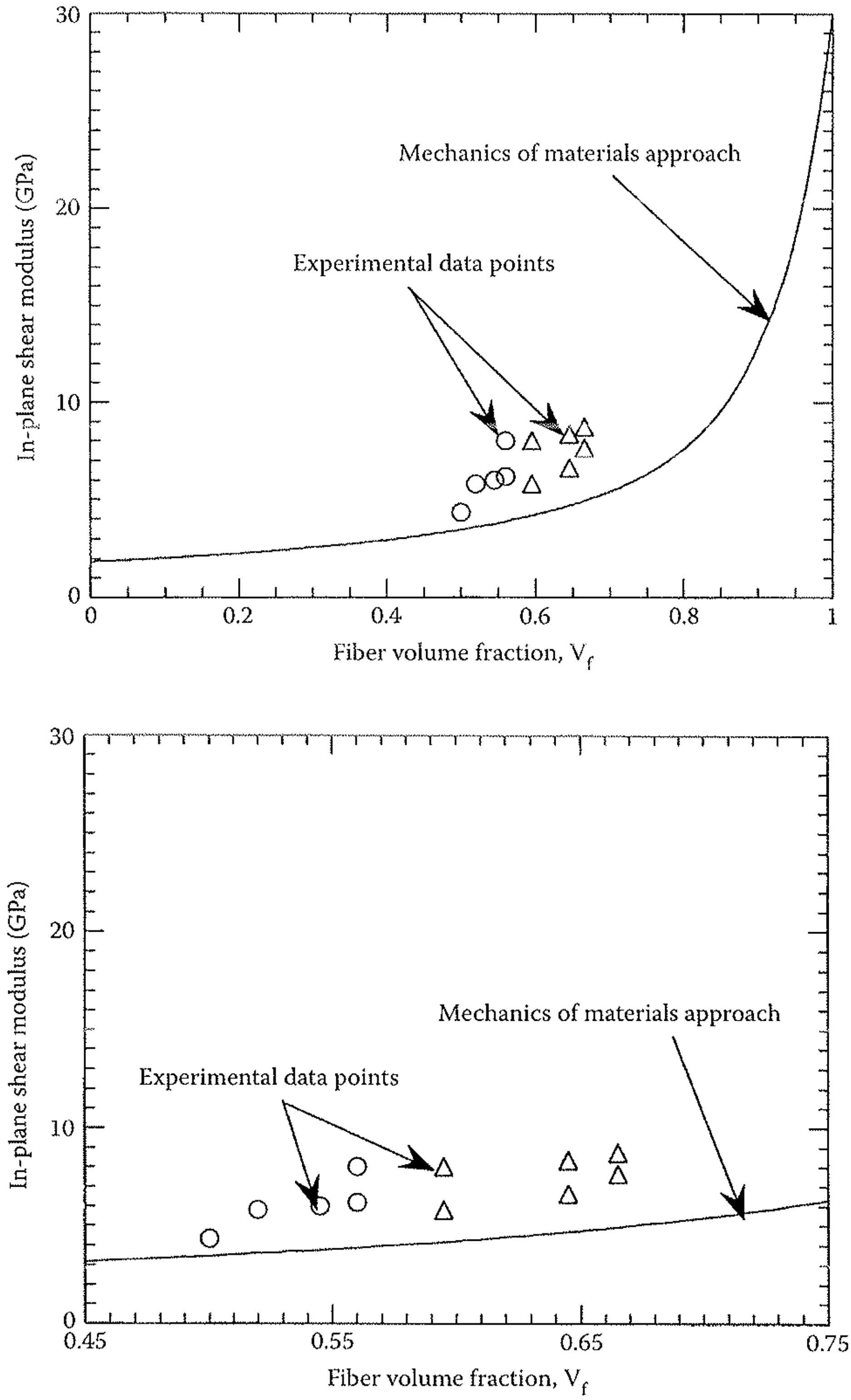

FIGURE 3.13
Theoretical values of in-plane shear modulus as a function of fiber volume fraction and comparison with experimental values for a unidirectional glass/epoxy lamina (G_f = 30.19 GPa, G_m = 1.83 GPa). Figure (b) zooms figure (a) for fiber volume fraction between 0.45 and 0.75. (Experimental data from Hashin, Z., NASA tech. rep. contract No. NAS1-8818, No

3.3.2.1 Longitudinal Young's Modulus

The Halphin–Tsai equation for the longitudinal Young's modulus, E_1, is the same as that obtained through the strength of materials approach — that is,

$$E_1 = E_f V_f + E_m V_m. \tag{3.56}$$

3.3.2.2 Transverse Young's Modulus

The transverse Young's modulus, E_2, is given by[6]

$$\frac{E_2}{E_m} = \frac{1 + \xi \eta V_f}{1 - \eta V_f}, \tag{3.57}$$

where

$$\eta = \frac{(E_f / E_m) - 1}{(E_f / E_m) + \xi}. \tag{3.58}$$

The term ξ is called the reinforcing factor and depends on the following:

- Fiber geometry
- Packing geometry
- Loading conditions

Halphin and Tsai[6] obtained the value of the reinforcing factor ξ by comparing Equation (3.57) and Equation (3.58) to the solutions obtained from the elasticity solutions. For example, for a fiber geometry of circular fibers in a packing geometry of a square array, $\xi = 2$. For a rectangular fiber cross-section of length a and width b in a hexagonal array, $\xi = 2(a/b)$, where b is in the direction of loading.[6] The concept of direction of loading is illustrated in Figure 3.14.

Example 3.7

Find the transverse Young's modulus for a glass/epoxy lamina with a 70% fiber volume fraction. Use the properties for glass and epoxy from Table 3.1 and Table 3.2, respectively. Use Halphin–Tsai equations for a circular fiber in a square array packing geometry.

Solution

Because the fibers are circular and packed in a square array, the $\xi = 2$. From Table 3.1, the Young's modulus of the fiber is E

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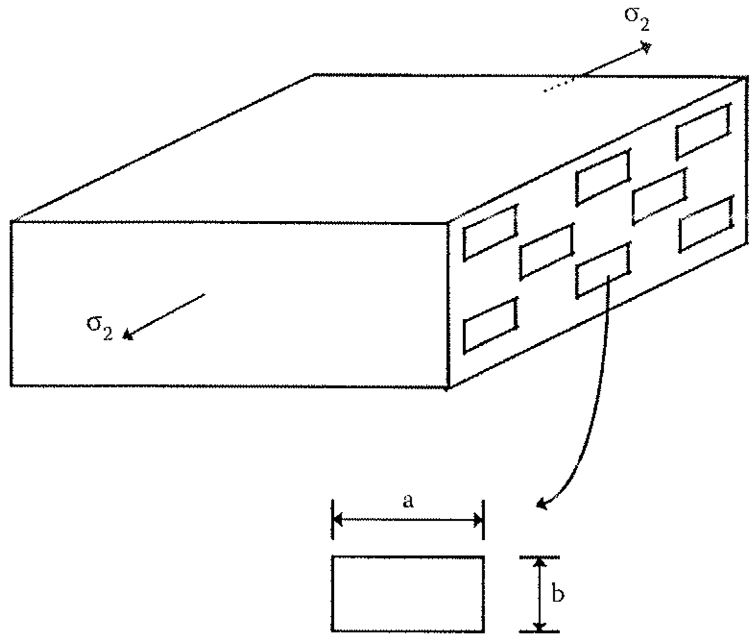

FIGURE 3.14
Concept of direction of loading for calculation of transverse Young's modulus by Halphin–Tsai equations.

From Table 3.2, the Young's modulus of the matrix is E_m = 3.4 GPa. From Equation (3.58),

$$\eta = \frac{(85/3.4)-1}{(85/3.4)+2}$$

$$= 0.8889.$$

From Equation (3.57), the transverse Young's modulus of the unidirectional lamina is

$$\frac{E_2}{3.4} = \frac{1+2(0.8889)(0.7)}{1-(0.8889)(0.7)}$$

$$E_2 = 20.20\ GPa.$$

For the same problem, from Example 3.4, this value of E_2 was found to be 10.37 GPa by the mechanics of materials approach.

Figure 3.15a and Figure 3.15b show the transverse Young's modulus as a function of fiber volume fraction for a typical boron/epoxy composite. The Halphin–Tsai equations (3.57) and the mechanics of materials approach Equation (3.41) curves are shown and compared to experimental (

As mentioned previously, the parameters ξ and η have a physic…
…mple,

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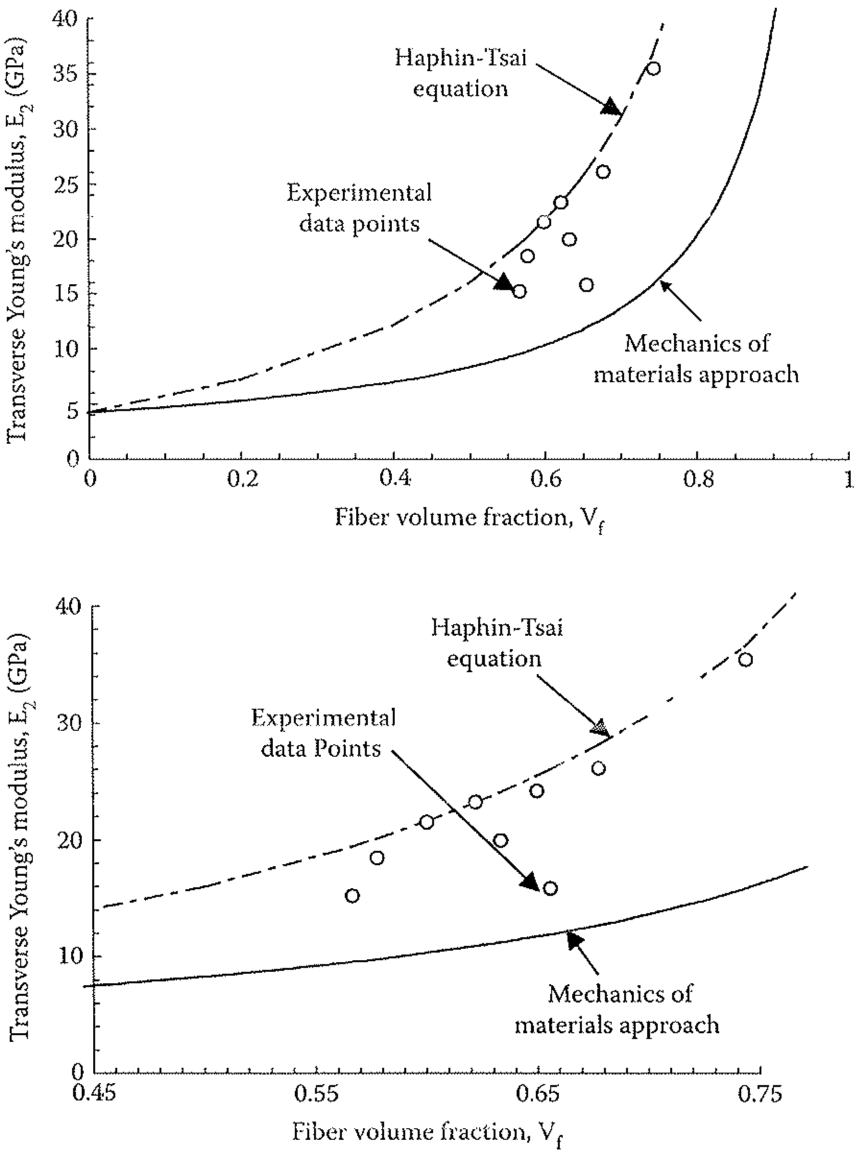

FIGURE 3.15
Theoretical values of transverse Young's modulus as a function of fiber volume fraction and comparison with experimental values for boron/epoxy unidirectional lamina ($E_f = 414$ GPa, $\nu_f = 0.2$, $E_m = 4.14$ GPa, $\nu_m = 0.35$). Figure (b) zooms figure (a) for fiber volume fraction between 0.45 and 0.75. (Experimental data from Hashin, Z., NASA tech. rep. contract no. NAS1-8818, November 1970.)

$$E_f/E_m = 1 \text{ implies } \eta = 0, \text{ (homogeneous medium)}$$
$$E_f/E_m \to \infty \text{ implies } \eta = 1 \text{ (rigid inclusions)}$$
$$E_f/E_m \to 0 \text{ implies } \eta = -\frac{1}{\xi} \text{ (voids)}$$

3.3.2.3 *Major Poisson's Ratio*

The Halphin–Tsai equation for the major Poisson's ratio, ν_{12}, is t
tained using the strength of materials approach — that is

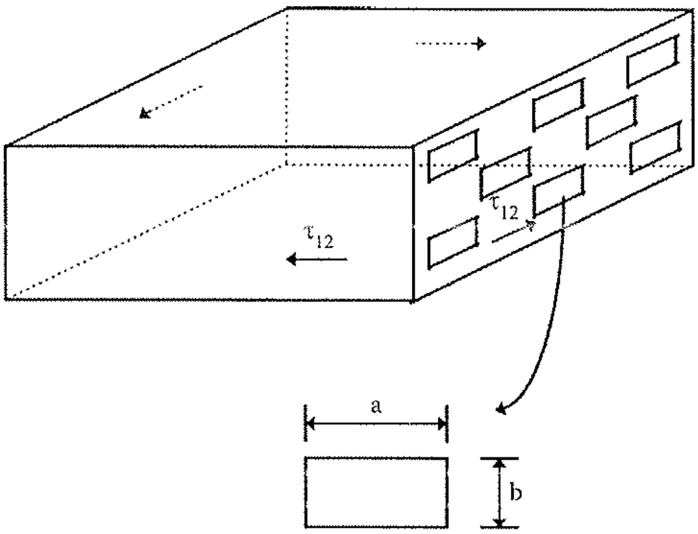

FIGURE 3.16
Concept of direction of loading to calculate in-plane shear modulus by Halphin–Tsai equations.

$$\nu_{12} = \nu_f V_f + \nu_m V_m. \tag{3.59}$$

3.3.2.4 In-Plane Shear Modulus

The Halphin–Tsai[6] equation for the in-plane shear modulus, G_{12}, is

$$\frac{G_{12}}{G_m} = \frac{1+\xi\eta V_f}{1-\eta V_f}, \tag{3.60}$$

where

$$\eta = \frac{(G_f / G_m) - 1}{(G_f / G_m) + \xi}. \tag{3.61}$$

The value of the reinforcing factor, ξ, depends on fiber geometry, packing geometry, and loading conditions. For example, for circular fibers in a square array, $\xi = 1$. For a rectangular fiber cross-sectional area of length a and width b in a hexagonal array, $\xi = \sqrt{3}\log_e(a/b)$, where a is the direction of loading. The concept of the direction of loading[7] is given in Figure 3.16.

The value of $\xi = 1$ for circular fibers in a square array gives reasonable results only for fiber volume fractions of up to 0.5. For example, for a typical glass/epoxy lamina with a fiber volume fraction of 0.75, the value of in-plane shear modulus using the Halphin–Tsai equation with $\xi = 1$ is 30% lower than that given by elasticity solutions. Hewitt and Malherbe[8] suggested choosing a function,

$$\xi = 1 + 40V_f^{10}.$$

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Example 3.8

Using Halphin–Tsai equations, find the shear modulus of a glass/epoxy composite with a 70% fiber volume fraction. Use the properties of glass and epoxy from Table 3.1 and Table 3.2, respectively. Assume that the fibers are circular and are packed in a square array. Also, get the value of the shear modulus by using Hewitt and Malherbe's[8] formula for the reinforcing factor.

Solution

For Halphin–Tsai's equations with circular fibers in a square array, the reinforcing factor $\xi = 1$. From Example 3.6, the shear modulus of the fiber is

$$G_f = 35.42 \text{ GPa}$$

and the shear modulus of the matrix is

$$G_m = 1.308 \text{ GPa}.$$

From Equation (3.61),

$$\eta = \frac{(35.42/1.308) - 1}{(35.42/1.308) + 1}$$

$$= 0.9288.$$

From Equation (3.60), the in-plane shear modulus is

$$\frac{G_{12}}{1.308} = \frac{1 + (1)(0.9288)(0.7)}{1 - (0.9288)(0.7)}$$

$$G_{12} = 6.169 \; GPa.$$

For the same problem, the value of G_{12} = 4.013 GPa was found by the mechanics of materials approach in Example 3.5.

Because the volume fraction is greater than 50%, Hewitt and Mahelbre[8] suggested a reinforcing factor (Equation 3.62):

$$\xi = 1 + 40V_f^{10}$$

$$= 1 + 40(0.7)^{10} \; .$$

$$= 2.130$$

, from Equation (3.61),

$$\eta = \frac{(35.42/1.308)-1}{(35.42/1.308)+2.130}$$

$$= 0.8928$$

From Equation (3.60), the in-plane shear modulus is

$$\frac{G_{12}}{1.308} = \frac{1+(2.130)(0.8928)(0.7)}{1-(0.8928)(0.7)}$$

$$G = 8.130\ GPa$$

Figure 3.17a and Figure 3.17b show the in-plane shear modulus as a function of fiber volume fraction for a typical glass/epoxy composite. The Halphin–Tsai equation (3.60) and the mechanics of materials approach, Equation (3.55) are shown and compared to the experimental[4] data points.

3.3.3 Elasticity Approach

In addition to the strength of materials and semi-empirical equation approaches, expressions for the elastic moduli based on elasticity are also available. Elasticity accounts for equilibrium of forces, compatibility, and Hooke's law relationships in three dimensions; the strength of materials approach may not satisfy compatibility and/or account for Hooke's law in three dimensions, and semi-empirical approaches are just as the name implies — partly empirical.

The elasticity models described here are called composite cylinder assemblage (CCA) models.[4,9–12] In a CCA model, one assumes the fibers are circular in cross-section, spread in a periodic arrangement, and continuous, as shown in Figure 3.18. Then the composite can be considered to be made of repeating elements called the representative volume elements (RVEs). The RVE is considered to represent the composite and respond the same as the whole composite does.

The RVE consists of a composite cylinder made of a single inner solid cylinder (fiber) bonded to an outer hollow cylinder (matrix) as shown in Figure 3.19. The radius of the fiber, a, and the outer radius of the matrix, b, are related to the fiber volume fraction, V_f, as

$$V_f = \frac{a^2}{b^2}. \tag{3.63}$$

Appropriate boundary conditions are applied to this composi ... on the elastic moduli being evaluated.

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$$\gamma_{23} = 0$$

$$\gamma_{31} = 0$$

$$\gamma_{12} = 0.$$

The Young's modulus in direction 1, E_1, is defined as

$$E_1 \equiv \frac{\sigma_1}{\varepsilon_1} = \frac{1}{S_{11}}. \tag{2.55}$$

The Poisson's ratio, ν_{12}, is defined as

$$\nu_{12} \equiv -\frac{\varepsilon_2}{\varepsilon_1} = -\frac{S_{12}}{S_{11}}. \tag{2.56}$$

In general terms, ν_{ij} is defined as the ratio of the negative of the normal strain in direction j to the normal strain in direction i, when the load is applied in the normal direction i.

The Poisson's ratio ν_{13} is defined as

$$\nu_{13} \equiv -\frac{\varepsilon_3}{\varepsilon_1} = -\frac{S_{13}}{S_{11}}. \tag{2.57}$$

Similarly, as shown in Figure 2.16b, apply $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 \neq 0$, $\tau_{23} = 0$, $\tau_{31} = 0$, $\tau_{12} = 0$. Then, from Equation (2.26) and Equation (2.39),

$$E_2 = \frac{1}{S_{22}} \tag{2.58}$$

$$\nu_{21} = -\frac{S_{12}}{S_{22}} \tag{2.59}$$

$$\nu_{23} = -\frac{S_{23}}{S_{22}}. \tag{2.60}$$

Similarly, as shown in Figure 2.16c, apply $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 \neq 0$, $\tau_{23} = 0$, $\tau_{31} = 0$, $\tau_{12} = 0$. From Equation (2.26) and Equation (2.39),

$$E_3 = \frac{1}{S_{33}}$$

$$\nu_{31} = -\frac{S_{13}}{S_{33}} \tag{2.62}$$

$$\nu_{32} = -\frac{S_{23}}{S_{33}} . \tag{2.63}$$

Apply, as shown in Figure 2.16d, $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 = 0$, $\tau_{23} \neq 0$, $\tau_{31} = 0$, $\tau_{12} = 0$. Then, from Equation (2.26) and Equation (2.39),

$$\varepsilon_1 = 0$$

$$\varepsilon_2 = 0$$

$$\varepsilon_3 = 0$$

$$\gamma_{23} = S_{44}\tau_{23}$$

$$\gamma_{31} = 0$$

$$\gamma_{12} = 0$$

The shear modulus in plane 2–3 is defined as

$$G_{23} \equiv \frac{\tau_{23}}{\gamma_{23}} = \frac{1}{S_{44}} . \tag{2.64}$$

Similarly, as shown in Figure 2.16e, apply $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 = 0$, $\tau_{23} = 0$, $\tau_{31} \neq 0$, $\tau_{12} = 0$. Then, from Equation (2.26) and Equation (2.39),

$$G_{31} = \frac{1}{S_{55}} . \tag{2.65}$$

Similarly, as shown in Figure 2.16f, apply $\sigma_1 = 0$, $\sigma_2 = 0$, $\sigma_3 = 0$, $\tau_{23} = 0$, $\tau_{31} = 0$, $\tau_{12} \neq 0$. Then, from Equation (2.26) and Equation (2.39),

$$G_{12} = \frac{1}{S_{66}} . \tag{2.66}$$

In Equation (2.55) through Equation (2.66), 12 engineering constants have been defined as follows:

Th--e Young's moduli, E_1, E_2, and E_3, one in each material axi

Six Poisson's ratios, ν_{12}, ν_{13}, ν_{21}, ν_{23}, ν_{31}, and ν_{32}, two for each plane
Three shear moduli, G_{23}, G_{31}, and G_{12}, one for each plane

However, the six Poisson's ratios are not independent of each other. For example, from Equation (2.55), Equation (2.56), Equation (2.58), and Equation (2.59),

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} . \tag{2.67}$$

Similarly, from Equation (2.55), Equation (2.57), Equation (2.61), and Equation (2.62),

$$\frac{\nu_{13}}{E_1} = \frac{\nu_{31}}{E_3} , \tag{2.68}$$

and from Equation (2.58), Equation (2.60), Equation (2.61), and Equation (2.63),

$$\frac{\nu_{23}}{E_2} = \frac{\nu_{32}}{E_3} . \tag{2.69}$$

Equation (2.67), Equation (2.68), and Equation (2.69) are called reciprocal Poisson's ratio equations. These relations reduce the total independent engineering constants to nine. This is the same number as the number of independent constants in the stiffness or the compliance matrix.

Rewriting the compliance matrix in terms of the engineering constants gives

$$[S] = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{13}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{21}}{E_2} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{31}}{E_3} & -\frac{\nu_{32}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} . \tag{2.70}$$

Inversion of Equation (2.70) would be the compliance matrix [C] and is given by

$$[C]=\begin{bmatrix} \frac{1-\nu_{23}\nu_{32}}{E_2E_3\Delta} & \frac{\nu_{21}+\nu_{23}\nu_{31}}{E_2E_3\Delta} & \frac{\nu_{31}+\nu_{21}\nu_{32}}{E_2E_3\Delta} & 0 & 0 & 0 \\ \frac{\nu_{21}+\nu_{23}\nu_{31}}{E_2E_3\Delta} & \frac{1-\nu_{13}\nu_{31}}{E_1E_3\Delta} & \frac{\nu_{32}+\nu_{12}\nu_{31}}{E_1E_3\Delta} & 0 & 0 & 0 \\ \frac{\nu_{31}+\nu_{21}\nu_{32}}{E_2E_3\Delta} & \frac{\nu_{32}+\nu_{12}\nu_{31}}{E_1E_3\Delta} & \frac{1-\nu_{12}\nu_{21}}{E_1E_2\Delta} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{12} \end{bmatrix}, \quad (2.71)$$

where

$$\Delta=\left(1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{13}\nu_{31}-2\nu_{21}\nu_{32}\nu_{13}\right)/\left(E_1E_2E_3\right). \quad (2.72)$$

Although nine independent elastic constants are in the compliance matrix [S] and, correspondingly, in the stiffness matrix [C] for orthotropic materials, constraints on the values of these constants exist. Based on the first law of thermodynamics, the stiffness and compliance matrices must be positive definite. Thus, the diagonal terms of [C] and [S] in Equation (2.71) and Equation (2.70), respectively, need to be positive. From the diagonal elements of the compliance matrix [S], this gives

$$E_1>0\ ,\ E_2>0\ ,\ E_3>0\ ,\ G_{12}>0\ ,\ G_{23}>0\ ,\ G_{31}>0 \quad (2.73)$$

and, from the diagonal elements of the stiffness matrix [C], gives

$$1-\nu_{23}\nu_{32}>0\ ,\ 1-\nu_{31}\nu_{13}>0\ ,\ 1-\nu_{12}\nu_{21}>0\ , \quad (2.74)$$

$$\Delta=1-\nu_{12}\nu_{21}-\nu_{23}\nu_{32}-\nu_{31}\nu_{13}-2\nu_{13}\nu_{21}\nu_{32}>0$$

Using the reciprocal relations given by Equation (2.67) through Equation (2.69),

$$\frac{\nu_{ij}}{E_i}=\frac{\nu_{ji}}{E_j} \text{ for } i\neq j \text{ and } i,j=1,2,3,$$

ı rewrite the inequalities as follows.

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For example, because

$$1 - \nu_{12}\nu_{21} > 0 \ ,$$

then

$$\nu_{12} < \frac{1}{\nu_{21}} = \frac{E_1}{E_2}\frac{1}{\nu_{12}}$$

$$|\nu_{12}| < \left|\frac{E_1}{E_2}\frac{1}{\nu_{12}}\right|$$

$$|\nu_{12}| < \sqrt{\frac{E_1}{E_2}} \ . \tag{2.75a}$$

Similarly, five other such relationships can be developed to give

$$|\nu_{21}| < \sqrt{\frac{E_2}{E_1}} \tag{2.75b}$$

$$|\nu_{32}| < \sqrt{\frac{E_3}{E_2}} \tag{2.75c}$$

$$|\nu_{23}| < \sqrt{\frac{E_2}{E_3}} \tag{2.75d}$$

$$|\nu_{31}| < \sqrt{\frac{E_3}{E_1}} \tag{2.75e}$$

$$|\nu_{13}| < \sqrt{\frac{E_1}{E_3}} \ . \tag{2.75f}$$

These restrictions on the elastic moduli are important in optim
...erties of a composite because they show that the nine independent
... be varied without influencing the limits of the others.

Example 2.5

Find the compliance and stiffness matrix for a graphite/epoxy lamina. The material properties are given as

$$E_1 = 181GPa\ ,\ E_2 = 10.3GPa\ ,\ E_3 = 10.3GPa$$

$$\nu_{12} = 0.28\ ,\ \nu_{23} = 0.60\ ,\ \nu_{13} = 0.27$$

$$G_{12} = 7.17GPa\ ,\ G_{23} = 3.0GPa\ ,\ G_{31} = 7.00GPa\ .$$

Solution

$$S_{11} = \frac{1}{E_1} = \frac{1}{181\times10^9} = 5.525\times10^{-12}\,Pa^{-1}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{10.3\times10^9} = 9.709\times10^{-11}\,Pa^{-1}$$

$$S_{33} = \frac{1}{E_3} = \frac{1}{10.3\times10^9} = 9.709\times10^{-11}\,Pa^{-1}$$

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{0.28}{181\times10^9} = -1.547\times10^{-12}\,Pa^{-1}$$

$$S_{13} = -\frac{\nu_{13}}{E_1} = -\frac{0.27}{181\times10^9} = -1.492\times10^{-12}\,Pa^{-1}$$

$$S_{23} = -\frac{\nu_{23}}{E_2} = -\frac{0.6}{10.3\times10^9} = -5.825\times10^{-11}\,Pa^{-1}$$

$$S_{44} = \frac{1}{G_{23}} = \frac{1}{3\times10^9} = 3.333\times10^{-10}\,Pa^{-1}$$

$$S_{55} = \frac{1}{G_{31}} = \frac{1}{7\times10^9} = 1.429\times10^{-10}\,Pa^{-1}$$

$$S_{66} = \frac{1}{G_{12}} = \frac{1}{7.17 \times 10^{9}} = 1.395 \times 10^{-10}\, Pa^{-1}\,.$$

Thus, the compliance matrix for the orthotropic lamina is given by

$$[S] =$$

$$\begin{bmatrix} 5.525\times10^{-12} & -1.547\times10^{-12} & -1.492\times10^{-12} & 0 & 0 & 0 \\ -1.547\times10^{-12} & 9.709\times10^{-11} & -5.825\times10^{-11} & 0 & 0 & 0 \\ -1.492\times10^{-12} & -5.825\times10^{-11} & 9.709\times10^{-11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.333\times10^{-10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.429\times10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.395\times10^{-10} \end{bmatrix} Pa^{-1}$$

The stiffness matrix can be found by inverting the compliance matrix and is given by

$$[C] = [S]^{-1}$$

$$[C] =$$

$$\begin{bmatrix} 0.1850\times10^{12} & 0.7269\times10^{10} & 0.7204\times10^{10} & 0 & 0 & 0 \\ 0.7269\times10^{10} & 0.1638\times10^{11} & 0.9938\times10^{10} & 0 & 0 & 0 \\ 0.7204\times10^{10} & 0.9938\times10^{10} & 0.1637\times10^{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3000\times10^{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6998\times10^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.7168\times10^{10} \end{bmatrix} Pa$$

The preceding stiffness matrix [C] can also be found directly by using Equation (2.71).

2.4 Hooke's Law for a Two-Dimensional Unidirectional Lamina

2.4.1 Plane Stress Assumption

A thin plate is a prismatic member having a small thickness, and it is the case for a typical lamina. If a plate is thin and there are no out-of-plane loads, it can be considered to be under plane stress (Figure 2.17). If the lower surfaces of the plate are free from external loads, then σ_3 = 0. Because the plate is thin, these three stresses within tl

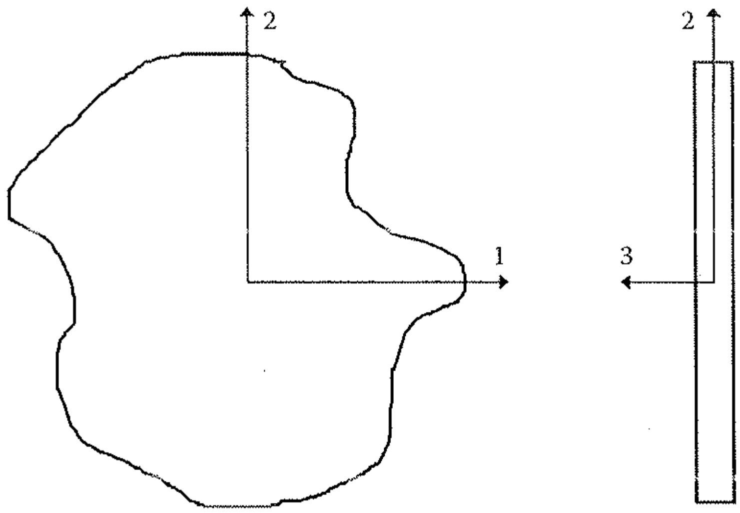

FIGURE 2.17
Plane stress conditions for a thin plate.

assumed to vary little from the magnitude of stresses at the top and the bottom surfaces. Thus, they can be assumed to be zero within the plate also. A lamina is thin and, if no out-of-plane loads are applied, one can assume that it is under plane stress. This assumption then reduces the three-dimensional stress–strain equations to two-dimensional stress–strain equations.

2.4.2 Reduction of Hooke's Law in Three Dimensions to Two Dimensions

A unidirectional lamina falls under the orthotropic material category. If the lamina is thin and does not carry any out-of-plane loads, one can assume plane stress conditions for the lamina. Therefore, taking Equation (2.26) and Equation (2.39) and assuming $\sigma_3 = 0$, $\tau_{23} = 0$, and $\tau_{31} = 0$, then

$$\varepsilon_3 = S_{13}\sigma_1 + S_{23}\sigma_2,$$

$$\gamma_{23} = \gamma_{31} = 0. \quad (2.76a,b)$$

The normal strain, ε_3, is not an independent strain because it is a function of the other two normal strains, ε_1 and ε_2. Therefore, the normal strain, ε_3, can be omitted from the stress–strain relationship (2.39). Also, the shearing strains, γ_{23} and γ_{31}, can be omitted because they are zero. Equation (2.39) for an orthotropic plane stress problem can then be written as

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix},$$

where S_{ij} are the elements of the compliance matrix. Note the four independent compliance elements in the matrix.

Inverting Equation (2.77) gives the stress–strain relationship as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}, \tag{2.78}$$

where Q_{ij} are the reduced stiffness coefficients, which are related to the compliance coefficients as

$$Q_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2},$$

$$Q_{12} = -\frac{S_{12}}{S_{11}S_{22} - S_{12}^2}, \tag{2.79a–d}$$

$$Q_{22} = \frac{S_{11}}{S_{11}S_{22} - S_{12}^2},$$

$$Q_{66} = \frac{1}{S_{66}}.$$

Note that the elements of the reduced stiffness matrix, Q_{ij}, are not the same as the elements of the stiffness matrix, C_{ij} (see Exercise 2.13).

2.4.3 Relationship of Compliance and Stiffness Matrix to Engineering Elastic Constants of a Lamina

Equation (2.77) and Equation (2.78) show the relationship of stress and strain through the compliance [S] and reduced stiffness [Q] matrices. However, stress and strains are generally related through engineering elastic constants. For a unidirectional lamina, these engineering elastics constants are

E_1 = longitudinal Young's modulus (in direction 1)

E_2 = transverse Young's modulus (in direction 2)

ν_{12} = major Poisson's ratio, where the general Poisson's ratio, ν_{ij} is defined as the ratio of the negative of the normal strain in direction j to the normal strain in direction i, when the only normal load is applied in direction i

G_{12} = in-plane shear modulus (in plane 1–2)

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R-15

Rajeev Gandhi Memorial College of Engineering & Technology
Autonomous
NANDYAL-518501
IV B. Tech, I-Semester Mid-II Examinations
MECHANICS OF COMPOSITE MATERIALS(A0326127)
(Mechanical Engineering)

Max. Marks: 25
Date: 7-3-2021
Time: 2 Hours

Note: 1.Answer *first* question compulsorily. (2 x 5 = 10Marks)
2. Answer any *three* from 2 to 5 questions.(5 x 3 = 15 Marks)

Q.1	a	What is angle ply lamina and state its significance?	2M	Co1
	b	Define the void fraction in two different ways?	2M	Co2
	c	State the Hooke's law for 2D element in terms of compliance matrix?	2M	Co3
	d	State the Betti reciprocal law and state its significance?	2M	Co1
	e	State the different failures of theories?	2M	Co2
Q.2	a	Derive an expression for in-plane shear modulus in micro-mechanics of composites using strength of materials approach?	3M	Co3
	b	Derive an expression for longitudinal modulus unidirectional composites of unidirectional lamina with strength of materials approach?	2M	Co4
Q.3	a	State the different theories of failures and explain?	3M	Co4
	b	A 45° angle lamina loaded under biaxial normal loading as $\sigma_x = -2\sigma_y = 2\sigma_o$ find σ_o, Basic strength properties of material are $(\sigma_1)_t^u = (\sigma_1)_c^u = 3(\sigma_2)_c^u = 5(\tau_{12})^u = 12(\sigma_2)_c^u = 600$MPa, check the inequalities using maximum stress theory?	2M	Co5
Q.4	a	Find the engineering constants for 30 degrees angle lamina, Use the following properties, E_1=204GPa, E_2=18.5 GPa, v_{12}=0.23, G_{12}= 5.59GPa	3M	Co4
	b	Write the number of independent elastic constants for anisotropic, isotropic, transversely isotropic, orthotropic materials?	2M	Co2
Q.5		Determine the E_1, E_2, G_{12}, v_{12} of carbon epoxy unidirectional lamina with the following properties? E_f=14.8GPa, E_m=3.45GPa, V_m=0.35, v_f=0.2, and v_m=0.5 by strength of materials approach?	5M	Co3

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College Code: 09

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NANDYAL-518501

R-15

IV B. Tech. I-Semester I-Mid Examinations

MECHANICS OF COMPOSITE MATERIALS (A0338158)

(Mechanical Engineering)

Max. Marks: 25 Date:29/12/2020 Time: 2 Hours

Note: 1.Answer *first* question compulsorily. (2 x 5 = 10Marks)

2. Answer any *three* from 2 to 5 questions.(5 x 3 = 15 Marks)

Q.1	a	Define composites?	2M	CO1
	b	State why boron fiber itself as a composites?	2M	CO1
	c	State the three disadvantages of hand layup process?	2M	CO1
	d	State the applications of pultrusion process?	2M	CO1
	e	State the advantages of PMCs?	2M	CO1
Q.2	a	State the types of composites? Explain any one composite?	3M	CO2
	b	State the different types of thermosets and explain any one ?	2M	CO1
Q.3	a	Explain briefly about carbon fiber and kevlar fibers?	2M	CO1
	b	State the different types of glass fibers?	3M	CO2
Q.4	a	Describe with neat sketches about spray layup technique?	2M	CO1
	b	Describe with neat sketches about resin transfer moulding technique?	3M	CO2
Q.5	a	Write short notes on ceramic matrix composite?	3M	CO2
	b	Write short notes on particulate composites?	2M	CO3

****

YOUR FUTURE DEPENDS ON THE WHAT YOU ARE DDOING IN THE PRESNT.....
By ......Mahatma Gandhi

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NANDYAL-518501

IV B. Tech. II-Semester Mid-II Examinations

MECHANICS OF COMPOSITE MATERIALS(A0326127)

(Mechanical Engineering)

Max. Marks: 25 Date: 7-3-2021 Time: 2 Hours

Note: 1.Answer *first* question compulsorily. (2 x 5 = 10Marks)

2. Answer any *three* from 2 to 5 questions.(5 x 3 = 15 Marks)

Q.1	a	What is angle ply lamina and state its significance?	2M	Co1
	b	Define the void fraction in two different ways?	2M	Co2
	c	State the Hooke's law for 2D element in terms of compliance matrix?	2M	Co3
	d	State the Betti reciprocal law and state its significance?	2M	Co1
	e	State the different failures of theories?	2M	Co2
Q.2	a	Derive an expression for in-plane shear modulus in micro-mechanics of composites using strength of materials approach?	3M	Co3
	b	Derive an expression for longitudinal modulus unidirectional composites of unidirectional lamina with strength of materials approach?	2M	Co4
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	b	A 45° angle lamina loaded under biaxial normal loading as $\sigma_x = -2\sigma_y = 2\sigma_o$ find σ_o, Basic strength properties of material are $(\sigma_1)_t^u = (\sigma_1)_c^u = 3(\sigma_2)_c^u = 5(\tau_{12})^u = 12(\sigma_2)_c^u = 600$MPa, check the inequalities using maximum stress theory?	2M	Co5
Q.4	a	Find the engineering constants for 30 degrees angle lamina, Use the following properties, E_1=204GPa, E_2=18.5 GPa, ν_{12}=0.23, G_{12}= 5.59GPa	3M	Co4
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RAJEEV GANDHI MEMORIAL COLLEGE OF ENGINEERING & TECHNOLOGY
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(ESTD-1995)

INTERNAL EXAMINATIONS ANSWER BOOKLET

NAME OF THE STUDENT: C. GURU PAVAN   Reg. No. 1 8 0 9 5 A 0 3 0 8

	1	2	3	4	5
A	2	2	2	2	
B	2	2	2	2	
C	2				
D	2				
E	2				
Total	10	4	4	4	
Grand Total :(In Figures)	22				
(in Words):					

NAME OF THE SUBJECT: M.C.M.

INTERNAL EXAM : I / II

Date of Exam: 29 - 12 - 2020 (FN/AN)

Course : B.Tech. / M.Tech./ MBA / MCA

Year : IV   Sem.: I

Branch: MECHANICAL

Signature of the Invigilator

(Start Writing From Here)

Q.1 * Composites:

a) Composites are defined as the combination of two materials which cannot dissolve and can distinguish each other.

Composite materials possess good strength and stiffness.

Composite materials are highly used in aircraft manufacturing Process.

Q.1 b) Boron fiber itself is a composite because in the metal matrix form of boron includes the material which are in internally occupied in the range of nano micron and it possess the great structural properties without any aid of external fiber.

So, that's the reason boron fiber itself called

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Q.1 c) * Advantages of Hand lay up Process: Q.4 a)

1. It is very economical process and less cost is required for this process.
2. The resin is uniformly distributed over entire fiber.
3. The hand lay up process is easy to operate.
4. By hand lay up process we can produce any structural requirements.

Q.1 d) * Applications of Pultrusion process

1. Textile industries
2. Aerospace industries
3. Used in industries where surface finish of the materials are major consideration.
4. Used in production of fabrications.
5. Used highly for any type of materials and are uniformly produce the fabrics.

Q.1 e) * Advantage of PMC'S

PMC - Polymer Matrix Composites

1. This PMC's posses good strength and good stiffness to withstand loads.
2. This composites have better structural properties.
3. The weight to density ratio of pol composites are less.
4. They are highly using composites in a

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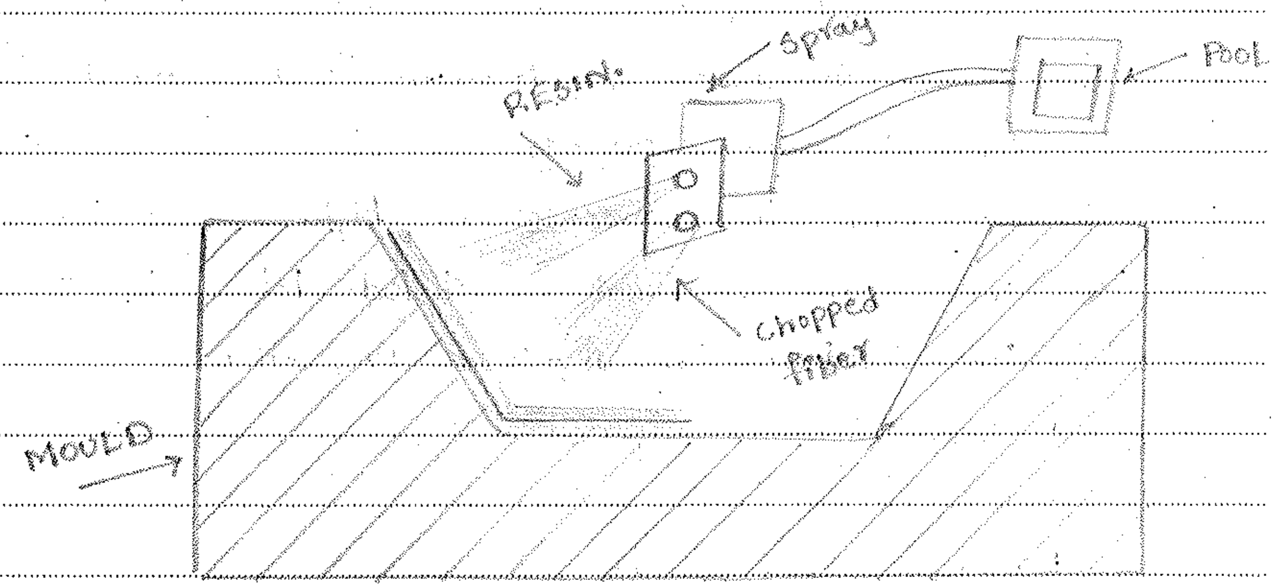
Q.4

a)



* SPRAY LAY UP PROCESS:

In spray lay up process the chopped fiber and resin is mixed in a pool. Here the impregnation of resin and chopped fiber is done by spraying the both at required proportion is to be happen. The sprayer will get the mixture from the pool as mentioned above. Here the resin and chopped fiber get sticked on the mould and after sometime the resin mixture get dried due to atmospheric conditions.

* Advantages:

1. It is more economical process.
2. Time required to perform the process is very less.
3. By this process only small and medium volume parts can be done.

* Disadvantages:

1. By this process surface finish is poor.
2. One side of the mould will get good

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and other side poor surface finish.

3. Large structural requirement parts cannot be done by this process.

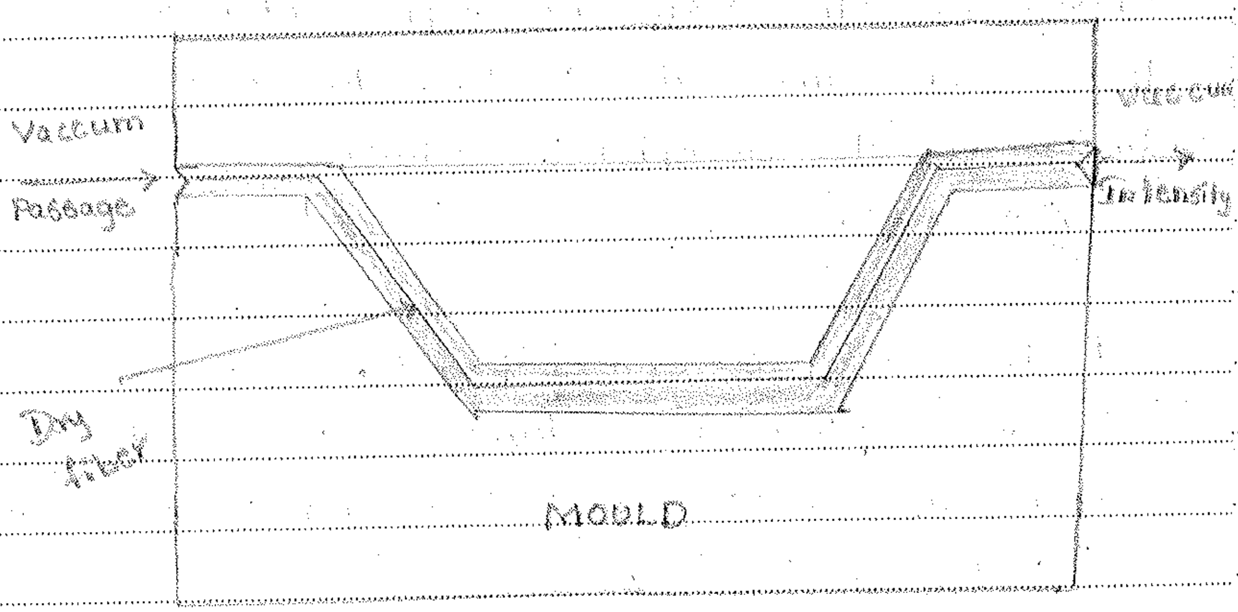
4. Cost of this process is high.

5. This process requires skilled labour to spray the mixture in required proportions.

* Applications:

1. Doors of cars
2. Chemical preserving tankers
3. Pipe lines
4. Autuomobile parts manufacturing.

Q.4 b) * Resin Transfer Moulding Technique:



In resin transfer moulding technique, and resin passed through a mould w intensity, so that the part from the will fit exactly to the mould walls and so that we can derive accurate parts. In this resin transfer moulding technique firstly the dry fiber is placed in the mould and then the vaccum resin intensity is passed through the mould. This process of transferring resin into the mould is called resin transfer moulding technique.

* Advantages:

1. Simple in process.
2. More economical process.
3. Good surface finish will be obtained.
4. Large structural components can be produced by this process.
5. No requirement of skilled labour.

* Disadvantages:

1. Vaccum pressure sending into the mould is to be uniform entire the process.
2. Mould will tends to vibrations.
3. Operated under specific pressure of vaccum.

* Applications:

1. Used in Aerospace industries
2. Used to produce superfish products.
3. Used in rail transport industries.
4. Used to produce mechanical component

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Q.2 * Types of Composites:

a)

1. Polymer Matrix Composites
2. Metal Matrix Composites
3. Ceramic Matrix composites
4. Carbon Carbon Composites.

* Polymer Matrix Composites:

Polymer matrix composites formed by the process of involving the polymer materials so as to get the desirable properties. The materials involved in the polymer matrix composites are polymers of Poly Vinyl and finolex majorly.
Since, these materials have great structural properties they are widely in the use of aerospace industries in the manufacturing process of aircraft.
These composites possess good ability to withstand loads, so as to maintain safe production.
This polymer matrix composite are very less density components but they have their own in built properties to accommodate the component.
These have the great bonding capability between the materials and can withstand with any type of material.
Since, these are highly useful compositices these will have applications in many indus manufacturing fields.

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Q.2 b) * Types of Thermosets

1. Vinylaptholene ethane
2. Polyfinyl axolene
3. Poly Vinyl acetylene.

Thermosets are those, once they can set into one mould it is difficult to deform the component either by aid of both pressure and temparature

* Polyvinyl acetylene:

This thermoset form have its own characteristics in the composites field. This thermosets have greater ability to set the component to any structural requirement. Since, these have the high usuage in olden times too these thermosets have wide range of applications in both industrial and manufacturing fields and also in the manufacturing of the aircraft parts. Due to their weight to density ratio these thermosets has good applications in any field.

These thermosets are possess good strength and stiffness to the components. So as that it can withstand heavy loads while operating.

* Advantages:

1. Highly used in aerospace industries.
2. Possess good strength and stiffness.
3. These can withstand heavy loads.
4. Possess good structural properties

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Q.3 b) * Types of Glass fibers:

1. S - Glass fibers
2. E - Glass fibers
3. C - Glass fibers
4. R - Glass fibers
5. D - Glass fibers
6. Ceramic Glass fibers.

The glass fibers above mentioned are the most commonly used fibers in the composite field. Due to glass properties these fibers are widely in the use of many industries and also in manufacturing too.
These fibers are less in weight to density ratio so as to attain its applications in the aerospace field too.

Q.3 a) * Carbon Fiber:

Carbon fibers are most usuage fiber in the aerospace field. Due to their own characteristics and properties carbon fiber has great applications in many of the industries.
These carbon fibers posses good strength and stiffness to the component.
While it comes to density and weight ratio it is very less. The weight of the bers are very less, so these fibers plications in aerospace field.

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* Kelvar fiber:

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Q.5 * Ceramic Matrix Composites:
a)

Ceramic matrix composites are derived from the glass fibers. The different glass fibers will possess different yield properties so as that ceramic matrix composites are derived by the both S & E type glass fibers. Ceramic matrix compositics have wide range of applications due to its temparature resistive properties. These Composites will possess good reliable strength to components.
But due to less stiffness it has also limitation to break easily so that it cannot withstand the high carrying sudden loads.
As apart from low stiffness it has all desirable properties for the development of the matrix composites in both industrial and aerospace fields.

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* Advantages:

1. Possess high reliable Strength
2. Used as the finished look component.
3. These have good temparature calibrating Properties.
4. Highly used in the Ceramic components.

Q.5 b) * Particulate Composites:

Particulate composites are derived from the fine particles of two different materials. This particulate composites posses good strength and stiffness because of deriving under the process of course grains ~~to~~of the high desirable property material to desirable component. This type of components are highly used in the manufacturing of machine engines, machinery, and space rocket engines. Due to the properties of this particulate ceramics these have applications in many other fields too. This particulate composites possess good strength and stiffness to the components that are derived.

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(ESTD-1995)

INTERNAL EXAMINATIONS ANSWER BOOKLET

NAME OF THE STUDENT: C. GURU PAVAN Reg. No. 1 8 0 9 5 A 0 3 0 8

	1	2	3	4	5
A	2	2	2	2	
B	2	2	2	2	
C	2				
D	2				
E	2				
Total	10	4	4	4	
Grand Total :(In Figures) 23					
(in Words):					

NAME OF THE SUBJECT: M.C.M.

INTERNAL EXAM : I / II

Date of Exam: 29-12-2020 (FN/AN)

Course : B.Tech. / M.Tech./ MBA / MCA

Year : IV Sem.: I

Branch: MECHANICAL

Signature of the Invigilator

(Start Writing From Here)

Q.1 * Composites:

a) Composites are defined as the combination of two materials which cannot dissolve and can distinguish each other.
Composite materials possess good strength and stiffness.
Composite materials are highly used in aircraft manufacturing process.

Q.1 b) Boron fiber itself is a composite because in the metal matrix form of boron includes the material which are in internally occupied in the range of nano micron and it possess the great structural properties without any aid of external fiber.
So, that's the reason boron fiber itself called a com

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Q.1 c) * Advantages of Hand lay up Process:

1. It is very economical process and less cost is required for this process.
2. The resin is uniformly distributed over entire fiber.
3. The hand lay up process is easy to operate.
4. By hand lay up process we can produce any structural requirements.

Q.1 d) * Applications of Pultrusion process

1. Textile industries
2. Aerospace industries
3. Used in industries where surface finish of the materials are major consideration.
4. Used in production of fabrications.
5. Used highly for any type of materials and are uniformly produce the fabrics.

Q.1 e) * Advantage of PMC'S

PMC - Polymer Matrix Composites

1. This PMC's posses good strength and good stiffness to withstand loads.
2. This composites have better structural properties
3. The weight to density ratio of polymer Matrix composites are less.
4. They are highly using composites

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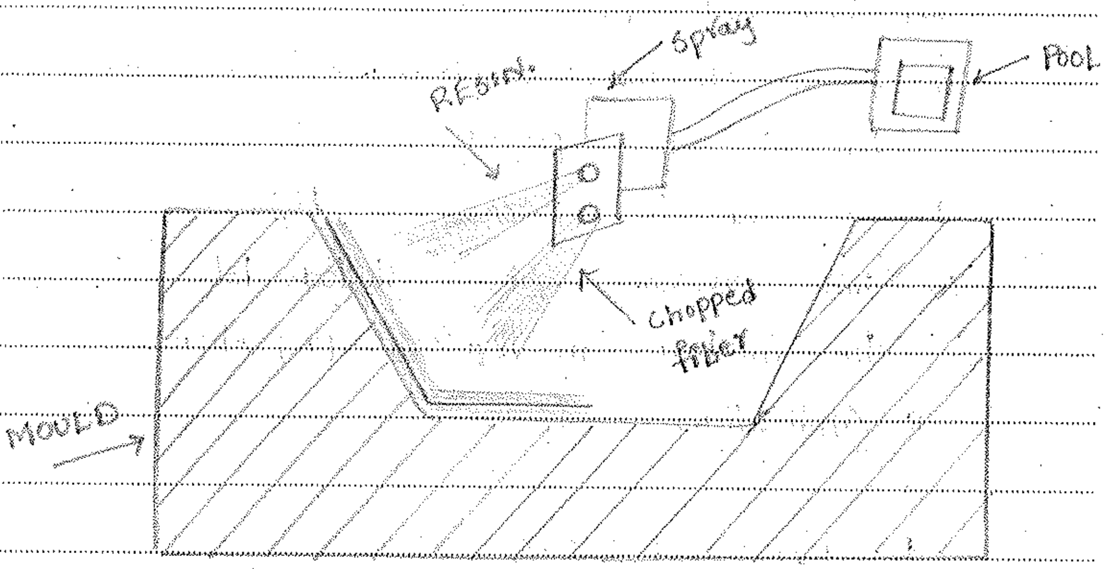
Q.4

a)



* SPRAY LAY UP PROCESS:

In Spray Lay up process the chopped fiber and resin is mixed in a pool. Here the impregnation of resin and chopped fiber is done by spraying the both at required proportion is to be happen. The sprayer will get the mixture from the pool as mentioned above. Here the resin and chopped fiber get sticked on the mould and after sometime the resin mixture get dried due to atmospheric conditions.

* Advantages:

1. It is more economical process.
2. Time required to perform the process is very less.
3. By this process only small and medium volume parts can be done.

* Disadvantages:

1. By this process surface finish is poor.
   ...ne side of the mould will get good surface

and other side poor surface finish.

3. Large structural requirement parts cannot be done by this process.

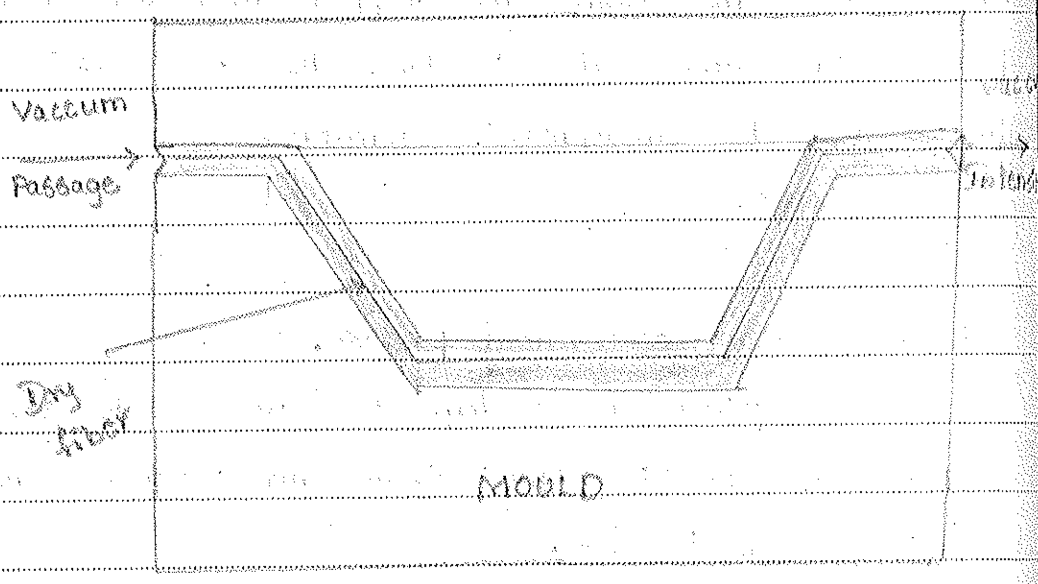
4. Cost of this process is high.

5. This process requires skilled labour to spray the mixture in required proportions.

* Applications:

1. Doors of cars
2. Chemical preserving tankers
3. Pipe lines
4. Autuomobile parts manufacturing.

Q.4 b) * Resin Transfer Moulding Tepknique :



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(348)

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Q.3 b) * Types of Glass fibers:

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The glass fibers above mentioned are the most Commonly used fibers in the Composite field. Due to glass properties these fibers are widely in the use of many industries and also in manufacturing too.
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* Advantages:

1. Possess high reliable strength
2. Used as the finished look component.
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INTERNAL EXAMINATIONS ANSWER BOOKLET

NAME OF THE STUDENT: T. Sai Dheeraj   Reg. No. 1 7 0 9 1 A 0 3 7 2

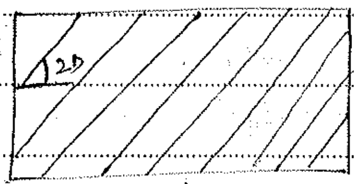
	1	2	3	4	5
A	)		✓	✓	3
B	✓		|	✓	
C	✓				
D	✓				
E	✓				
Total	9		3	4	3
Grand Total :(In Figures)	19				
(in Words):					

NAME OF THE SUBJECT: MCM
INTERNAL EXAM : I / II ✓
Date of Exam: 07/03/2021. (FN/AN) ✓
Course : B.Tech. ✓ / M.Tech./ MBA / MCA
Year : IV   Sem.: I
Branch: MECHANICAL ENGG.

D. Abhishek 7/3/21
Signature of the Invigilator

(Start Writing From Here)

Q1) ⓐ Angle ply lamina :- The angle by which the fibres are oriented in the direction with the matrix composite. That angle is called angle ply lamina.



plane

The significance of angle ply lamina is it makes the matrix more stronger and make the matrix to bind stronger. Therefore this is the significance of angle ply lamina.

Q1) ⓑ Void fraction : Void fraction is defined as the ratio of volume of void to matrix. Therefore this is called

$V_f = \frac{V_v}{V_m}$

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Q1) (c) Hooks law for 2D element can be defined by two ways
i) Compliance matrix
ii) Stiffness matrix

Reduced Reverse Compliance matrix :-

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{xy} \end{Bmatrix}$$

Therefore this is the Hookes law for 2D element in terms of compliance matrix.

Q1) (d) Betti-Reciprocal law: It is nothing but the reciprocal of the engineering constants of best standards in theoretical & practical aspect at approximate ratios this is the Betti-Reciprocal law and its significance.

Q1) (e) There are different failures of theories
i) Maximum principal strain theory
ii) Tsai Hills theory
iii) Tsai theory.

Q4) (a) ~~2α~~ = Given data

angle ~~to~~ of lamina, 2α = 30°

$E_1 = 204 GPa = 204 \times 10^9 MPa$

$E_2 = 18.5 GPa = 18.5 \times 10^9 MPa$

$\nu_{12} = 0.23$

$G_{12} = 5.59 GPa = 5.59 \times 10^9 MPa$

Q4) (e)

Q5)

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Q4) (b) The number of independent elastic constants for anisotropic material = 1

For isotropic material = 2

For transversely isotropic = 3.

For orthotropic material = 9.

Q5) Given data

$E_f = 14.8 \text{ GPa}$

$E_m = 3.45 \text{ GPa}$

$V_m = 0.35$

$V_f = 0.2$

$V_m = 0.5$

we have to determine ~~E_1, E_2, G~~

$E_1 = ?$

$E_2 = ?$

$G_{12} = ?$

$V_{12} = ?$

Given material → carbon epoxy unidirectional lamina

$$E_1 = \frac{E_f}{E_m} = \frac{14.8}{3.45} = 3.8$$

$$E_2 = \frac{E_m}{E_f} = \frac{3.45}{14.8} = 0.33.$$

$$G_{12} = \frac{E_1}{E_2} = \frac{3.80}{0.33} = 9.6$$

$$V_{12} = \frac{V_f}{V_m} = \frac{0.2}{0.5} = \frac{2}{5} = 0.4.$$

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Q3) (a) There are different theories of failures

i) Maximum principal strain theory.

ii) Tsai-Hill theory.

iii) Tsai theory.

Q3) (b) Given data

angle of lamina = 45°

$\sigma_x = -2\sigma_y = 2\sigma_0$

$(\sigma_1)_t^u = 600 MPa$

$(\sigma_1)_c^u = 600 MPa$

$(\sigma_2)_c^u = 200 MPa$

$5(\tau_{12})^u = 600$

$(\tau_{12})^u = 120 MPa$

$(\sigma_2)_t^u = \frac{600}{12} = 50 MPa$

All theories

$-600 < \tau_{12} < 600$ ✓

$-120 < \sigma_1 < 50$ ✓

$-50 < \sigma_2 < 200$ ✓.

So, the conditions all are true, so there are no inequalities, we have found this by maximum stress theory.

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R-15

Rajeev Gandhi Memorial College of Engineering & Technology
(Autonomous)
NANDYAL-518501
IV B. Tech. I-Semester I-Mid Examinations
MECHANICS OF COMPOSITE MATERIALS (A0338158)
(Mechanical Engineering)

Max. Marks: 25 Date:24-08-2019 Time: 2 Hours

Note: 1.Answer *first* question compulsorily. (2 x 5 = 10Marks)
2. Answer any *three* from 2 to 5 questions.(5 x 3 = 15 Marks)

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	e	State the principle of pultrusion?	2M
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Q.5	a	Write short notes on carbon-carbon matrix composite?	3M
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	b	Write short notes on particulate composites?	2M

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Code: A0338158S0721

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RGM COLLEGE OF ENGINEERING & TECHNOLOGY (AUTONOMOUS)

22nd July-2021

IV B.Tech I Semester (R15) End Examinations (Supplementary)

MECHANICS OF COMPOSITE MATERIALS

MECH

Time: 3 Hrs | Total Marks: 70

Note 1:Answer Question No.1 (Compulsory) and 4 from the remaining

2:All Questions Carry Equal Marks

1a Give names of various fibers used in advanced polymer composites.

b Define Orthotropic material and give the number of independent constants in macro mechanics.

c What are the assumptions made in the strength of materials approach?

d List strength failure theories of an angle lamina.

e List the factors to be considered while selecting the most efficient manufacturing process for composites.

f Give four examples of naturally found composites.

g Mention the types of glass fiber.

2 Find the four elastic moduli of a unidirectional glass/epoxy lamina with a 70% fiber volume fraction. Use mechanics of materials approach. Take E_f = 85 GPa, E_m=3.4 GPa, G_f = 35.42 GPa, G_m = 1.308 GPa. v_f = 0.25 and v_m = 0.5. (14)

3 Find the strains in the 1-2 coordiante system (Local axes) in a uni-directional boron/epoxy lamina, if the stresses in the 1-2 coordinate system applied are σ_1=4Mpa, σ_2= 2Mpa and τ_{12}= -3Mpa. Use the following properties.
E_1= 204 Gpa, E_2= 18.5Gpa, v_{12}= 0.23, G_{12}= 5.59 Gpa. 14)

4 a) What is pultrusion? With a neat sketch explain pultrusion technique. (10)
b) List its advantages, disadvantages and applications. (4)

5 a) Briefly explain ceramic matrix composites. (8)
b) Discuss their salient features, advantages, limitations and applications. (6)

6 a) What are the various types of reinforcement materials used in metal matrix composites? (7)
b) Discuses how reinforcement materials selected in metal matrix composites. (7)

7 Explain Hooke's law for
a) Anisotropic (6)
b) Monoclinic (4)
c) Isotropic materials. (4)

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22nd July - 2021

IV B.Tech I Semester (R15) End Exams (Supply.)

Sub: Mechanics of Composite materials

Branch: Mechanical Engg.

Scheme prepared by: Dr M Ashok kumar, Assoc. prof

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1a Fibers

- Natural Fiber
  - Ex:- Coir fiber, Sisal fiber, Bamboo fibers, Banana fiber, Hemp fibers — 1M
- Synthetic fibers
  - Ex:- Glass fibers, Kevlar fibers, Carbon fibers, Nylon fiber, Silica fibers — 1M

1b

- For orthotropic material properties are same at every 90°

3 ⊥r planes of Symmetry

3 modulus of Elasticities — 2M

3 Shear modulus

Totally 9 independent elastic constants are required

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix} C_{11}^{(1)} & C_{12}^{(2)} & C_{13}^{(3)} & 0 & 0 & 0\\ C_{12} & C_{22}^{(4)} & C_{23}^{(5)} & 0 & 0 & 0\\ C_{13} & C_{32} & C_{33}^{(6)} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44}^{(7)} & 0 & 0\\ 0 & 0 & 0 & & (8) & \\ 0 & 0 & 0 & & & \end{bmatrix} \begin{Bmatrix}\epsilon_1\\ \epsilon_2\\ \epsilon_3\\ \gamma_{23}\\ \gamma_{31}\\ \end{Bmatrix}$$

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1c Assumptions made in strength of materials approach :-

(1) Material is homogenious

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(2) Material is isotropic ~~mat~~ nature

(3) weight of the material should be neglected

(4) principle of superposition is considered as valid. — (2M)

1d Strength failure theories are 4 types

(a) Max. ~~shear~~ stress failure theory

(b) Max. strain failure Theory

(c) Sai Hill failure Theory

(d) Sai Wu failure theory. — (2M)

1e factors considered while selecting manufacturing process for composites

(a) strength of a fiber / matrix

(b) stiffness of a fiber / matrix

(c) Type of process

(d) Fiber orientation

(e) Fiber & matrix weight — (2m)

(f) Magnitude of defects at the end of the process

(g) size of the fiber (or diameter)

1f Naturally found composites examples :-

(1) Bone

(2) wood — (2M)

(3) Granite stone

(4) Tooth

(5) Flesh of animal

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Types of glass fibers

(a) A – glass fibers

(b) E – glass fiber

(c) S – glass fiber

(d) D – glass fiber

(e) C – Glass fiber — (2m)

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(2) Given Data

$E_f = 85$ GPa ; $E_m = 3.4$ GPa ; $G_f = 35.42$ GPa ; $G_m = 1.308$ GPa

$\nu_f = 0.25$ and $\nu_m = 0.5$; $V_f = 70\% = 0.7$

$V_m = 1 - 0.7 = 0.3$ — (2m)

Sol:

① Longitudinal Modulus (E_1) $= E_f V_f + E_m V_m$

$= 85 \times 0.7 + 3.4 \times 0.3 = 60.52$ GPa.

$$\boxed{E_1 = 60.52 \text{ GPa}}$$ — (2M)

② Transverse modulus (E_2)

$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

$$\frac{1}{E_2} = \frac{0.7}{85} + \frac{0.3}{3.4} = 96.47 \times 10^{-3}$$

$$E_2 = \frac{1}{96.47 \times 10^{-3}} = 10.36 \text{ GPa}$$

$$\boxed{E_2 = 10.36 \text{ GPa}}$$ — (2m)

③ In plane shear modulus (G_{12})

$$\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}$$

$$G_f = \frac{E_f}{2(1+\nu_f)} = \frac{85}{2(}$$

$(G_f = 53.12 \text{ GPa})$

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$$G_m = \frac{E_m}{2(1+\nu_m)} = \frac{3.4}{2(1+0.5)} = 2.55\ GPa.$$

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$$\boxed{G_m = 2.55\ GPa}$$

$$\frac{1}{G_{12}} = \frac{0.7}{53.12} + \frac{0.3}{2.55} = 130.82 \times 10^{-3}$$

$$G_{12} = \frac{1}{130.82 \times 10^{-3}} = 7.64\ GPa.$$

$$\boxed{G_{12} = 7.64\ GPa}$$ — (2m)

(4) Major poisson ratio (ν_{12})

$$\nu_{12} = \nu_f V_f + \nu_m V_m$$

$$= 0.25 \times 0.7 + 0.5 \times 0.3$$

$$\boxed{\nu_{12} = 0.325}$$ — (2 m)

(3) Given Data

$\sigma_1 = 4\ MPa$; $\sigma_2 = 2\ MPa$; $\tau_{12} = -3\ MPa$

$E_1 = 204\ GPa$; $E_2 = 18.5\ GPa$;

$\nu_{12} = 0.23$; $G_{12} = 5.59\ GPa.$ — (2m)

2D Stress strain relation is given as

$$\{\epsilon\} = [S]\{\sigma\} \quad \text{Short form}$$

1.

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \text{ or } \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}_{3\times3} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \text{ or } \sigma_6 \end{Bmatrix}$$ — (2m)

$[S]$ = Compliance matrix for a lamina

$$S_{11} = \frac{1}{E_1} = \frac{1}{204 \times 10^9} = 4.90 \times 10^{-12}\ Pa^{-1}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{18.5 \times 10^9} = 5.405 \times 1$$

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$S_{66} = \frac{1}{G_{12}} = \frac{1}{5.59\times10^9} = 1.788\times10^{-10}\ pa^{-1}$ (2½)

$S_{21} = -\frac{\nu_{12}}{E_1} = \frac{-0.23}{204\times10^9} = 1.129\times10^{-12}\ pa^{-1}$ (5/14)

$$\boxed{S_{21} = 1.129\times10^{-12}\ pa^{-1}}$$

Compliance matrix $= [S] = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}$

$$= \begin{bmatrix} 4.90\times10^{-12} & 1.129\times10^{-12} & 0 \\ 1.129\times10^{-12} & 5.405\times10^{-11} & 0 \\ 0 & 0 & 1.788\times10^{-10} \end{bmatrix} pa^{-1}$$

Now we have to calculate Minor poisson's ratio (ν_{21})

$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j} \qquad i \neq j$$

let, $i = 1$
$j = 2$

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$

$$\nu_{21} = \frac{E_2}{E_1}\nu_{12} = \frac{18.5}{204}\times0.23$$

$$\boxed{\nu_{21} = 0.02085}$$ — (5M)

Reduced Stiffness matrix $= [Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} = \frac{204}{(1-0.23\times0.020)}$$

$$= 204.98\times10^9\ Pa \text{ (or) } 204.98\ GPa$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = \frac{18.5}{(1-0.23\times0.02085)}$$

$$= 18.589\ GPa.$$

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$$Q_{12} = Q_{21} = \frac{E_2 \nu_{12}}{1 - \nu_{12}\nu_{21}} = \frac{18.5 \times 0.23}{(1 - 0.23 \times 0.2085)}$$

$Q_{21} = 4.27$ GPa.

$Q_{66} = G_{12} = 5.59$ GPa

6/14

$$[Q] = \begin{bmatrix} 204.98 & 4.27 & 0 \\ 4.27 & 18.58 & 0 \\ 0 & 0 & 5.59 \end{bmatrix} \text{GPa}$$

We know that

$$\{\sigma\} = [Q]\{\epsilon\}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

$$\sigma_1 = Q_{11}\epsilon_1 + Q_{12}\epsilon_2 \Rightarrow 204.98 \times 10^9 \times \epsilon_1 + 4.27 \times 10^9 \times \epsilon_2$$

$$4 \times 10^6 = 204.98 \times 10^9 \epsilon_1 + 4.27 \times 10^9 \epsilon_2 \quad \text{--- (1)}$$

$$\sigma_2 = Q_{21}\epsilon_1 + Q_{22}\epsilon_2$$

$$2 \times 10^6 = 4.27 \times 10^9 \epsilon_1 + 18.58 \times 10^9 \times \epsilon_2 \quad \text{--- (2)}$$

$$\tau_{12} = Q_{66} \times \gamma_{12}$$

$$-3 \times 10^6 = 5.59 \times 10^9 \gamma_{12} \quad \text{--- (3)}$$

$$\boxed{\gamma_{12} = 536.67 \times 10^{-6}}$$ — (5M)

$$\boxed{\epsilon_1 = 1.563 \times 10^{-5}}$$

$$\boxed{\epsilon_2 = -2.943 \times 10^{-4}}$$

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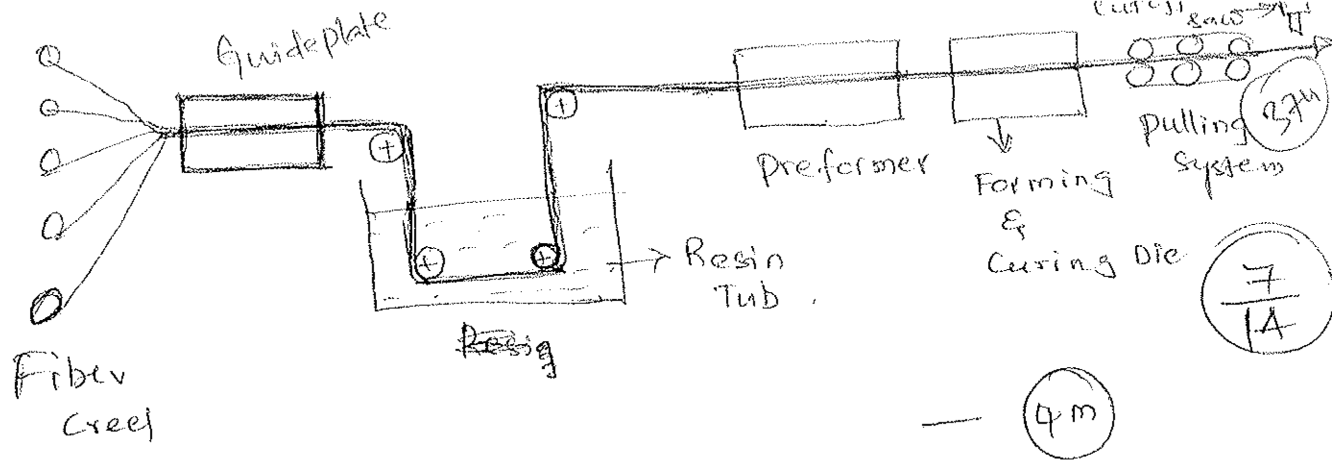

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— 4m

Pultrusion process is a highly automated continuous fiber laminating process producing high fiber volume profiles with a constant cross section.

From the in feed area the impregnated reinforcement is pulled into the heated pultrusion die.

The resin matrix is such that solidifies and cures with in the die.

<u>principal parts of The pultrusion</u>

1. Fiber creels : supply the fiber continuous
2. Resin Tub : provides impregnation (soaking) of fiber with matrix material
3. preformer ; It decides final shape of the product
4. Curing die : Cure the soft material to solid mate
5. pulling mechanism ; It pulls the material from the die
6. Cut off die : Cut the finished product at pre-determined ~~shae~~ size by length,
7. <u>Guide plate</u> : It combines all fiber ~~reels~~ filaments into strands.

— 3m

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4b) Advantages of pultrusion

(a) It is a continuous places

(b) Const. C/s of any len size may be made

8/14

(c) very fast process

(d) Resin content may be controlled accurately

(e) fiber cost is minimized since majority of the fiber is taken from the creels. — 3M

Disadvantages of pultrusion

(a) It's suitable for only constant c/s not suitable for tapered objected

(b) Control of fiber orientation is not possible

(c) Quick curing system typically have low strength.

(d) voids may form if excess opening is given at the die opening

(e) High initial investment. — 2M

Applications

(a) slatted floors
(b) Car cases
(c) PVC - windows
(d) Stiffening bars
(e) Air craft components
(f) Flag sticks
(g) Tent
(h) Hovel construction — 2M
(i) panels

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5 a Ceramic Matrix Composites :-

(a) Ceramic matrix composites (CMC) are sub group of composite materials and sub group of ceramics.

(b) They consist of ceramic fibers embedded in a ceramic matrix.

(c) Fibers and matrix both can consists of any ceramic material where carbon & carbon fibers can also be regarded as a ceramic material.

Properties — (2m)

1. High fracture toughness or crack resistance
2. High ~~flexat~~ flexural strength
3. High tensile strength
4. Low density of fiber.

Typical fiber m/ls are — (3m)

(a) Carbon, C
(b) Silicon Carbide, SiC
(c) Alumina, Al_2O_3
(d) Mullite (or) Alumina silica, $Al_2O_3 - SiO_2$

These fibers are temp. resistant up $1800^\circ C$

Reinforcing fibers are like human hair size and in case of nano-fibers are even ~~thi~~ tinier. They are often woven into fabric or tape like materials for inclusion in CMC part.

In a typical CMC's fibers are coated with boron-nitride and then passed through a matrix slurry bath, resulting in a prepeg tape. ~~as~~ is stored at $0^\circ C$ Centigrate.

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Short fiber, whisker, or continuous fiber reinforcement used in CMC composites

Commonly used fiber/matrix combinations in CMC's

(a) C/C

(b) C/SiC

(c) SiC/SiC

(d) Al_2O_3/Al_2O_3 — (2m)

10/1A

5b **Salient features of CMC's :—**

(a) They are hard and stable at higher temp.

(b) They are ultra-light weight
1/3 wt. of nickel superalloys

(c) Possess greater fracture toughness

(d) High thermal shock resistance.

(e) Retain high mechanical strength at elevated temp.

(f) Excellent stiffness & very good stability

(g) Elongation rupture of CMC's are upto 1%

(h) They are not susceptible to fracture like traditional ceramic materials.

(i) High corrosion resistance.

(j) Handle dynamic loads very well.

(k) ↑ Durable — (3m)

Advantages

(1) Light weight

(2) Much better fuel η

(3) less pollution

(4) Can operate at high temp.

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Applications of CMCs

Heat exchangers

Turbine blade

Stator vanes

High performance braking system,

Immersion burner tubes

Bullet proof armor

Heating elements

Gas - ~~fib~~ fired burner parts, etc. —

Limitations

(a) Low impact resistance

(b) Brittle fracture

(c) part size & shape limitation

(d) Defect size effect

(e) limited load level during sliding, — (3M)

6a Reinforcement materials used in metal matrix composites (MMC's) :—

Continuous fibers

Short fibers

Whiskers

Equiaxed particles

Interconnected networks. — (7M)

6b Al - silicon carbide (particles

Properties

- Reduced weight
- High strength

wear ~~St~~ resistance — (3M)

Application : pistons.

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Aluminum - silicon carbide (whisker)

- High wear resistance
- Reduced weight
- Reduced reciprocating mass
- High sp. strength & stiffness

Applications

~~Brake rotors~~, ~~calipers, liners~~, sprockets, pulleys, ~~piston rings~~.

12/1A

Aluminum - aluminum oxide (short fibers) :-

Wear resistance
High running temp.
Reduced reciprocating mass
High creep & fatigue resistance

Applications

Connecting rods

Copper - Graphite

Low friction & wear
Low co-eff of thermal expansion

Aluminum - Graphite

Gall resistance
Reduced friction, wear resistance

Applns

Cylinders,
liner platons,
Bearings. — 4M

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7 Explain Hooke's law for (13/14)

(a) Anisotropic Materials :-

- Anisotropic material has '21' elastic constants at a point.
- once this constants are found for a particular point, stress - strain relations are developed.

$\{\epsilon\} = [S]\{\sigma\}$ — Hooke's law

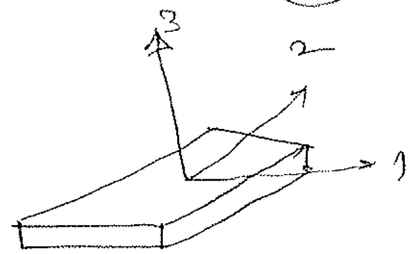
$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix}_{6\times 6}$$

21 — Constants from the symmetry $S_{ij} = S_{ji}$

Ex : $S_{12} = S_{21}$

— 4M

(b) Monoclinic Materials :-



- It has one plane of symmetry ie 1-2 plane is the plane of symmetry
- '3' direction is ⊥r to the plane of symmetry
- Shear strain $\gamma_{23} = 0$; $\gamma_{31} = 0$
- It has '13' independent elastic co-efficients
- If a linear elastic solid has one plane of symmetry then its called monoclinic material.

— 6M

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{62} & C_{63} & 0 & 0 & \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \\ \end{Bmatrix}$$

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(c) Isotropic Materials :-

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \left(\frac{C_{11}-C_{12}}{2}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & \left(\frac{C_{11}-C_{12}}{2}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & \left(\frac{C_{11}-C_{12}}{2}\right) \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}$$

- It has same property in all direction
- It has 'two' independent elastic constant totally.
- properties are directionally independent.
- ~~At~~ Material contains infinite no. of planes of material property symmetry passing through a point.

— (4M)

× × ×

Scheme is Prepared by
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24/7/24

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RGM COLLEGE OF ENGINEERING & TECHNOLOGY (AUTONOMOUS)

28th February-2020

IV B.Tech I Semester (R15) End Examinations (Supplementary)

MECHANICS OF COMPOSITE MATERIALS

MECH

Time: 3 Hrs **Total Marks: 70**

Note 1:Answer Question No.1 (Compulsory) and 4 from the remaining

2:All Questions Carry Equal Marks

1a What is meant by fiber wash?

b Write expression to determine the in-plane shear modulus using Halphin-Tsai criteria.

c Mention some of the applications of carbon-carbon composites?

d Mention two different combinations of matrices and reinforcements for metal matrix composites.

e Define macro mechanics in the analysis of composites.

f List strength failure theories of an angle lamina.

g What are the limitations of hand lay-up technique?

2 The properties of unidirectional Glass/Epoxy lamina are E_1=38.6GPa, E_2=8.27GPa, ν_{12} = 0.26 and G_{12} = 4.14 GPa. Find the following for a 60° angle lamina of Glass/Epoxy, if the applied stresses are σ_x=4Mpa, σ_y=2Mpa, τ_{xy}=-3Mpa

a) Transformed compliance matrix (5)

b) Transformed reduced stiffness matrix (5)

c) Global strains. (4)

3 With the help of neat sketch, explain the following processes for manufacturing of composites

a) Pultrusion (7)

b) Resin Transfer Molding (RTM) (7)

4 a) Find the strains in the 1-2 co-ordinate system in a uni-directional boron/epoxy lamina, if the stresses in the 1-2 co-ordinate system applied are,

σ_1= 4MPa, σ_2= 2 MPa, τ_{12} = -3 MPa.

Use the following properties,

E_1 = 204GPa, E_2 = 18.5 GPa, ν_{12} = 0.23, G_{12} = 5.5GPa. (8)

b) For the above 1-2 co-ordinate system in a uni-directional boron/epoxylamina, find the stiffness matrix [C]. (6)

5 a) Enumerate desirable characteristics of fibers in fiber reinforced composites. (8)

b) What are the different types of glass fibers? Explain. (6)

6 a) What do you exactly mean by 'Composite Material'? What advantages does it possess compared to the conventional materials? (8)

b) What are the typical mechanical properties of polymer matrix composites? Explain. (6)

7 Using Halphin–Tsai equations, find the longitudinal modulus, transverse modulus and shear modulus of a glass/epoxy unidirectional lamina with 40% fiber volume fraction. Take E_{glass}=85GPa, E_{epoxy}=3.4GPa, G_{glass}=35.42 GPa and G_{epoxy}=1.308GPa. (14)

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IV B.Tech I Sem (R15) End Examinations (Supply.)
SUB: MECHANICS OF COMPOSITES
Time: 3 Hrs BRANCH: ME Max. Marks: 70

(25M)

1a Fiber wash is defined as cleaning of fiber with NaOH chemical in order to remove dust, rust materials of the fiber.

Objectives of cleaning

1. Improve the strength of fiber
2. To reduce dust / rust on the fiber surfaces — (1)
3. To remove lignin / Cellulose / starch material which is actually organic material decreases strength of a composite. — (1)

2 M

1b

G_{12} = Inplane shear modulus

$$G_{12} = \frac{1}{G_f} V_f + \frac{1}{G_m} V_m$$ — (1)

where

G_f = Shear modulus of fiber
G_m = " " " matrix — (1)
V_f = Vol. fraction of fiber
V_m = " " " matrix.

2 M

1c Applications of Carbon-Carbon Composites

It's mainly used in missiles, military, air-craft,

- Rocket nozzles
- Exit cones for strategic missiles,

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1d Composite material with at least two ~~const~~ constituents parts, one being a metal necessarily, the other material may be different m/l or another material, such as ceramic or organic compound.

When 3 materials are present it's called hybrid composite.

① — Reinforcement (particulate, fiber), SiC, Al_2O_3, B_4C, TiC, TiB_2, graphite

① — Matrix — Aluminium, Be, Mg, Ti, Ni, Co, and Ag

1e <u>2M</u>

Macromechanics is the study of composite materials behaviour wherein the material is presumed to be homogeneous, and the effects of the constituents materials are detected only as averaged apparent macroscopic properties of composites.

— Laminates are used for macroscopic analysis

— Laminate consist of fiber in unidirection with matrix

2M

1f <u>Failure Theories for angle lamina</u>

1. Max stress failure Theory }— 1
2. Max. strain failure Theory
3. Tsai - Hill failure Theory }— 1
4. Tsai - Wu failure Theory

2M

1g. <u>limitations of</u> hand lay-up technique.

1. Long time to produce
2. Skill is required — 2M
3. Surface finish is obtained on

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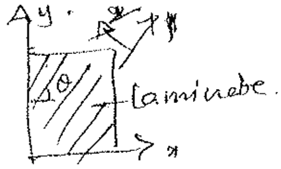
2. Given Data

$E_1 = 38.6$ GPa.

$E_2 = 8.27$ GPa

$\nu_{12} = 0.26$ $\theta = 60°$

$G_{12} = 4.14$ GPa.



Sol:

$$S_{11} = \frac{1}{E_1} = \frac{1}{38.6 \times 10^9} = 2.59 \times 10^{-11}\ Pa^{-1}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{8.27 \times 10^9} = 1.21 \times 10^{-10}\ Pa^{-1}$$

$$S_{12} = -\frac{\nu_{12}}{E_1} = \frac{-0.26}{38.6 \times 10^9} = 6.73 \times 10^{-12}\ Pa^{-1}$$

$$S_{66} = \frac{1}{G_{12}} = \frac{1}{4.14 \times 10^9} = 2.41 \times 10^{-10}\ Pa^{-1}$$

	S_{11}	S_{22}	S_{12}	S_{66}
S_{xx}	m^4	n^4	$2m^2n^2$	m^2n^2
S_{yy}	n^4	m^4	$2m^2n^2$	m^2n^2
S_{xy}	m^2n^2	m^2n^2	m^4+n^4	$-m^2n^2$
S_{ss}	$4m^2n^2$	$4m^2n^2$	$-8m^2n^2$	$(m^2-n^2)^2$
S_{xs}	$2m^3n$	$-2mn^3$	$2(mn^3-m^3n)$	(mn^3-m^3n)
S_{ys}	$2mn^3$	$-2m^3n$	$2(m^3n-mn^3)$	(m^3n-mn^3)

— (7 M)

$$Q_{xx} = m^4 Q_{11} + n^4 Q_{22} + 2m^2n^2 Q_{12} + 4m^2n^2 Q_{66}$$

$$Q_{yy} = n^4 Q_{11} + m^4 Q_{22} + 2m^2n^2 Q_{12} + 4m^2n^2 Q_{66}$$

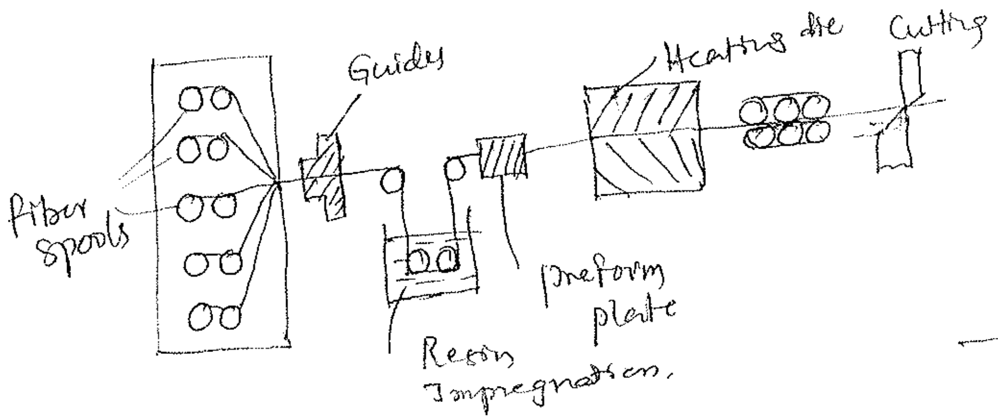
$$Q_{xy} = m^2n^2 Q_{11} + m^2n^2 Q_{22} + (m^4+n^4) Q_{12} - 4m^2n^2 Q_{66}$$

$$Q_{ss} = m^2n^2 Q_{11} + m^2n^2 Q_{22} - 2(m^2-n^2) Q_{12} + (m^2-n^2) Q_{66}$$

$$Q_{xs} = m^3n Q_{11} + mn^3 Q_{22} + (mn^3-m^3n) Q_{12} + 2(mn^3-m^3n) Q_{66}$$

3. a) Pultrusion is continuous process for manufacturing of composite materials with constant cross-section.

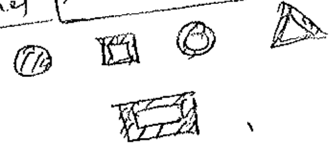
Pulltrusion = Pull + Extrusion



— (4 M)

1- Fiber Spools/creels
2- Resin impregnation
3- preformer
4- Heating dies
5. pulling Mechanism.
6. Cut off saws

Final products:



A rack of holding cylinder or cone holding threads or spools when spinning travelling from Then it allows the reinforcement the creels down into the bath and the fibers comes out through a resin is coated guide bar.

Preform plates are critical components of Pulltrusion sys as it properly aligns and feeds the reinforcement to the heated die.

— (3 M)

Reinforcements
Glass (E-glass & S-glass)
Carbon
Aramid

Matrix Materials
Unsaturated polyester
epoxy
vinyl ester
phenolic resins

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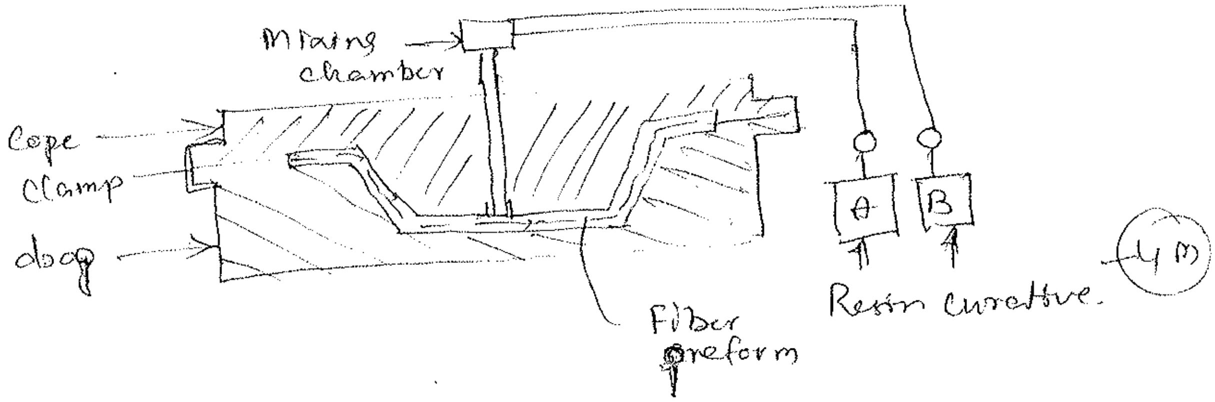
3b



Mould interior surfaces may be gel-coated. If desired the mold is first preloaded with a reinforcing ~~agent~~ fiber matrix or preform.

Resin transfer moulding uses the liquid thermoset resin to saturate a fiber preform placed in a closed mould. This process is versatile and can fabricate product with embedded objects such as foam cores or other components in addition to fiber preforms. (3M)

4a)

$\sigma_1 = 4MPa$	$E_1 = 204 GPa$	$G_{12} = 5.5 GPa.$
$\sigma_2 = 2 MPa$	$E_2 = 18.5 GPa$	
$\tau_{12} = -3 MPa$	$\nu_{12} = 0.23$	

(1M)

$$\{\epsilon\} = [S]\{\sigma\}$$

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 (\gamma_{12}) \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$S_{11} = \frac{1}{E_1} = \frac{1}{204 \times 10^9} = 4.9 \times 10^{-12} \; Pa^{-1}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{18.5 \times 10^9} = 5.40 \times 10^{-11} \; Pa^{-1}$$

$$S_{66} = \frac{1}{G_{12}} = \frac{1}{5.5 \times 10^9} = 1.818 \times 10^{-10} \; Pa^{-1}$$

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$$[S] = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} = \begin{bmatrix} 4.9\times10^{-12} & 1.12\times10^{-12} & 0 \\ 1.12\times10^{-12} & 5.4\times10^{-11} & 0 \\ 0 & 0 & 1.818\times10^{-10} \end{bmatrix} Pa^{-1}$$

To calculate minor poissons ratio, ν_{21}

Betti Reciprocal Raw has to be used

$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j} \quad \Bigg| \quad \frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} = \frac{0.23}{204} = \frac{\nu_{21}}{18.5}$$

$$\nu_{21} = \frac{18.5}{204} \times 0.23$$

$$\nu_{21} = 0.020$$

We know that

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} \quad \text{— (5M)}$$

$$\{\sigma\} = [Q]\{\epsilon\}$$

where $[Q]$ = Reduced stiffness matrix

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} = \frac{204\times10^9}{1-0.23\times0.020} = 2.048\times10^{11} \text{ GPa}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = \frac{18.5\times10^9}{1-0.23\times0.020} = 1.85\times10^{10}$$

$$Q_{66} = G_{12} = 5.5\times10^9 \text{ Ma}.$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{0.23\times18.5\times10^9}{1-0.23\times0.020}$$

$$= 4.27\times10^9 \text{ Pa}$$

4 b)

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} = \begin{bmatrix} 2.048\times10^{11} & 4.27\times10^9 & 0 \\ 4.27\times10^9 & 1.85\times10^{10} & 0 \\ 0 & 0 & 5.5\times10^9 \end{bmatrix} \quad \text{— (3M)}$$

$$\begin{Bmatrix} \sigma_1 \end{Bmatrix} \quad \begin{bmatrix} Q_{11} & Q_{12} & 0 \end{bmatrix} \begin{Bmatrix} \epsilon_1 \end{Bmatrix}$$

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$$\begin{Bmatrix} 4\times10^6 \\ 2\times10^6 \\ -3\times10^6 \end{Bmatrix} = \begin{bmatrix} 2.048\times10^{11} & 4.27\times10^9 & 0 \\ 4.27\times10^9 & 1.55\times10^{10} & 0 \\ 0 & 0 & 5.5\times10^9 \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

$$4\times10^6 = 2.048\times10^{11}\times\epsilon_1 + 4.27\times10^9\times\epsilon_2 + 0$$

$$2\times10^6 = 4.27\times10^9\times\epsilon_1 + 1.55\times10^{10}\times\epsilon_2 + 0$$

$$-3\times10^6 = 0 + 0 + 5.5\times10^9\,\gamma_{12}$$

$$\gamma_{12} = -\frac{-3\times10^6}{5.5\times10^9} = -5.45\times10^{-4}$$

$$\epsilon_1 = 1.568\times10^{-5}$$

$$\epsilon_2 = 2.945\times10^{-4}$$

5. (a) Desirable characteristics of fibers

1) Improves the strength to weight ratio
2) ↑ Fiber length to width ratio
3) Good strength ~~to~~ and flexibility
4) less diameter of the fiber
5) fiber cohesiveness
6) fiber elasticity
7) uniform c/s area
8) Adequate strength (Tenacity) (7M)
9) Cohesiveness
10) uniformity

5 (b) Types of Glass fibers

A glass → also called alkali glass

C- Glass — Offers resistance to chemical impact
— also called Chemical glass

E - glass → Also called electrical glass
→ Known for mechanical properties

AE - Glass → Alkali resistance glass

S - glass → structured glass and known for mechanical properties

Fiber glass comes from

1. Fiber glass tape
2. Fiber glass cloth
3. Fiberglass rope.

(7M)

(6) (a) Composite : process of combining two or more materials in order to produce new material

+ Constituent materials have different physical and chemical properties
+ A new material has different properties from the individual materials.
+ Constituent 1 - Matrix
  Constituent 2 - Reinforcement

Adv

1. Higher sp. strength than metals & non-metals
2. lower sp. gravity
3. Improved stiffness of the material
4. Maintain their wt. even at high temp.
5. Toughness is improved
6. Fabrication & production is cheaper
   ... strength is better

(7M)

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6(b) Mechanical properties of polymer matrix composites (PMC's) (392)

1. High strength to weight ratio
2. High sp. stiffness to weight ratio.
3. High Stiffness
4. High durability
5. High corrosion resistance
6. properties can be tailored
7. Better fatigue performance than metals in tension
8. No furnace is required. — (7M)

(7)

$E_{glass} = E_f = 85\,GPa$

$E_{epoxy} = E_m = 3.4\,GPa$

$G_{glass} = G_f = 35.42\,GPa$

$G_{epoxy} = G_m = 1.308\,GPa$

$V_f = 0.4$ $\quad$ $[\because V_f + V_m = 1]$

$V_m = 1 - 0.4 = 0.6$ — (7M)

Find

1) $E_1 = E_f V_f + E_m V_m = 85\times10^9 \times 0.4 + 3.4\times10^9 \times 0.6$
$= (85\times0.4 + 3.4\times0.6)\times10^9$
$= 3.604\times10^{10}\,Pa$

2) $\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m} = \frac{0.4}{85\times10^9} + \frac{0.6}{3.4\times10^9}$ = ~~Ans~~ $5.52\times10^9\,Pa$

3) $\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m} = \frac{0.4}{35.42\times10^9} + \frac{0.6}{1.308\times10^9}$

= ~~2.42 Pa~~ $4.7\times10^9\,Pa$

— (7M)

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RGM COLLEGE OF ENGINEERING & TECHNOLOGY (AUTONOMOUS)

27th February-2019

IV B.Tech I Semester (R15) End Examinations (Supplementary)

MECHANICS OF COMPOSITE MATERIALS

MECH

Time: 3 Hrs **Total Marks: 70**

Note 1:Answer Question No.1 (Compulsory) and 4 from the remaining

2:All Questions Carry Equal Marks

1 a What is a angle ply lamina? What is its significance?

b What are the metal matrix composites?

c What is a failure Envelope?

d List few advantages of Vacuum Bag molding?

e What are the typical mechanical properties of carbon fiber?

f How many independent elastic constants required to define an orthotropic material?

g State generalized Hooke's law for 2D cross ply lamina.

2 a) Write the number of independent elastic constants for anisotropic, orthotropic, monoclinic, transversely isotropic and isotropic materials. (6)

b) Find the relationship between the engineering constants and its compliance matrix for an orthotropic material. (8)

3 a) With the help of neat sketch, explain Spray lay-up process for manufacturing of composites. (10)

b) List advantages, drawbacks and applications of Spray lay-up process. (4)

4 a) What are particulate composites? Enumerate their salient features, advantages, limitations. (8)

b) What are the different reinforcements used in ceramic matrix composites? Explain. (6)

5 The properties of unidirectional graphite/epoxy lamina are E_1=181 GPa, E_2=10.3 GPa, ν_{12} = 0.28 and G_{12} = 7.17 GPa. Find the following for a 60° angle lamina of graphite/epoxy

a) Transformed compliance matrix (7)

b) Transformed reduced stiffness matrix. (7)

6 Find the four elastic moduli of a unidirectional glass/epoxy lamina with a 70% fiber volume fraction. Use mechanics of materials approach. Take E_f = 85 GPa, E_m=3.4 GPa, G_f = 35.42 GPa, G_m = 1.308 GPa. ν_f = 0.25 and ν_m = 0.5. (14)

7 Give the complete classification of composite materials? Briefly explain each type of composites citing one example in each category.

- xxx -

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Nop 5/3/18.
21, 22, 23, 24
25, 29, 31, 32, 33
38

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IV B.Tech I-Sem (R15) End Exams (Supply)

Mechanics of Composite Materials

MECH Faculty: Dr. M. Ashok Kumar

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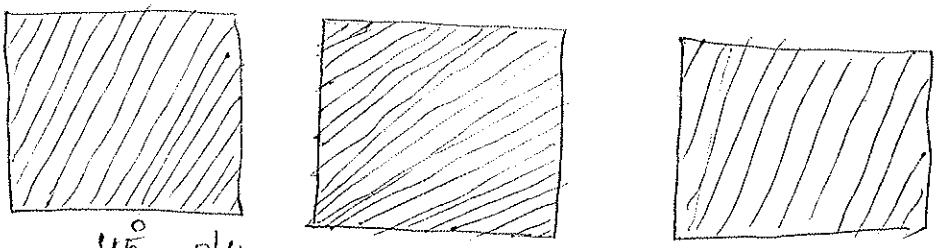
1(a) Angle ply is defined as the fibers are oriented from 0° to 90° orientation in the fiber matrix it is known as angle ply.

Ex: — (1)



45° -ply 30° -ply 60° -ply — (1)

2

It's significance is to strength in all directions other than x & y axis.

1(b) Metal Matrix Composites. (MMC)

MMC's Consists of at least two constituent parts One being metal necessarily, another metal may be a different material such as Ceramic (or) organic Compound

When at least 3- materials are present is called Hybrid composite. — (1)

Material 1 - Matrix ⇒ monolithic Material In which reinforcement is embedded and is completely Continuous

Ex: Aluminum, magnesium, titanium — (1)

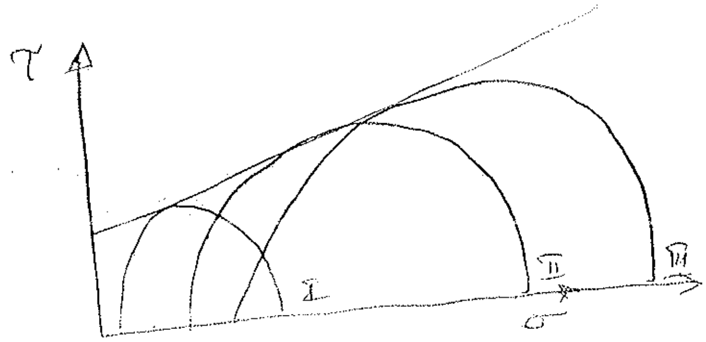
Cobalt, Cobalt- nickel alloy

2

Material - 2 - 'Reinforcement'. Is embedded into a matrix which improves the strength

Ex: Carbon fibers, Alumina, Silicon carbide

1(c)



— ①

It is a plane on which max. shear stress acts or the ~~ratio~~ plane where the ratio of shear stress to normal stress is maximum,

the mohr's ~~cone~~ circles are plotted and a line tangential to the Mohr's ~~Circles~~ circles are called Mohr-Coulomb failure envelop.

— ① (P)

2

1(d)

Advantages

1. Higher fiber content laminates can be achieved with standard wet lay up techniques — 0.5
2. Low void contents — 0.5
3. Better fiber wet-out — 0.5
4. Healthy & ~~safety~~ safety. — 0.5

2

1(e)

Mechanical properties of 'CF'

1. High strength to weight ratio — 0.5
2. Rigidity — 0.5
3. High fatigue resistance, — 0.5
4. Good tensile strength but brittle — 0.5
5. Fire resistance — 2
6. Low thermal Co-eff of expan

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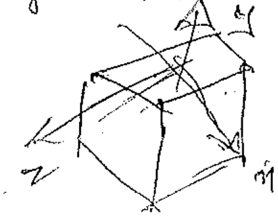
1(f) Orthotropic materials have different properties along 3 mutually ⊥r two fold axes of (x, y, z) ~~that co-ordinate which contains~~ rotational symmetry.

These are subset of anisotropic materials, because their properties change when measured from different directions.

Ex: wood



0.5

This material consists of 3 ⊥lar planes of symmetry — 0.5

3 modulus of elasticities 0.5

3 shear modulus, 0.5

2

For evaluating orthotropic material, we ~~required~~ Nine independent constants to have a stress-strain relation

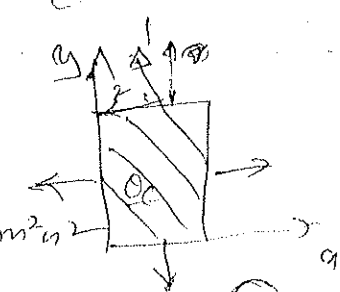
1(g)

Generalized Hooke's law for 2D cross ~~of~~ ply lamina

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

where

$Q_{xx} = m^4 Q_{11} + n^4 Q_{22} + 2m^2n^2 Q_{12} + 4m^2n^2 Q_{66}$



$Q_{yy} = n^4 Q_{11} + m^4 Q_{22} + 2m^2n^2 Q_{12} + 4m^2n^2 Q_{66}$

1

$Q_{xy} = m^2n^2 Q_{11} + m^2n^2 Q_{22} + (m^4+n^4) Q_{12} - 4m^2n^2 Q_{66}$

$m = \cos\theta$

$n = \sin\theta$

$Q_{ss} = m^2n^2 Q_{11} + m^2n^2 Q_{22} - 2(m$

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$Q_{xs} = m^3n\, Q_{11} + mn^3\, Q_{22} + (mn^3 - m^3n)\, Q_{12} + 2(mn^3 - m^3n)\, Q_{66}$

$Q_{ys} = mn^3\, Q_{11} - m^3n\, Q_{22} + (m^3n - mn^3)\, Q_{12} + 2(m^3n - mn^3)\, Q_{66}$.

Where

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} \qquad \text{— (1)}$$

$$Q_{12} = Q_{21} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}$$

2

2(a)

Anisotropic Material

$$\begin{bmatrix} S_{11} & \cdots & S_{16} \\ S_{21} & \cdots & S_{26} \\ S_{31} & \cdots & S_{36} \\ S_{41} & \cdots & S_{46} \\ S_{51} & \cdots & S_{56} \\ S_{61} & \cdots & S_{66} \end{bmatrix} \qquad \text{— (3)}$$

For defining anisotropic material '21' material constants are required.

From the symmetry $S_{ij} = S_{ji}$

Ex: $S_{12} = S_{21}$

Orthotropic Material

3 ⊥r planes of symmetry

3 Modulus of elasticity — (3)

3 shear modulus.

6

For evaluating orthotropic & ... constants

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2(b) Orthotropic Material

$$\epsilon_1 = \frac{\sigma_1}{E_1} - \frac{\nu_{21}\sigma_2}{E_2} - \nu_{31}\frac{\sigma_3}{E_3}$$

$$\epsilon_2 = -\nu_{12}\frac{\sigma_1}{E_1} + \frac{\sigma_2}{E_2} - \nu_{32}\frac{\sigma_3}{E_3}$$

$$\epsilon_3 = -\nu_{13}\frac{\sigma_1}{E_1} - \nu_{23}\frac{\sigma_2}{E_2} + \frac{\sigma_3}{E_3}$$

$$\gamma_{23} = \epsilon_4 = \frac{\tau_{23}}{G_{23}} \qquad \tau_{23} = \sigma_4$$ — (3)

$$\gamma_{31} = \epsilon_5 = \frac{\tau_{31}}{G_{31}} \quad \because \tau_{31} = \sigma_5$$

$$\gamma_{12} = \epsilon_6 = \frac{\tau_{12}}{G_{12}} \quad \because \tau_{12} = \sigma_6$$

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$ — (1)

$$\{\epsilon\} = [S]\{\sigma\}$$ — (3)

$[S]$ → Compliance matrix (6×6)

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & \\ 0 & 0 & 0 & 0 & 0 & S \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{12} \end{Bmatrix}$$ — (2)

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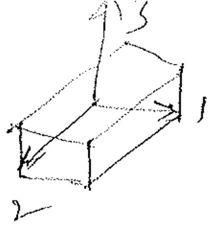
Monoclinic Materials



It has one plane of symmetry (1-2)

3 directions is ⊥r to the plane of symmetry

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \tau_{23}\\ \tau_{32}\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16}\\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26}\\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36}\\ 0 & 0 & 0 & C_{44} & C_{45} & 0\\ 0 & 0 & 0 & C_{45} & C_{55} & 0\\ C_{66} & C_{62} & C_{63} & 0 & 0 & C_{66}\end{bmatrix} \begin{Bmatrix}\epsilon_{11}\\ \epsilon_{22}\\ \epsilon_{33}\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{Bmatrix}$$

(13) - constants

Transversely Isotropic Material

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0\\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & \left(\frac{C_{11}-C_{12}}{2}\right)\end{bmatrix} \begin{Bmatrix}\epsilon_{11}\\ \epsilon_{22}\\ \epsilon_{33}\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{Bmatrix}$$

'5' Independent const. are required.

Isotropic Material

$$\begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0\\ 0 & 0 & 0 & \left(\frac{C_{11}-C_{12}}{2}\right) & 0 & 0\\ 0 & 0 & 0 & 0 & \left(\frac{C_{11}-C_{12}}{2}\right) & \end{bmatrix}$$

2 - Elastic ~~Conta~~ Constants are required

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Equate eqn ① & eqn ②

$S_{11} = \frac{1}{E_1}$; $S_{12} = -\frac{\nu_{21}}{E_2}$; $S_{13} = -\frac{\nu_{31}}{E_3}$

$S_{21} = -\frac{\nu_{12}}{E_1}$; $S_{22} = \frac{1}{E_2}$; $S_{23} = -\frac{\nu_{32}}{E_3}$

$S_{31} = -\frac{\nu_{13}}{E_1}$; $S_{32} = -\frac{\nu_{23}}{E_2}$; $S_{33} = \frac{1}{E_3}$

$S_{44} = \frac{1}{G_{23}}$; $S_{55} = \frac{1}{G_{13}}$; $S_{66} = \frac{1}{G_{12}}$

Inverse of Compliance matrix is called Stiffness matrix, Sometimes it is also called modulus matrix (or) Elasticity matrix, commonly denoted by 'C'

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \left[\frac{E}{(1+\nu)(1-2\nu)}\right] \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & (\frac{1}{2}-\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & (\frac{1}{2}-\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & (\frac{1}{2}-\nu) \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

②

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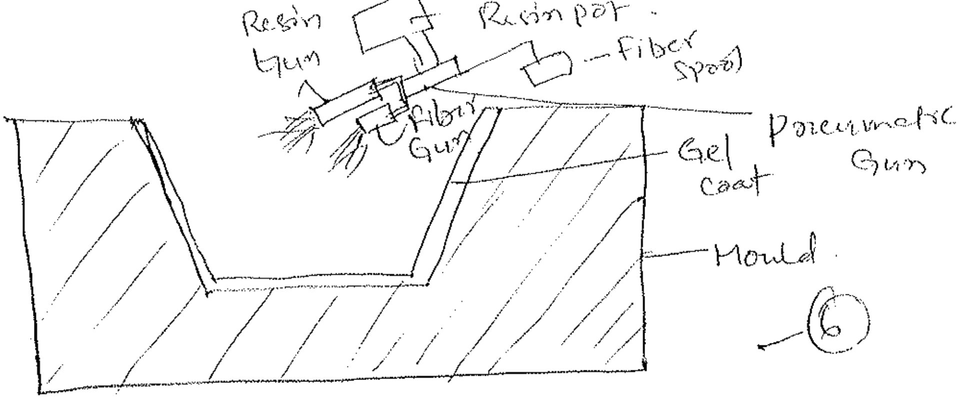
3(a)



Polyester resin with glass fiber rovings are used

It's used when one side is required finishing

Large quantities are made cheaply. (4)

core m/l may be added.

10

3(b) Advantages

(1) Suitable for small & medium volumes

(2) It's a very economical process for making small and large parts

(3) Needs low cost tooling & low cost materials. (1½)

Disadvantages

(1) Not suitable for parts required high structural requirements

(2) Difficult to control fiber and resin volume fraction (2)

(3) Due to open mould emission is a concern

Applications

Bath tubs
Boat hulls
Storage tanks
Swimming pools
Furniture components. (1)

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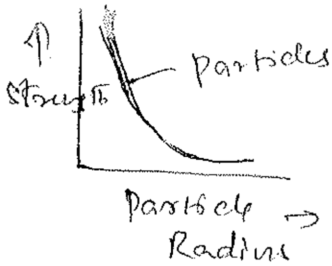
4(a) As particulates are smaller of size 0.01 - 0.1 μm in size strengthening occurs at atomic / molecular level. 10M

Ex:

① Thorium (ThO_2) is dispersed in Ni-alloys

② Sintered aluminium powder + Small flakes of alumina (Al_2O_3)

Particles are dispersed randomly — ④



Particulate Composites
- Large particles
- Dispersion

— ①

Adv

① provides reinforcement to the Matrix
② Improved mechanical properties
③ Tailored m/c properties
④ Manufacturing flexibility
⑤ High creep resistance — ②
⑥ High tensile strength at elevated temp.
⑦ High toughness
⑧ High strength to weight ratio

Disadv — ①

① It used for high strength materials
② Poor ductility
③ Strength depends on uniform dispersion of particles

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4(b) Different reinforcements used in Ceramic Matrix Composites

Ceramic matrix Composites are sub-group of composite materials

They consists of Ceramic fibers ~~for~~ embedded in a ceramic matrix.

The following improvements over Ceramics

1. Degree of anisotropy on incorporation of fibers
2. Increased fracture toughness
3. Elongation to rupture up to %
4. Higher dynamic load capacity — (3)

Common fibers used in Ceramic matrix composites

1. Silicon carbide (SiC)
2. Carbon fibers
3. Zirconia fibers
4. ~~Alumina~~ Boron fiber
5. Boron Carbide fibers
6. Titanium Boride (TiB_2)
7. Aluminum nitride (AlN)
8. Zirconium oxide (ZrO_2)

① SiC Fiber

SiC is Ceramic material yet light weight and covalently bonded material.

High structural stability

Low thermal co-eff. of expansion

High strength and hardness

High melting point 2730°C

High thermal shock resistance.

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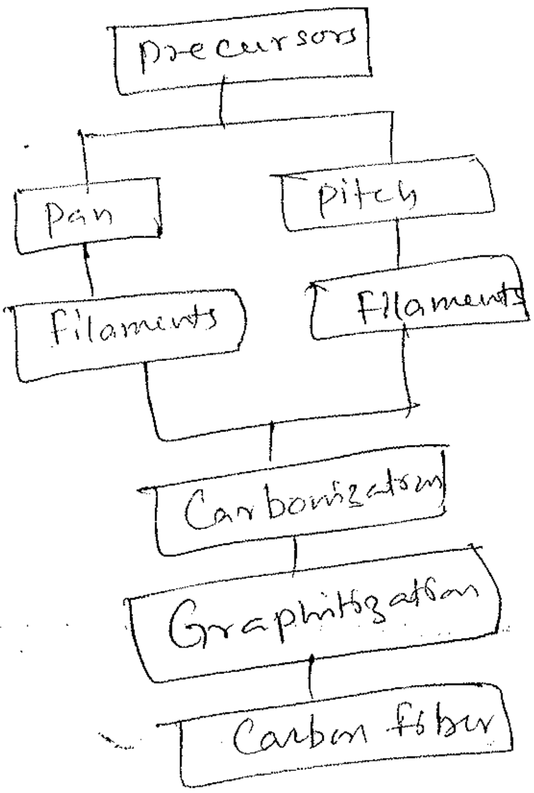
(5) Carbon fibers

Properties

1. High tensile strength
2. High extension at break.
3. High stiffness
4. Low thermal co-eff of expn.
5. low density
6. High wear resistance
7. Long working life
8. Five times stronger and two times stiffer than steel

Fabrication process



Disadvantages

1. Costly
2. Bit harmful
3. Brittleness

Applications

Rackets
golf stickers
Mobile cases
Recharge battery
Fuel cells

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(3) Boron fibers

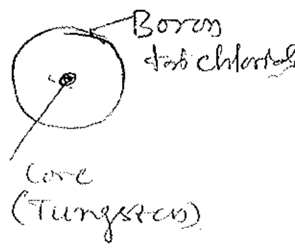
chemical vapour deposition process is used to produce boron fibers

These are Ceramic mono-filament fibers

Fibers itself composites

Circular c/s

fibers dia. range b/w 33–400 μm



Boron fiber properties

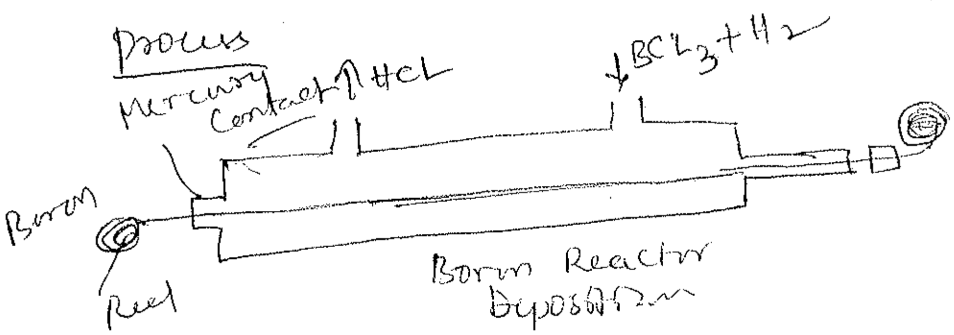
(1) High Tensile strength (400 GPa)

(2) High compressive strength (6900 MPa)

(3) Low thermal co-eff of expansion $\alpha = 4.5\ ppm/^\circ c$

(4) Low density $\rho = 2.57\ g/cm^3$

Process



(2)

6

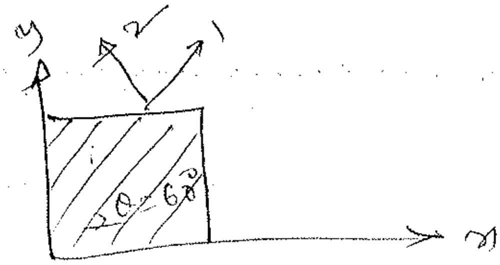
(5) $E_1 = 181\ GPa$

$E_2 = 10.3\ GPa$

$\nu_{12} = 0.28$

$G_{12} = 7.17\ GPa.$

$\theta = 60^\circ$



(1)

Solution $m = \cos 60^\circ = 0.5$

$n = \sin 60^\circ = 0.866$

(1)

WKT

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xs} \\ S_{yx} & S_{yy} & S_{ys} \\ S_{sx} & S & \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \end{Bmatrix}$$

(2)

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Transformed Reduced stiffness matrix

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	Q_{11}	Q_{22}	Q_{12}	Q_{66}
Q_{xx}	m^4	n^4	$2m^2n^2$	$4m^2n^2$
Q_{yy}	n^4	m^4	$2m^2n^2$	$4m^2n^2$
Q_{xy}	m^2n^2	m^2n^2	m^4+n^4	$-4m^2n^2$
Q_{ss}	m^2n^2	m^2n^2	$-2(m^2-n^2)$	$(m^2-n^2)^2$
Q_{xs}	m^3n	mn^3	(mn^3-m^3n)	$2(mn^3-m^3n)$
Q_{ys}	mn^3	$-m^3n$	(m^3n-mn^3)	$2(m^3n-mn^3)$

— (2)

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$ (2)

~~$Q_{xx} = m^4 Q_{11}$~~

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}$$

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$

$$Q_{11} = \frac{181\times10^9}{1-0.28\times0.0159}$$

$$= 1.818\times10^{11}\ \text{Pa}$$

$$= 181.8\times10^9\ \text{Pa} = 181.8\ \text{GPa}.$$

$$\nu_{21} = \frac{E_2}{E_1}\nu_{12}$$

$$= \frac{10.3}{181}\times0.28$$

$$= 15.9\times10^{-3}$$

$$\nu_{21} = 0.0159$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = \frac{10.3\times10^9}{1-0.28\times0.0159)} =$$

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$S_{11} = \frac{1}{E_1} = \frac{1}{181\times10^9} = 5.524\times10^{-12}\ Pa^{-1}$

$S_{22} = \frac{1}{E_2} = \frac{1}{10.3\times10^9} = 9.708\times10^{-11}\ Pa^{-1}$

$S_{12} = -\frac{\nu_{12}}{E_1} = \frac{-0.28}{181\times10^9} = -1.546\times10^{-12}\ Pa^{-1}$

$S_{66} = \frac{1}{G_{12}} = \frac{1}{7.17\times10^9} = 1.404\times10^{-10}\ Pa^{-1}$ — (1)

Reduced transformed compliance Matrix

	S_{11}	S_{22}	S_{12}	S_{66}
S_{xx}	m^4	n^4	$2m^2n^2$	m^2n^2
S_{yy}	n^4	m^4	$2m^2n^2$	m^2n^2
S_{xy}	m^2n^2	m^2n^2	m^4+n^4	$-m^2n^2$
S_{ss}	$4m^2n^2$	$4m^2n^2$	$-8m^2n^2$	$(m^2-n^2)^2$
S_{xs}	$2m^3n$	$-2mn^3$	$2(mn^3-m^3n)$	(mn^3-m^3n)
S_{ys}	$2mn^3$	$-2m^3n$	$2(m^3n-mn^3)$	(m^3n-mn^3)

— (3)

$S_{xx} = 8.08\times10^{-11}\ Pa^{-1}$

$S_{yy} = 3.506\times10^{-11}\ Pa^{-1}$

$S_{xy} = -8.05\times10^{-12}$

$S_{ss} = 1.109\times10^{-11}\ Pa^{-1}$

$S_{xs} = -1.176\times10^{-12}\ Pa^{-1}$

$S_{ys} = -3.843\times10^{-11}\ Pa^{-1}$

$S_{ss} = S_{11}4c^2s^2 + S_{22}4c^2s^2 - S_{12}8c^2s^2 + S_{66}(c^2-s^2)$

$S_{xs} = S_{11}2c^3s - S_{22}cs^3 + S_{12}2(cs^3 - sc^3) + S_{66}(cs^3 - c^3s)$

$$\begin{bmatrix} S_{xx} & S_{xy} & S_{xs} \\ S_{yx} & S_{yy} & S_{ys} \\ S_{sx} & S_{sy} & S_{ss} \end{bmatrix} = \begin{bmatrix} 8.08\times10^{-11} & -8.05\times10^{-12} & -1.176\times10^{-12} \\ -8.05\times10^{-12} & 3.506\times10^{-11} & -3.843\times10^{-11} \\ -1.176\times10^{-12} & -3.843\times10^{-11} & 1.109\times10^{-11} \end{bmatrix}$$

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410/540

$$Q_{12} = Q_{21} = \frac{E_2 \nu_{12}}{1-\nu_{12}\nu_{21}} = \frac{10.3\times10^9 \times 0.28}{1-0.28\times0.0159}$$

$$= 2.896\times10^9 = 2.89\,GPa.$$

$$Q_{66} = G_{12} = 7.17\,GPa$$

m = 0.5
n = 0.866

$$Q_{xx} = m^4 Q_{11} + n^4 Q_{22} + 2m^2n^2 Q_{12} + 4m^2n^2 Q_{66}$$

$$= (0.5)^4 \times 181.8 + (0.866)^4\, 10.34 + 2(0.5)^2(0.866)^2 \times 2.89$$
$$+ 4\times(0.5)^2\times(0.866)^2 \times 7.17.$$

$$Q_{yy} = n^4 Q_{11} + m^4 Q_{22} + 2m^2n^2 Q_{12} + 4m^2n^2 Q_{66}$$

$$= (0.866)^4 \cdot 181.8 + (0.5)^4 \times 10.34 + 2\times(0.5)^2\times(0.866)^2 \times 2.89$$
$$+ 4\times(0.5)^2 (0.866)^2 \times 7.17$$

$$Q_{xy} = m^2n^2 Q_{11} + m^2n^2 Q_{22} + (m^4+n^4) Q_{12} - 4m^2n^2 Q_{66}$$

$$= (0.5)^2(0.866)^2 \times 181.8 + (0.5)^2\times(0.866)^2\, 10.34 + (0.5^4+0.866^4)\, 2.89$$
$$- 4\times(0.5)^2\times(0.866)^2 \times 7.17.$$

$$Q_{ss} = -m^2n^2 Q_{11} + m^2n^2 Q_{22} - 2(m^2-n^2) Q_{12} + (m^2-n^2)^2 Q_{66}$$

$$= -(0.5)^2\times(0.866)^2\, 181.8 + (0.5)^2\times(0.866)^2 \times 10.34 - 2((0.5)^2 - 0.866^2)$$
$$\times 10.34 + ((0.5)^2 - (0.866)^2)\, 10.178$$

(2)

$$Q_{xs} = m^3 n\, Q_{11} + mn^3 Q_{22} + (mn^3 - m^3 n) Q_{12} + 2(mn^3 - m^3 n) Q_{66}$$

$$= (0.5)^3 \times 0.866 \times 181.18 + 0.5\times(0.866)^3 \times 10.34 + (0.5\times0.866^3$$
$$- (0.5)^3 \times 0.866) + 2(0.5\times(0.866$$

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(6) Given Data

$E_f = 55 GPa$

$E_m = 3.4 GPa$

$G_f = 35.42 GPa$

$G_m = 1.308 GPa$

$\nu_f = 0.25$

$\nu_m = 0.5$

$V_f = 0.7$; $V_m = 0.3$ — (2)

$V_f + V_m = 1$

$V_m = 1 - 0.7 = 0.3$

(1) $E_1 = E_f V_f + E_m V_m$ — (3)

$= 55 \times ~~0.25~~ 0.7 + 3.4 \times 0.3$

$E_1 = 60.52 GPa$

(2) $$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

$$\frac{1}{E_2} = \frac{0.7}{55} + \frac{0.3}{3.4}$$ — (3)

$E_2 = 10.36 GPa$

(3) $$\frac{1}{G_{12}} = \frac{\nu_f}{G_f} + \frac{\nu_m}{G_m}$$

$$\frac{1}{G_{12}} = \frac{0.25}{35.42} + \frac{0.5}{1.308}$$ — (3)

$G_{12} = 2.568 GPa$

(4) $\nu_{12} = \nu_f V_f + \nu_m V_m$

$= 0.25 \times 0.7 + 0.5 \times 0.3$

$\boxed{\nu_{12} = 0.325}$ — (3)

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(7) (4M)

Classifications of composites

- Polymer Matrix Composites (PMC's)
- Metal Matrix Composites (MMC's)
- Ceramic Matrix composites (CMC's)
- Carbon-Carbon Matrix composites (CCMC's)

(3)

Polymer Matrix composites

Most commonly used composites are polymer composite matrix, these composites consists of polymer as a matrix material reinforced by fiber.

Ex: Glass fiber, Kevlar fiber, Carbon fiber, Boron fibers, Nylon fibers, Graphite fibers

Reasons

(3)

1. Low cost
2. Non furnace required
3. light weight and high strength
4. Good corrosion & chemical resistance
5. Good Electrical & mechanical properties

Metal matrix composites

Matrix is a soft ductile material (3)

Ex: Al, Mg, Ti, Cu, etc.

Typical fibers includes Boron, Carbon, Silicon Carbide fiber.

These composites are used at higher service temp. than these base metals.

Reinforcements may be in the form fiber (continuous, discontinuous) with

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Reinforcement properties

Specific strength
Stiffness
Abrasion resistance
Creep resistance
Thermal conductivity
Thermal stability

MMC's are more exp. than PMC's

Adv

1. Higher service temp
2. High elastic properties
3. Insensitive to moisture.
4. Higher thermal conductivity.

Ceramic Matrix Composites (CMC's)

CMC's have Ceramic matrix materials such as Alumina, Calcium alumino-silicate reinforced by fibers such as carbon, silicon carbide etc.

(3)

Adv

1. High strength
2. Hardness
3. High service temp limits
4. chemical resistance

Carbon-Carbon Matrix Composites (CCMC's)

(3)

These are high temp. resistance composites in which carbon itself as fiber and matrix, in order to reduce thermal stresses.

Used High abrasion resistance.
High temp resistance.

Heat shields,
air crafts,

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Code: A0338158R0321

RGM COLLEGE OF ENGINEERING & TECHNOLOGY (AUTONOMOUS)

31st March-2021

IV B.Tech I Semester (R15) End Examinations (Regular)

MECHANICS OF COMPOSITE MATERIALS

MECH

Time: 3 Hrs **Total Marks: 70**

Note 1:Answer Question No.1 (Compulsory) and 4 from the remaining

2:All Questions Carry Equal Marks

1a Define mass volume fraction.

b Mention the applications of spray layup process?

c Mention two types of thermoplastic resins.

d Mention the advantages of Invariant Form of Stiffness and Compliance Matrices for an Angle Lamina.

e What are the functions of reinforcements in polymeric composites?

f Differentiate between a lamina and isotropic homogeneous material.

g What are semi-epirical models?

2 a) What is a composite material? Differentiate composite material from metallic alloy. (8)

b) Explain potential applications of composites in the fields of marine, electronics, aerospace and automobile. (6)

3 Explain the mechanics of materials approach to determine four elastic moduli of a composite lamina. (14)

4 a) Explain Resin Transfer Molding with a neat sketch. (10)

b) Discus Advantages, disadvantages and applications of Resin Transfer Molding. (4)

5 a) Explain clearly different types of matrix materials. (6)

b) Discuss about the following: (8)

i) Silicon carbide fiber

ii) Boron carbide fiber

6 The Engineering constants for an orthotropic material are found to be E_1= 40Gpa, E_2= 9Gpa, E_3= 9Gpa, ν_{12}= 0.26, ν_{23}= 0.21, ν_{13}= 0.21 G_{12}= 4.41Gpa, G_{23}= 3.8Gpa, G_{13}= 3.8Gpa. Find the stiffness matrix[C] and compliance matrix [S] for the above orthotropic material. (14)

7 Find the Engineering constants for a 30^0 angle ply lamina. Use the following properties.

E_1= 204 Gpa, E_2= 18.5Gpa, ν_{12}= 0.23, G_{12}= 5.59 Gpa. (14)

- xxx -

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$S_{xx} = 7.27 \times 10^{-11}\ pa^{-1}$

$S_{yy} = 6.38 \times 10^{-11}\ pa^{-1}$

$S_{xy} = -2.312 \times 10^{-11}\ pa^{-1}$

$S_{ss} = 4.52 \times 10^{-10}$

$S_{xs} = -8.59 \times 10^{-11}$

$S_{ys} = 4.33 \times 10^{-11}$

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RGMCET (Autonomous) Prepared by

31st March 2021 Dr. M. Ashok Kumar

IV B.Tech I Sem R15 End Exams (Regular) Cell: 944111585

Sub: MCM ; Branch : ME Time : 3Hrs

Scheme of Evaluation Max. Marks: 70

Code: A0338158R0321

1a) Defination of mass fraction

- It's also known as mass percentage or percentage by mass
- It's the ratio of mass of the constituent to that of the total mass of the composite — (1)

W_m = Mass fraction of the ~~composite~~ matrix,

W_f = Mass fraction of the fiber

$$W_m = \frac{w_m}{w_c}$$

$$W_f = \frac{w_f}{w_c}$$ — (1)

Where, w_m = ~~wet~~ mass of the ~~composite~~ matrix.

w_f = mass of the fiber

w_c = mass of the composite

$$\boxed{W_m + W_f = 1}$$

1b

Applications of Spray lay up

- Making of custom parts
- Bath tubs,
- Boat hulls,
- Storage tanks (1)
- Furniture components
- Swimming pools.

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1c Thermoplastic materials

- polycarbonate (PC)
- polystyrine (PS) } — ①
- poly-vinyl-chloride (PVC)
- Nylon (polyamides. } — ①

1d

Advantages

- stresses and strains on principal axes are computed
- stiffnesses are also calculated along the axes (Moduli)
- poisson's ratios can be calculated along the given planes. } — ①
- Engg. constants can also be calculated

1e Reinforcements in Polymer Matrix composites (PMC)

- Natural ①
  - Ex: Coir fiber, Sisal fiber, Banana fiber, Hemp fiber
- synthetic ①
  - Ex: Glass fiber, Carbon fiber, Kevlar fiber, Silica fiber

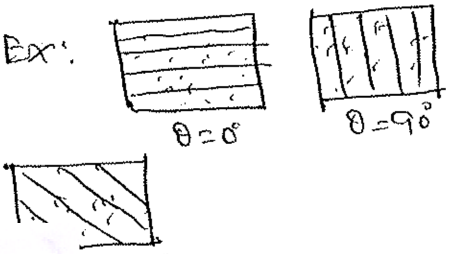
① + ① = ②

1f

Lamina ①

It's a layer of fiberous material arranged in a plane with matrix material in one particular direction

Ex:



Isotropic Homogeneous material ①

Homogeneous refers uniformity of the structure of a material, but isotropic materials are having same properties in all directions

If the properties are same in all directions in any location of the material it is homogeneous

Ex: Steel

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2) Semi-empirical model

- Developed by Halphin-Tsai

(a) E_1 = young's modulus along the longitudinal axis

$$= E_f V_f + E_m V_m$$

(b) E_2 = young's modulus along the transverse axis

$$= \left[\frac{1 + \xi \eta V_f}{1 - \eta V_f}\right] \times E_m$$

where

$$\eta = \left[\frac{\frac{E_f}{E_m} - 1}{\frac{E_f}{E_m} + \xi}\right] \quad \text{--- (1)}$$

(c) G_{12} = Inplane shear modulus

$$= \left[\frac{1 + \xi \eta V_f}{1 - \eta V_f}\right] \times G_m$$

where

$$\eta = \left[\frac{\frac{G_f}{G_m} - 1}{\frac{G_f}{G_m} + \xi}\right] \quad \text{--- (1)}$$

G_m, G_f = Inplane shear modulus of matrix and fiber resp.

E_m, E_f = Youngs modulus of matrix and fiber resp.

V_f, V_m = Volume fractions of fiber and matrix resp.

(1) + (1) = (2)

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2 a) Composite is a new material which is produced by combining two or more materials by process

(or)

Composite is material fiber is embeded in a matrix material. — (2)

- It is made up of two or more materials
- Matrix — material one
- Reinforcement — material two
- Matrix binds the other constituents
- Reinforcement improves the strength and stiffness of the material, protects from the environment (2)

Types of Composites

- Polymer matrix composite
  - ↓ Polymer is matrix material
  - ↓ Around 250°C temp resistance
- Metal matrix composites
  - ↓ Metal alloy is matrix material
  - ↓ Around 700°C temp. resistance
- Ceramic matrix composites
  - ↓ Ceramic is the matrix material
  - ↓ Around 2500°C temp. resistance
- Carbon–Carbon matrix composites (2)
  - ↓ Carbon is matrix material
  - ↓ Around 3000°C temp. resistan

(2)

2b **Field wise applications of composites**

Marine field

- Fishing boats
- Life boats
- Anti-marine ships
- Rescue ships
- Hover crafts

- Hulls (2)
- Decks
- Pri
- Ru
- Ya

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(5)

4/20

Electronic field

- Switches
- optical fibers
- Led TV's
- Mother boards
- Circuit boards
- wires
- Sinks

Aerospace

Gliders
Helicopter blades
Transmission shafts
Elevators
spoilers
Rocket boosters
Nozzles
Antenna covers
Fuselage, Doors, seats
Landing gears

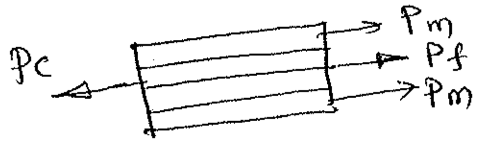
(2)

Automobile

- leaf springs
- Bumpers
- Body components
- chassis components
- Engine components
- Engine bonnet
- Mud wings
- Lamp heads
- Cabins
- Instrument panels
- window frames,

(2)

3. (a) Longitudinal modulus (E_1)



To determine this the following assumptions are made

(a) strain experienced by the composite is equal to fiber and matrix

$$\epsilon_c = \epsilon_f = \epsilon_m$$

(b) Load applied on the composite is shared by fiber and matrix

$$P_c = P_m + P_f$$

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$$\epsilon_c \sigma_c A_c = \epsilon_f E_f A_f + \epsilon_m E_m A_m \qquad \sigma = \frac{P}{A}$$

$$P = \sigma A$$

$$\sigma = \epsilon E$$

$$E_c = E_f\left(\frac{A_f}{A_c}\right) + E_m\left(\frac{A_m}{A_c}\right) \qquad \because \epsilon_c = \epsilon_f = \epsilon_m$$

$$E_c = E_f V_f + E_m V_m \quad (\text{or})$$

$$\because V_f = \frac{A_f}{A_c} ; \quad V_m = \frac{A_m}{A_c}$$

$$V_m + V_f = 1$$

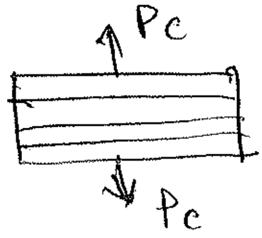
$$\therefore V_m = 1 - V_f$$

$$E_c = E_f V_f + E_m(1 - V_f)$$

$$E_1 = E_f V_f + E_m(1 - V_f) \qquad \because E_c = E_1 \quad \text{—(4)}$$

(b) <u>Transverse modulus (E_2)</u>

<u>Assumptions</u>



(A) $\sigma_c = \sigma_f = \sigma_m$ —(1)

(B) $t_c = t_f + t_m$ —(2)

(C) $\delta_c = \delta_f + \delta_m$ —(3)

$$\epsilon = \frac{\delta}{t} = \frac{\Delta L}{L}$$

$$\delta = \epsilon t \quad \text{(4)}$$

$\therefore$ eqn-(B) is modified as

$$\epsilon_c t_c = \epsilon_f t_f + \epsilon_m t_m$$

$$\epsilon_c = \epsilon_f\left(\frac{t_f}{t_c}\right) + \epsilon$$

$$\epsilon_c = \epsilon_f V_f + \epsilon$$

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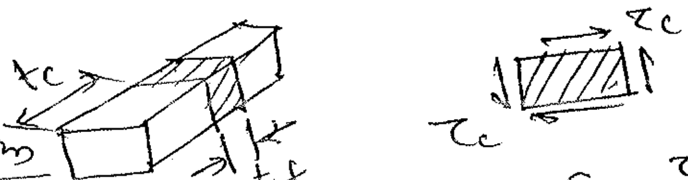
$$\frac{\sigma_c}{E_c} = \frac{\sigma_f}{E_f} V_f + \frac{\sigma_m}{E_m} V_m \qquad \because \epsilon = \frac{\sigma}{E}$$

$$\frac{1}{E_c} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

$$E_c = \frac{E_f E_m}{V_f E_m + V_m E_f}$$

$$E_2 = \left[\frac{E_f E_m}{V_f E_m + V_m E_f}\right] \qquad \because E_c = E_2 \quad \text{--- (4)}$$

(c) In-plane shear modulus (G_{12})



Assumptions

(a) $\tau_c = \tau_f = \tau_m$

$$G = \frac{\tau}{\gamma} \qquad \gamma = \frac{\delta}{t}$$

$\delta_c, \delta_f, \delta_m$ are the deformations in the composites, fiber & matrix resp.

$$\delta_c = \delta_f + \delta_m \quad \text{--- (1)}$$

$$\gamma_c t_c = \gamma_f t_f + \gamma_m t_m \quad \text{--- (2)}$$

$$G_{12} = \frac{\tau_c}{\gamma_c} \Rightarrow \gamma_c = \frac{\tau_c}{G_{12}} \quad \text{--- (3)}$$

$$\gamma_f = \frac{\tau_f}{G_f} \text{ (4)}, \quad \gamma_m = \frac{\tau_m}{G_m} \text{ --- (5)}$$

Sub. eqn (3), (4), (5) in eqn (2)

~~$\frac{\tau_c}{\gamma_c} t_c = $~~

$$\frac{\tau_c}{G_{12}} t_c = \frac{\tau_f}{G_f} t_f + \frac{\tau_m}{G_m}$$

$$\frac{1}{\ } = \frac{1}{\ } \cdot \frac{t_f}{\ } +$$

$$\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}$$

where $\therefore V_f = \frac{t_f}{t_c}$

Where

V_f = Vol. fraction of fibre $\therefore V_m = \frac{t_m}{t_c}$

V_m = Vol. fraction of matrix,

— (4)

(d) Major poisson's ratio (ν_{12})

$$\nu_{12} = \nu_f V_f + \nu_m V_m$$

Where

ν_f = poisson's ratio of fibre

ν_m = poisson's ratio of matrix

V_m, V_f = Vol. fractions of matrix & fibre resp.

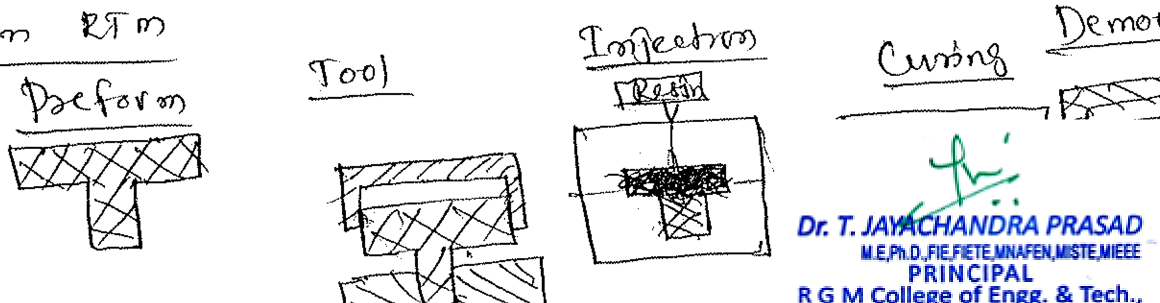
— (2)

4(a) Resin transfer mould (RTM)

- RTM is an intermediate volume moulding process for producing composites
- In RTM resin is injected under pressure in mould cavity.
- This process produces parts with two finished surfaces.

— (2)

Steps in RTM



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Advantages

- low skilled labour is required
- Low tooling cost
- Low volatile emission
- Required design tailorability
- Good surface finish
- Very large complex shapes can be made
- less material wastage
- Good dimensional tolerances
- fast production, — (2)
- less emission due to closed mould

Disadvantages

- preforms are labour intensive
- Waste may be high
- Chances of moisture entrapment
- Distortion of fiber during injection of resin due to fiber wash — (1)
- Control of resin uniformity is difficult.

Applications

- Complex structures can be produced
- Automotive body parts, big containers, bath tubs, helmets etc
- Vehicle panels (1)
- Boat hulls,
- Wind turbine blades
- Aerospace parts,

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5(a) 4 types of matrix materials

1) polymers
2) Metals
3) ceramics
4) Carbon

Matrix
- PMC's (250°C)
  - Thermoplastics
  - Thermoset plastics
- MMC's (700°C)
- CMC's (2500°C)
- CCMC's (3000°C)

polymers

– Two different matrix materials are used in polymer matrix composites (PMC's)

~~The thermoplastics are~~ Thermosets

Ex: – Epoxy
– polyester
– vinylester

Thermoplastics

polyethylene (PE)
polycarbonate (PC)
polystyrene (PS)
polyvinyl chloride (PVC)

(2)

Metals

Cermets, Tic, TicN,
~~Cement~~ Cemented carbides

(2)

Ceramics

Ceramics
Al_2O_3, SiC
Application: tool materials,

Carbon

Carbon, graphite

Application: ~~Brake mat~~ Brake pads

(2)

5(b) Silicon Carbide fiber (SiC) (1)

(U26)

- High strength at elevated temp.
- High oxidation resistance
- High micro-structural stability
- High stiffness
- High tensile strength
- Low thermal expansion
- Low weight

— (4)

Boron - Carbide fiber

- Extreme hardness
- Difficult to sinter to high relative densities
- Good chemical resistance
- Good nuclear properties
- It's elastic modulus is close to diamond.

(4)

(6) Given Data

$E_1 = 40\,GPa$	$G_{12} = 4.41\,GPa$	$\nu_{12} = 0.26$
$E_2 = 9\,GPa$	$G_{23} = 3.8\,GPa$	$\nu_{23} = 0.21$
$E_3 = 9\,GPa$	$G_{13} = 3.8\,GPa$.	$\nu_{13} = 0.21$

Find

(a) Compliance matrix [S]

(b) Stiffness matrix [C]

(2)

Given material

orthotropic material

Sol:- Using Betti Reciprocal law

$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}$$

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \qquad \nu_{21} = \frac{E}{E}$$

$= \frac{9}{4\ldots}$

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$$\frac{\nu_{31}}{E_3} = \frac{\nu_{13}}{E_1}$$

$$\nu_{31} = \frac{E_3}{E_1}\nu_{13} = \frac{9}{40} \times 0.21$$

$$\boxed{\nu_{31} = 0.047}$$

$$\frac{\nu_{23}}{E_2} = \frac{\nu_{32}}{E_3}$$

$$\nu_{32} = \frac{E_3}{E_2}\nu_{23} = \frac{9}{9} \times 0.21$$

$$\boxed{\nu_{32} = 0.21} \quad (4)$$

WKT

$$\{\epsilon\} = [S]\{\sigma\}$$

where $[S]$ = compliance matrix.

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_2 \\ \tau \\ \tau \end{Bmatrix}$$

$$S_{11} = \frac{1}{E_1} = \frac{1}{40 \times 10^9} = 2.5 \times 10^{-11}\ Pa^{-1}$$

$$S_{12} = -\frac{\nu_{21}}{E_2} = -\frac{0.0585}{9 \times 10^9} = -6.5 \times 10^{-12}\ Pa^{-1}$$

$$S_{12} = S_{21} = -\frac{\nu_{21}}{E_2} = 6.5 \times 10^{-12}\ Pa^{-1}$$

$$S_{13} = -\frac{\nu_{31}}{E_3} = -\frac{0.047}{9 \times 10^9} = -5.22 \times 10^{-12}\ Pa^{-1}$$

$$S_{13} = S_{31} = -\frac{\nu_{31}}{E_3} = -5.22 \times 10^{-12}\ Pa^{-1}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{9 \times 10^9} = 1.11 \times 10^{-10}\ Pa^{-1}$$

$$S_{23} = -\frac{\nu_{23}}{E_2} = -\frac{0.21}{9 \times 10^9} = -$$

$$S_{23} = -\frac{\nu_{23}}{} = -2.33 \times 10^{-11}\ Pa^{-1}$$

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$S_{33} = \frac{1}{E_3} = \frac{1}{9\times10^9} = 1.11\times10^{-10}\ Pa^{-1}$ (13)

$S_{44} = \frac{1}{G_{23}} = \frac{1}{3.8\times10^9} = 2.63\times10^{-10}\ Pa^{-1}$

$S_{55} = \frac{1}{G_{13}} = \frac{1}{3.8\times10^9} = 2.63\times10^{-10}\ Pa^{-1}$

$S_{66} = \frac{1}{G_{12}} = \frac{1}{4.41\times10^9} = 2.26\times10^{-10}\ Pa^{-1}$

$$[S] = \begin{bmatrix} 2.5\times10^{-11} & -6.5\times10^{-12} & -5.22\times10^{-12} & 0 & 0 & 0 \\ -6.5\times10^{-12} & 1.11\times10^{-10} & -2.33\times10^{-11} & 0 & 0 & 0 \\ -5.25\times10^{-12} & -2.33\times10^{-11} & 1.11\times10^{-10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.63\times10^{-10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.63\times10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.26\times10^{-10} \end{bmatrix}$$ — (4)

WKT

Stiffness matrix is given by

$$\{\sigma\} = [Q]\{\epsilon\} \text{ or } \{\sigma\} = [C]\{\epsilon\}$$

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

$$C_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} = \frac{40\times10^9}{(1-0.26\times0.0585)}\ Pa = 4.061\times10^{10}\ Pa$$

$$C_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = \frac{9\times10^9}{(1-0.26\times0.0585)}\ Pa$$

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$$C_{33} = \frac{E_3}{1-\nu_{13}\nu_{31}} = \frac{9\times10^9}{(1-0.21\times0.047)} Pa = 9.089\times10^9 Pa$$
$= 9.089 GPa$

$C_{44} = G_{23} = 3.8\times10^9 Pa = 3.8 GPa$

$C_{55} = G_{13} = 3.8 GPa$

$C_{66} = G_{12} = 4.41 GPa$

$$C_{12} = \frac{\nu_{12}E_{12}}{1-\nu_{12}\nu_{21}} = \frac{0.26\times9\times10^9}{(1-0.26\times0.0585)} Pa = 2.37\times10^9 Pa$$

$= 2.377 GPa$

$$C_{13} = \frac{\nu_{13}E_3}{1-\nu_{13}\nu_{31}} = \frac{0.21\times9\times10^9}{(1-0.21\times0.047)} Pa = 1.90\times10^9 Pa$$

$C_{13} = 1.90 GPa \quad \because C_{13} = C_{31}$

$$C_{21} = \frac{\nu_{21}E_1}{1-\nu_{21}\nu_{12}} = C_{12} = 2.37\times10^9 Pa$$

$$C_{23} = \frac{\nu_{23}E_3}{1-\nu_{23}\nu_{32}} = \frac{0.21\times9\times10^9}{1-0.21\times0.21} Pa = 1.977\times10^9 Pa$$

$C_{23} = 1.977 GPa$

$C_{23} = C_{32} \quad \because C_{ij} = C_{ji}$

$$[C] = \begin{bmatrix} 40.61 & 2.37 & 1.90 & 0 & 0 & 0 \\ 2.37 & 9.13 & 1.977 & 0 & 0 & 0 \\ 1.89 & 1.977 & 9.089 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.41 \end{bmatrix} GPa$$

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Given Data

$\theta = 30^\circ$

$E_1 = 204\ GPa$

$E_2 = 18.5\ GPa$

$\nu_{12} = 0.23$

$G_{12} = 5.59\ GPa$

(2)

$S_{11} = \frac{1}{E_1} = \frac{1}{204\times10^9} = 4.901\times10^{-12}\ Pa^{-1}$ (1.5)

$S_{22} = \frac{1}{E_2} = \frac{1}{18.5\times10^9} = 5.405\times10^{-11}\ Pa^{-1}$

$S_{12} = \frac{-\nu_{12}}{E_1} = \frac{-0.23}{204\times10^9} = -1.127\times10^{-12}$

$S_{66} = \frac{1}{G_{12}} = \frac{1}{5.59\times10^9} = 1.78\times10^{-10}$

WRT

$$\begin{Bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xs}\\ S_{yx} & S_{yy} & S_{ys}\\ S_{sx} & S_{sy} & S_{ss}\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}$$

↓ Reduced transformed compliance matrix

$C = \cos\theta = \cos 30^\circ$

$S = \sin\theta = \sin 30^\circ$

where

$S_{xx} = ~~m^4 S_{11} + n^4 S_{22} + 2\times m^2 n^2 S_{12} + m^2 n^2 S_{66}~~$

$= C^4 S_{11} + S^4 S_{22} + 2C^2S^2 S_{12} + C^2S^2 S_{66}$

$S_{xx} = (0.866)^4 \times 4.901\times10^{-12} + (0.5)^4 \times 5.405\times10^{-11}$
$+ 2\times(0.866)^2\times(0.5)^2 \times(-1.127\times10^{-12}) + (0.866)^2\times(0.5)$
$\times 1.788\times10^{-10}$

$S_{xx} = ~~3.922\times10^{-11}\ Pa^{-1}~~$ $7.27\times10^{-11}\ Pa^{-1}$

$S_{xy} = C^2S^2 S_{11} + C^2S^2 S_{22} + (C^4+S^4) S_{12} - C^2S^2 S_{66}$

$= (0.866)^2\times(0.5)^2\times4.901\times10^{-12} + (0.866)^2\times(0.5)^2\times5.405\times10^{-11}$
$+ ((0.866)^4+(0.5)^4)\times-1.127\times10^{-12} - (0.866)^2\times(0.5)^2\times1.788\times10^{-10}$

$S_{xy} = ~~-2.317\times10^{-11}\ Pa^{-1}~~$ $= -2.317\times10^{-11}\ Pa^{-1}$

$S_{yy} = S^4 S_{11} + C^4 S_{22} + 2C^2S^2 S_{12} + C^2S^2 S_{66}$

$= (0.5)^4 \times 4.901\times10^{-12} + (0.866)^4\times5.405\times10^{-11}$
$+ 2\times(0.866)^2\times(0.5)^2(-1.127\times10^{-12})$
$+ (0.866)^2\times(0.5)^2 \times 1.788\times10^{-10}$

(2)

$S_{yy} = ~~3.63\times10^{-11}\ Pa^{-1}~~$

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$$S_{ss} = 4c^2s^2 S_{11} + 4c^2s^2 S_{22} - 8c^2s^2 S_{12} + (c^2-s^2)^2 S_{66}$$

$$= 4\times(0.866)^2\times(0.5)^2\times 4.901\times10^{-12} + 4\times(0.866)^2\times(0.5)^2\times 5.405\times10^{-11}$$
$$- 8\times(0.866)^2\times(0.5)^2\times(-1.127\times10^{-12})$$
$$+ [(0.866)^2-(0.5)^2]^2\times 1.788\times10^{-10}$$

$S_{ss} =$ ~~2.24×10^{-10} pa^{-1}~~ 4.52×10^{-10} pa^{-1}

$$S_{xs} = 2c^3s S_{11} - 2cs^3 S_{22} + 2(cs^3-c^3s) S_{12} + (cs^3-c^3s)$$

$$= 2\times(0.866)^3\times0.5\times4.901\times10^{-12} - 2\times0.866\times(0.5)^3\times 5.405\times10^{-11}$$
$$+ 2(0.866\times(0.5)^3 - (0.866)^3\times0.5)\times(-1.127\times10^{-12})$$
$$+ (0.866\times0.5^3 - (0.866)^3\times0.5)\times 1.788\times10^{-10}$$

$S_{xs} = -$ ~~4.67×10^{-11} pa^{-1}~~ -8.59×10^{-11} pa^{-1}

$$S_{ys} = 2cs^3 S_{11} - 2c^3s S_{22} + 2(c^3s-cs^3) S_{12} + (c^3s-cs^3) S_{66}$$

$$= 2\times0.866\times(0.5)^3\times4.901\times10^{-12} - 2\times(0.866)^3\times0.5\times5.40\ldots\times10^{\ldots}$$
$$+ 2((0.866)^3\times0.5 - 0.866\times(0.5)^3)\times1.788\times10^{-10}$$

$S_{ys} = 4$~~$.336\times10^{-11}$ pa~~ (or) 4.336×10^{-11} pa^{-1}

$$\begin{Bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{bmatrix}\frac{1}{E_x} & -\frac{\nu_{xy}}{E_y} & \frac{\eta_{xs}}{G_{xy}}\\ -\frac{\nu_{yx}}{E_x} & \frac{1}{E_y} & \frac{\eta_{sy}}{G_{xy}}\\ \frac{\eta_{sx}}{E_x} & \frac{\eta_{ys}}{E_y} & \frac{1}{G_{xy}}\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} \quad -(1)$$

$$\begin{Bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{bmatrix}S_{xx} & S_{xy} & S_{xs}\\ S_{yx} & S_{yy} & S_{ys}\\ S_{sx} & S_{sy} & S_{ss}\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \ldots\end{Bmatrix} \quad (2)$$

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Eqn ① & Eqn ② are same.

Let us equate ① & ②

Eqn ① = Eqn ② in terms of compliance matrix

$$\begin{bmatrix} S_{xx} & S_{xy} & S_{xs} \\ S_{yx} & S_{yy} & S_{ys} \\ S_{sx} & S_{sy} & S_{ss} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & \frac{-\nu_{yx}}{E_y} & \frac{\eta_{xs}}{G_{xy}} \\ \frac{-\nu_{xy}}{E_x} & \frac{1}{E_y} & \frac{\eta_{sy}}{G_{xy}} \\ \frac{\eta_{sx}}{E_x} & \frac{\eta_{ys}}{E_y} & \frac{1}{G_{xy}} \end{bmatrix}$$

$$S_{xx} = \frac{1}{E_x} \Rightarrow E_x = \frac{1}{S_{xx}} = \frac{1}{7.27\times10^{-11}} = 1.37\times10^{10}\ Pa$$

$$S_{yy} = \frac{1}{E_y} \Rightarrow E_y = \frac{1}{S_{yy}} = \frac{1}{6.38\times10^{-11}} = 1.56\times10^{10}\ Pa$$

$$S_{ss} = \frac{1}{G_{xy}} \Rightarrow G_{xy} = \frac{1}{S_{ss}} = \frac{1}{4.52\times10^{-10}} = 2.22\times10^{9}\ Pa$$

(2)

$$S_{xy} = \frac{-\nu_{yx}}{E_y}$$

$$\nu_{yx} = -E_y \times S_{xy}$$

$$= -(1.56\times10^{10}) \times -2.317\times10^{-11}$$

$\nu_{yx} = 0.3614$ $\quad \because \nu_{ij} = \nu_{ji}$

$\nu_{yx} = \nu_{xy} = 0.3614$

$$S_{xs} = \frac{\eta_{xs}}{G_{xy}} \Rightarrow \eta_{xs} = S_{xs} \times G_{xy}$$

(2)

$$= -8.59\times10^{-11} \times 2.22\times10^{9}$$

$\eta_{xs} = -0.188$ $\quad (\because \eta_{ij} = \eta_{ji})$

$\eta_{xs} =$ ..

$\eta_{sx} = -0.188$

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$$S_{ys} = \frac{\eta_{sy}}{G_{xy}} \Rightarrow \eta_{sy} = G_{xy} \times S_{ys}$$

$$\eta_{sy} = 2.22 \times 10^{9} \times 4.33 \times 10^{-11}$$

$$\eta_{sy} = 0.095$$

$$\because \eta_{ij} = \eta_{ji}$$

$$\boxed{\eta_{sy} = \eta_{ys} = 0.095}$$

(2)

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THE END

1/4/21

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Code: A0338158R1118

RGM COLLEGE OF ENGINEERING & TECHNOLOGY (AUTONOMOUS)

15 th November-2018

IV B.Tech I Semester (R15) End Examinations (Regular)

MECHANICS OF COMPOSITE MATERIALS

MECH

Time: 3 Hrs **Total Marks: 70**

Note 1:Answer Question No.1 (Compulsory) and 4 from the remaining

2:All Questions Carry Equal Marks

1a What is major Poisson's ratio?

b What are the typical mechanical properties of carbon fiber?

c What are the typical mechanical properties of ceramic matrix composites?

d What is maximum stress failure theory?

e List the factors to be considered while selecting the most efficient manufacturing process for composites.

f Differentiate between a lamina and isotropic homogeneous material.

g List two physical properties that can be estimated using rule of mixtures.

2 The Engineering constants for an orthotropic material are found to be E_1= 40Gpa, E_2= 9Gpa, E_3= 9Gpa, ν_{12}= 0.26, ν_{23}= 0.21, ν_{13}= 0.21 G_{12}= 4.41Gpa, G_{23}= 3.8Gpa, G_{13}= 3.8Gpa. Find the stiffness matrix[C] and compliance matrix [S] for the above orthotropic material. (14)

3 Obtain an expression for E_1, E_2, ν_{12} and G_{12} in terms of material properties with respect to principal material directions using strength of material approach. (14)

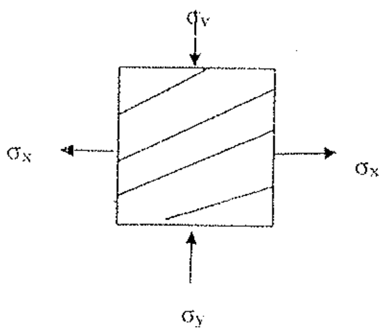
4 a) What is reinforcement? Explain the purpose of reinforcements? (6)

b) Describe different types of reinforcements used in polymer composites. (8)

5 What are the two types of filament winding? Explain them with the help of neat sketches. Mention their applications. (14)

6 Give the complete classification of composite materials? Briefly explain each type of composites citing one example in each category.

7 An off axis laminais loaded as shown. Determine σ_x = $-\sigma_y$ =F_0 at failure using the Tsai-Hill and max. Stress failure criteria for a material of the following properties. (14)



F_{1t} = 2280MPa

F_{2t} =59 MPa

F_6=69 MPa

F_{1C} = 1450MPa

F_{2C} = 228MPa

- xxx -

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RGMCET (Nandyal) - Autonomous (436) 1
IV B.Tech I-Sem R-15, End Exams (Reg.)
Mechanics of Composite Materials
Code: A0338158R1118 ; (MECH.) Total Marks: 70

1a.

The major poisson's ratio for local plane '12' is found by taking negative lateral strain in the local plane '12' and dividing it by the axial strain in the direction of normal to the local plane '12' for an axially loaded member. — (1M)

$$\therefore \nu_{12} = \frac{E_1}{E_2}$$

$\nu_{12} = \frac{E_T}{E_L}$ where ν_{12} = Major poisson's ratio

ν_{21} = Minor poisson's ratio

E_1 = Young's modulus in the longitudinal direction

E_2 = Young's modulus in the transverse direction.

(or)

$$\nu_{12} = \nu_f V_f + \nu_m V_m$$ — (1M)

where V_f = Vol. fraction of fiber

V_m = Vol. fraction of matrix

ν_f = Poisson ratio of fiber

ν_m = Poisson ratio of matrix

1b. Mechanical properties of Carbon fiber

High strength to wt. ratio, Rigidity, Corrosion resistance, High Electrical conductivity, Fatigue Resistance, Good tensile strength — (1M)

Low co-eff of thermal expn.

High thermal conductivity.

High stiffness & High wear res

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1c Mechanical properties of CMC

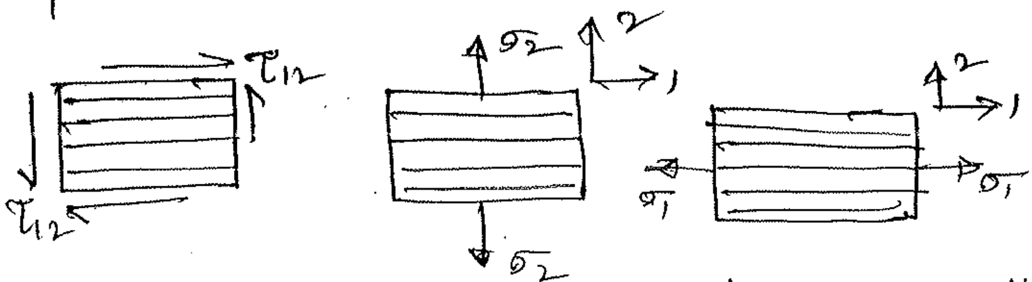
(a) High strength to wt. ratio
(b) High toughness
(c) High stiffness
(d) High strength elevated temps.
} — (1M)

(e) High thermal shock resistance
(f) Low density
(g) High fatigue life.
} — (1M)

1d. Max. Stress failure theory:

It states that failure will occur if any one of the stresses induced by the applied loads in the principal material axis exceed the corresponding allowable stress. — (1M)

Therefore, to avoid the failure the following inequalities must be satisfied.



$\sigma_1 < (\sigma_1)^u_c$	$\sigma_1 < (\sigma_1)^u_t$
$\sigma_2 < (\sigma_2)^u_c$	$\sigma_2 < (\sigma_2)^u_t$
$\tau_{12} < (\tau_{12})^u_c$	$\tau_{12} < (\tau_{12})^u_t$

— (1M)

$\sigma_1, \sigma_2, \tau_{12}$ = Stresses produced by the applied loads

$(\sigma_1)^u, (\sigma_2)^u (\tau_{12})^u$ = correspondin

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1e

(a) Strength to weight ratio
(b) Density of the composite
(c) voids needs to be less
(d) Surface finish on both sides
} → 1M

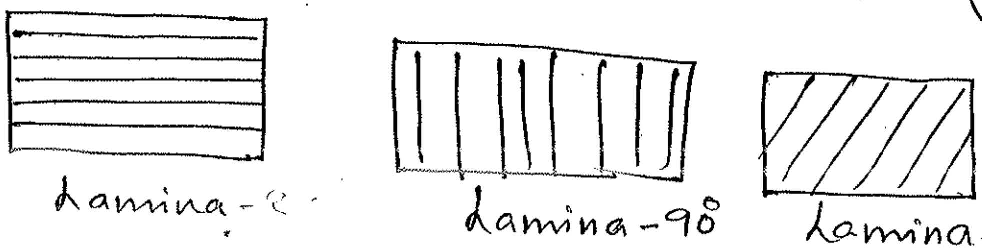
(e) High corrosion resistance
(f) High electrical & thermal conductivity
(g) Selection of sp. fiber & matrix
} → 1M

1f

Lamina : It's plane surface area in which fiber is arranged in uni-direction held with matrix. Material whose thickness is 0.125 mm

Ex:

→ 1M



→ 1M

Isotropic Homogeneous material :

- characterized by infinite no. of planes of m/l
- properties are same in all directions
- 2 elastic constants are req.
- properties are directionally dependent.

} → 1M

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \left(\frac{C_{11}-C_{12}}{2}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & \left(\frac{C_{11}-C_{12}}{2}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}$$

1M

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19. (a) volume fraction

$$V_f = \frac{v_f}{v_c}$$

$$V_m = \frac{v_m}{v_c}$$

$$V_f + V_m = 1$$ — (0.5M)

(b) weight fraction

$$W_f = \frac{w_f}{w_c}$$

$$W_m = \frac{w_m}{w_c}$$

$$W_f + W_m = 1$$ — ($\frac{1}{2}$M)

(c) Density of the composites

$$\rho_c = \left[\frac{1}{\frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}}\right]$$

(or)

$$\rho_c = \rho_f V_f + \rho_m V_m$$ — ($\frac{1}{2}$M)

(d) Void fraction

$$V_v = \frac{\rho_T - \rho_e}{\rho_T}$$ — ($\frac{1}{2}$M)

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2. Given Data

5 (440)

$E_1 = 40 GPa$

$E_2 = 9 GPa$

$E_3 = 9 GPa$

$\nu_{12} = 0.26$

$\nu_{23} = 0.21$

$\nu_{13} = 0.21$

$G_{12} = 4.41 GPa$

$G_{23} = 3.8 GPa$

$G_{13} = 3.8 GPa$

WKT

$$\{\epsilon\} = [S]\{\sigma\}$$

where $[S]$ = Compliance Matrix

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}$$

$S_{11} = \frac{1}{E_1}$; $S_{12} = -\frac{\nu_{21}}{E_2}$; $S_{13} = -\frac{\nu_{31}}{E_3}$

$S_{21} = -\frac{\nu_{12}}{E_1}$; $S_{22} = \frac{1}{E_2}$; $S_{23} = -\frac{\nu_{32}}{E_3}$ — (3M)

$S_{31} = -\frac{\nu_{13}}{E_1}$; $S_{32} = -\frac{\nu_{23}}{E_2}$; $S_{33} = \frac{1}{E_3}$

$S_{44} = \frac{1}{G_{23}}$; $S_{55} = \frac{1}{G_{13}}$; $S_{66} = \frac{1}{G_{12}}$

where

$$S_{11} = \frac{1}{E_1} = \frac{1}{40\times10^9} = 2.5\times10^{-11}\ Pa^{-1}$$

$$S_{12} = -\frac{\nu_{21}}{E_2} = \frac{-0.0585}{9\times10^9} = 6.5\times10^{-12}\ Pa^{-1}$$

$$S_{13} = -\frac{\nu_{31}}{E_3} = \frac{-0.047}{9\times10^9} = 5.22\times10^{-12}\ Pa^{-1}$$

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$$S_{21} = -\frac{\nu_{12}}{E_1} = \frac{-0.26}{40\times10^9} = -6.5\times10^{-12}\ Pa^{-1}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{9\times10^9} = 1.11\times10^{-10}\ Pa^{-1}$$

$$S_{23} = -\frac{\nu_{32}}{E_3} = \frac{-0.21}{9\times10^9} = -2.33\times10^{-11}\ Pa^{-1}$$

$$S_{31} = -\frac{\nu_{13}}{E_1} = \frac{-0.21}{40\times10^9} = -5.25\times10^{-12}\ Pa^{-1}$$

$$S_{32} = -\frac{\nu_{23}}{E_2} = \frac{-0.21}{9\times10^9} = -2.33\times10^{-11}\ Pa^{-1}$$

$$S_{33} = \frac{1}{E_3} = \frac{1}{9\times10^9} = 1.11\times10^{-10}\ Pa^{-1}$$

$$S_{44} = \frac{1}{G_{23}} = \frac{1}{3.8\times10^9} = 2.63\times10^{-10}\ Pa^{-1}$$

$$S_{55} = \frac{1}{G_{13}} = \frac{1}{3.8\times10^9} = 2.63\times10^{-10}\ Pa^{-1}$$

$$S_{66} = \frac{1}{G_{12}} = \frac{1}{4.41\times10^9} = 2.26\times10^{-10}\ Pa^{-1}$$

(4M)

$$[S] = \begin{bmatrix} 2.5\times10^{-11} & -6.5\times10^{-12} & -5.22\times10^{-12} & 0 & 0 & 0 \\ -6.5\times10^{-12} & 1.11\times10^{-10} & -2.33\times10^{-11} & 0 & 0 & 0 \\ -5.25\times10^{-12} & -2.33\times10^{-11} & 1.11\times10^{-10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.63\times10^{-10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.63\times10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.26\times10^{-10} \end{bmatrix}$$

From Betti - Reciprocal Law

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$

$$\nu_{21} = \frac{E_2}{E_1}\nu_{12} = \frac{9}{40}\times0.26$$

$$\nu_{21} = 0.0585$$

$$\frac{\nu_{31}}{E_3} = \frac{\nu_{13}}{E_1}$$

$$\nu_{31} = \frac{E_3}{E_1}\nu_{13} = \frac{9}{40}\times0.21$$

$$\nu_{31} = 0.047$$

$$\frac{\nu_{23}}{E_2} = \frac{\nu_{32}}{E_3}$$

$$\nu_{32} = \frac{E_3}{E_2}\nu_{23} = \frac{9}{9}\times0.21$$

$$\nu_{32} = 0.21$$

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Stiffness matrix is given by

$\{\sigma\} = [Q]\{\epsilon\}$ (or) $\{\sigma\} = [C]\{\epsilon\}$

→ Stiffness matrix

$$[C] \text{ (or) } [Q] = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \quad \text{— (3M)}$$

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} = \frac{40\times10^9}{(1-0.26\times0.0585)} \text{ Pa} = 4.061\times10^{10} \text{ Pa}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = \frac{9\times10^9}{1-0.26\times0.0585)} \text{ Pa} = 9.13\times10^9 \text{ Pa}$$

$$Q_{33} = \frac{E_3}{1-\nu_{13}\nu_{31}} = \frac{9\times10^9}{(1-0.21\times0.047)} \text{ Pa} = 9.089\times10^9 \text{ Pa}$$

$$Q_{44} = G_{23} = 3.8\times10^9 \text{ Pa} = 3.8\times10^9 \text{ Pa}$$

$$Q_{55} = G_{13} = 3.8\times10^9 \text{ Pa} = 3.8\times10^9 \text{ Pa}$$

$$Q_{66} = G_{12} = 4.41\times10^9 \text{ Pa} = 4.41\times10^9 \text{ Pa}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{0.26\times9\times10^9}{(1-0.26\times0.0585)} \text{ Pa} = 2.37\times10^9 \text{ Pa}$$

$$Q_{13} = \frac{\nu_{13}E_3}{1-\nu_{13}\nu_{31}} = \frac{0.21\times9\times10^9}{(1-0.21\times0.047)} \text{ Pa} = 1.90\times10^9 \text{ Pa}$$

$$Q_{21} = \frac{\nu_{21}E_1}{1-\nu_{21}\nu_{12}} = \frac{0.0585\times40\times10^9}{(1-0.0585\times0.26)} \text{ Pa} = 2.37\times10^9 \text{ Pa}$$

$$Q_{23} = \frac{\nu_{23}E_3}{1-\nu_{23}\nu_{32}} = \frac{0.21\times9\times10^{11}}{\ldots}$$

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$$Q_{31} = \frac{\nu_{31} E_1}{1-\nu_{31}\nu_{13}} = \frac{0.047 \times 40 \times 10^9}{(1-0.047 \times 0.21)} \, Pa = 1.89 \times 10^9 \, Pa$$

$$Q_{32} = \frac{\nu_{32} E_2}{1-\nu_{32}\nu_{23}} = \frac{0.21 \times 9 \times 10^9}{(1-0.21 \times 0.21)} \, Pa$$

$$= 1.977 \times 10^9 \, Pa$$

$$[Q] = \begin{bmatrix} 40.61 & 2.37 & 1.90 & 0 & 0 & 0 \\ 2.37 & 9.13 & 1.977 & 0 & 0 & 0 \\ 1.89 & 1.977 & 9.089 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.41 \end{bmatrix} \, GPa. \quad \text{— (4M)}$$

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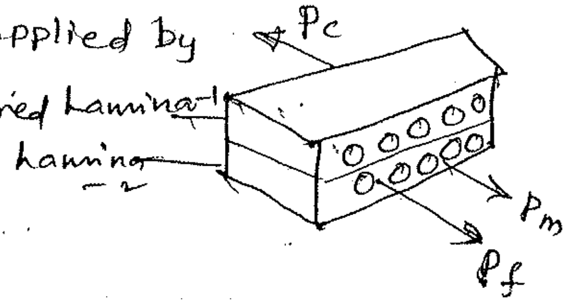
3. (a) Derivation for longitudinal Modulus (E_1)

When composite be applied by a load 'P_c' which is shared by the fibers & matrix



Mathematically,

$$P_c = P_f + P_m \quad -(1)$$

P_c, P_f, P_m are the loads acting on the composite, fiber, m

Strain experienced by the fibers & matrix are equal

Mathematically

$$\epsilon_c = \epsilon_f = \epsilon_m \quad -(2)$$

— (2M)

where

$\epsilon_c, \epsilon_f, \epsilon_m$ = are the strains acting on the composite, fiber and matrix resp.

WKT

$$P_c = P_f + P_m$$

$$\sigma_c A_c = \sigma_f A_f + \sigma_m A_m \qquad \because P = \sigma A$$

$$E_c \epsilon_c A_c = E_f \epsilon_f A_f + E_m \epsilon_m A_m \qquad \because \sigma = E\epsilon$$

$$E_c = E_f \left[\frac{\epsilon_f A_f}{\epsilon_c A_c}\right] + E_m \left[\frac{\epsilon_m A_m}{\epsilon_c A_c}\right] \qquad \because \epsilon_c = \epsilon_m = \epsilon_f$$

$$E_c = E_f \frac{A_f}{A_c} + E_m \left(\frac{A_m}{A_c}\right)$$

— (2M)

$$= E_f V_f + E_m V_m \qquad V_f = \frac{N_f}{V_c} = \frac{A_f}{A_c}$$

$$E_c = E_f V_f + E_m V_m$$

$$E_1 = E_c = \sum^{n} E_i V_i$$

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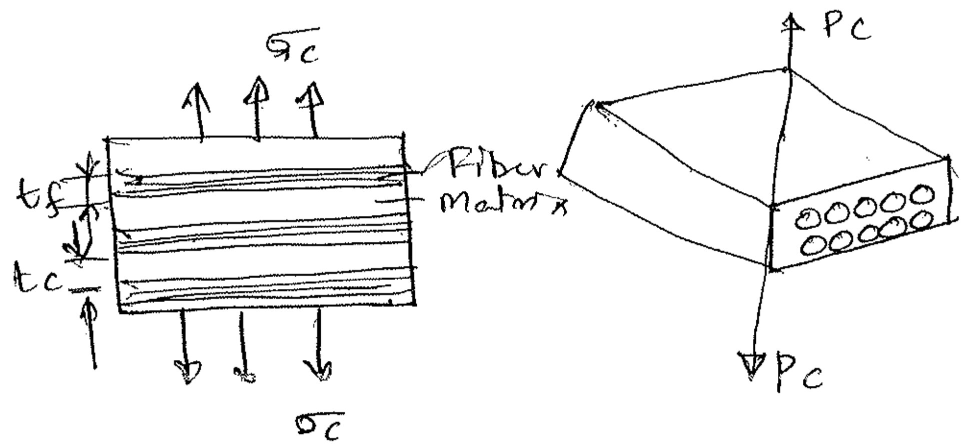
(b) Determination of transverse modulus (E_2)



WKT

$$t_c = t_f + t_m \quad — (1)$$

Where

t_c = thickness of the composite

t_f = thickness of the fiber

t_m = Thickness of the matrix

When force is applied along the transverse direction a little consideration will show that elongation in the composite is equal to algebraic sum fiber & matrix

Mathematically, $\delta_c = \delta_f + \delta_m \quad (2)$ — (2M)

Fibers and matrix experience equal stress, mathematically

$$\sigma_c = \sigma_f = \sigma_m \quad — (3)$$

WKT $\epsilon = \frac{\delta}{t} = \frac{\Delta L}{L}$

$$\delta = \epsilon t \quad — (4)$$

Sub eqn (4) in eqn (2)

$$\epsilon_c t_c = \epsilon_f t_f + \epsilon_m t_m$$

$$\epsilon_c = \epsilon_f \left(\frac{t_f}{t_c}\right) + \epsilon_m \left(\frac{t_m}{t_c}\right) \quad — (2M)$$

$$\epsilon_c = \epsilon_f V_f + \epsilon_m V_m \quad ; V_f = \frac{v_f}{v_c}$$

$$\frac{\sigma_c}{E_c} = \frac{\sigma_f}{E_f} V_f + \frac{\sigma_m}{E_m} V_m$$

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$$\frac{1}{E_c} = \frac{V_f}{E_f} + \frac{V_m}{E_m} \quad \because \sigma_c = \sigma_m = \sigma_f$$

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$$E_c = \frac{E_f E_m}{V_f E_m + V_m E_f}$$

In general

$$E_c = \frac{1}{\sum_{i=1}^{n} \frac{V_i}{E_i}}$$

(c) Major poisson's ratio (ν_{12})

$$\nu_{12} = \frac{(\epsilon_c)T}{(\epsilon_c)L}$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m \quad \text{— (3M)}$$

V_f, V_m = vol. fractions of fiber & matrix resp

ν_f, ν_m = poisson's ratio of fiber & matrix

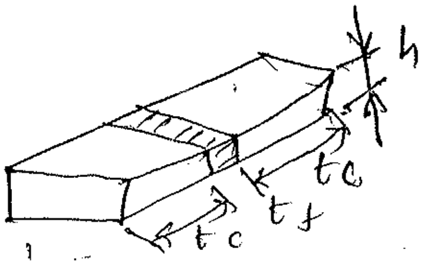
(d) In-plane shear modulus (G_{12})

$$S_c = S_f + S_m \quad \text{—(1)}$$

$$S_c = \gamma_c t_c \quad \text{—(2)}$$

$$S_f = \gamma_f t_f \quad \text{—(3)}$$

$$S_m = \gamma_m t_m \quad \text{—(4)}$$



$$G_{12} = \frac{\tau_c}{\gamma_c}$$

$$\gamma_c = \frac{\tau_c}{G_{12}}$$

$$\gamma_f = \frac{\tau_f}{G_f}$$

$$\gamma_m = \frac{\tau_m}{G_m}$$

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Sub. eqns ②, ③, ④ in eqn ①

$$\gamma_c t_c = \gamma_f t_f + \gamma_m t_m$$

$$\frac{\tau_c}{G_{12}} t_c = \frac{\tau_f}{G_f} t_f + \frac{\tau_m}{G_m} t_m$$

WKT $\tau_c = \tau_f = \tau_m$

$$\frac{t_c}{G_{12}} = \frac{t_f}{G_f} + \frac{t_m}{G_m}$$

$$\frac{1}{G_{12}} = \frac{1}{G_f}\left(\frac{t_f}{t_c}\right) + \frac{1}{G_m}\left(\frac{t_m}{t_c}\right)$$

$\because V_f = \frac{t_f}{V_c}$

$\because V_m = \frac{t_m}{t_c}$

$$\boxed{\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}}$$

→ (3M)

4ⓐ <u>Reinforcement</u>: It's a something which builds strength in the composite is known as reinforcement. → (2M)

Reinforcements are different for different matrices.

Polymer Matrix composite (PMC)

<u>PMC's</u>

Reinforcements	Matrices
Ex: Glass fibers	Ex:
Kevlar fibers	Epoxy
Carbon fibers	polyester
Silica fibers	Vinylester
Natural fibers	Poly-vinylchloride
	Poly Carbonate

Metal Matrix comp (MMC)

Reinforcing agent	Matrix
# Carbon fibers	Al, Mg,
# SiC Fibers	Titan

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Ceramic Matrix Composite (CMCs)

They consist of ceramic fibers em in a ceramic matrix.

Reinforcements	Matrix
Carbon	Carb
Al_2O_3	sna
SiC	llite
Mullite (Al_2O_3 SiO_2)	

Purpose of reinforcement

I. * To increase the mechanical properties of the neat resin (matrix) such as
 * To redu the strength
 * To ncrease the toughness
 * To reduce the brittleness
 * To increase the fatigue life

II. To increase thermal properties such as
 * Glass transition temp.
 * Thermal stability
 * Thermal shock.

III. To increase the corrosion resistance

IV. To increase the electrical and magnetic properties

V. To increase the stability of the composite

VI. To increase the hardness of

VII. To carry the load

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following fibers are used in the polymer composite

a. Glass fibers
b. Kevlar fibers
Carbon fibers
Silica fibers
Boron fiber
Iron carbide fibers
? fibers ← (1M)

a. Glass fibers

made up of direct-melt process.

Mixing, heating, mixing, fusion, drawing, quenching, Coating are the different process used for producing glass-fibers

The following types are used in Glass-fibers

E-Glass : High electrical conductivity
S-Glass : High Strength (stiffness)
C-Glass : High corrosion resistance
D-Glass : Dielectric properties
R-Glass : High Mechanical properties. — (1M)

b. Kev-lar fibers

Applns: Car washers, Food processing, Dock & marine, Aero-space & defence application

- Strong & heat resistant
- Maintain strength & resilience up to −196°C
- Slightly stronger at lower temp.
- less prone to breake
- High tensile strength

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Applications

Bullet proof vests
Bicycle tires
Racing sails
armors
Cricket bats
Helmets

C. Carbon fibers

Carbon atoms are bonded together to form long chain
produced by PAN or pitch
It's a super strong material and too light wt.
Five time stronger & Two times stiffer than steel.
It need some safety precautions as they produce skin irritation due to dust.
It has high wear resistance
It has low thermal co-eff. of expn.
less weight
Long working life
High tensile strength and extension at break. — (1M)
High stiffness.

Applis

Rackets
golf sticks
Automobile bodies
Mobile cases
fuel cells.

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d. Silica fibers

- # Silica fibers are made of sodium silicate (water glass)
- # Used for heat protection applications
- # These have high mechanical strength against pulling and bending
- # These have very good optical properties
- # Used in sound, light and guiding applications

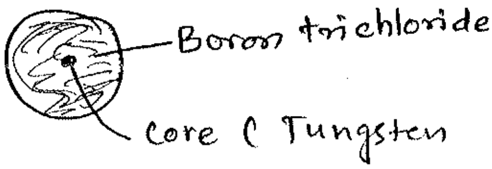
Appl's

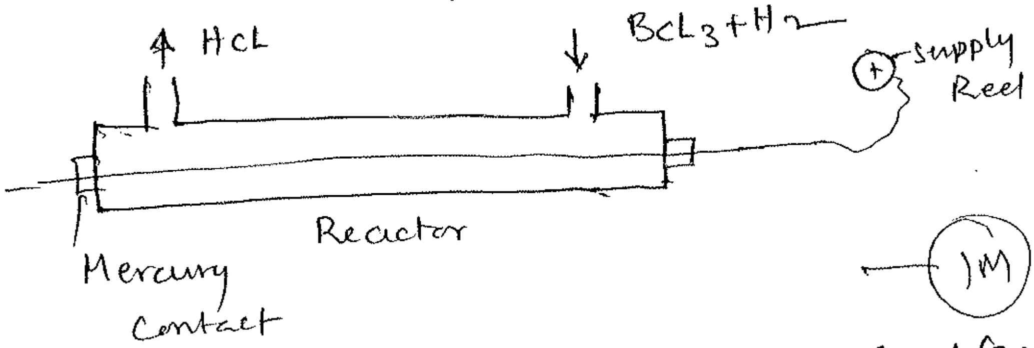
Smart cameras
LED - TV's
Blue Ray disc.
Solar cells
LED light bulbs

— 1M

e. Boron fibers (BF)

- # First introduced in the year of 1959
- # Chemical vapour deposition process is used to produce BF.
- #





— 1M

$$2\,BCl_3\,(g) + 3H_2\,(g) \rightarrow 2B\,(s) + 6HCl\,(g)$$

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f. Boron Carbide Fiber (B_4C)

It is also called black dimond and dark gray color.

Hardest material after dimond.

$$2B_2O_3 + 7C \rightarrow B_4C + 6CO$$

$2B_2O_3$ → Boron trichloride; $7C$ → Carbon; B_4C → Boron Carbide; $6CO$ → Carbon monoxide

High fracture toughness upto 3.5 MPa/m²

Low thermal conductivity

Susceptiable to thermal shock failure

Extremly brittle — (1M)

Good thermal neutron capture ability

Applications

Cutting tools & dies

Abrasives

Solid fuels

Brake lining materials

Wear resistant coatings

High pr water Jet nozzle cutters

armor plating.

g. SiC Fibers

Properties

i) ↓ ρ

ii) ↑ E

iii) ↓ α

iv) ↑ Thermal Conductivity

v) ↑ Hardness

vi) ↑ Elastic Modulus

vii) High Thermal shock resistance

Applns

pumps & rockets in [illegible] engines

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4ब.

5. Filament winding :

It's a automatic method for creating composite structures by winding filament under tension over a rotating mandrel (tool).

Fiber placement is guided by a m/c with two or more axis of motion as it can be seen in the simple schematic diagram.

Filament winding is used to manufacture range of products such as pipes, pipe joints, drive shafts, masts, pressure vessels, storage tanks.

Two types — (4M)

1. continuous FW
2. Discontinuous FW

1. Continuous FW

In which fiber is continuous fed on to the mandrel with the help of carriage

It helps us to produce uniform thickness — (3M)

2. Discontinuous FW

Different layers of different fiber can be achived.

Strength of the product cannot maintained at all places.

— (3M)

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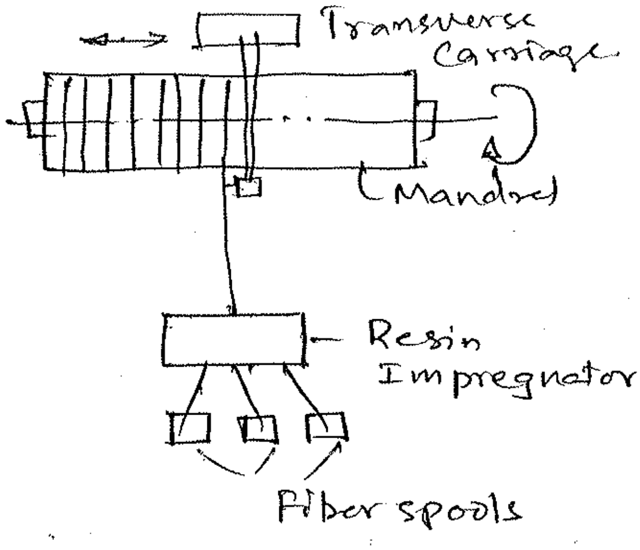
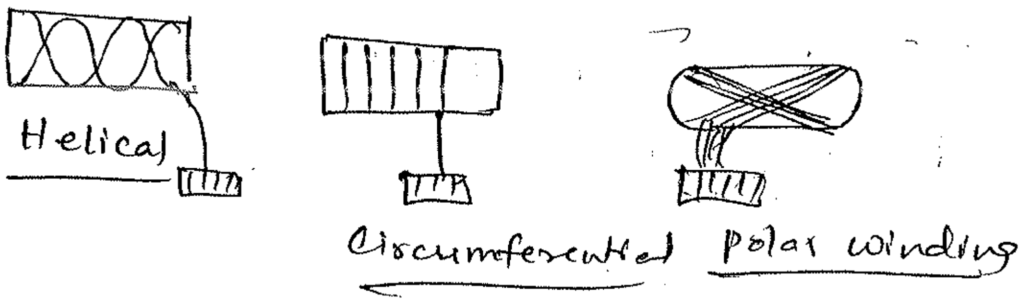


Patterns — 2M



Applications

Big pipes, Missiles,
Chemical tanks
pressure vessels,
Sporting goods. — 2M

(6) Composite : It is defined as the process of combining two or more constituents macroscopically to produ or yield useful material.

(or)

Fiber material embedded in a matrix material is known as composite

Composite consists of two different material.

① Reinforcement ② Matrix

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Function of matrix

(a) It binds the reinforcement
(b) Transfer the load from matrix to reinforcement
(c) protect the reinforcement from adverse temperatures
(d) Reduces moisture absorption
(e) Low shrinkage
(f) Low co-eff of thermal expansion
(g) good flowability
(h) Resist heat & electricity

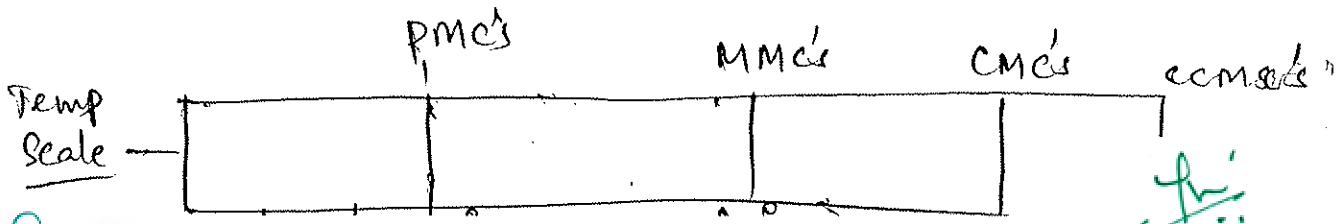
Function of Reinforcing agent

1. Stiffer
2. Stronger
3. Cost effective
4. Chemical inert
5. Heat resistant
6. Good Electrical properties

Types of Composites

(3M)

- Polymer Matrix Composites (PMC's)
- Metal Matrix Composites (MMC's)
- Ceramic Matrix Composites (CMC's)
- Carbon-Carbon Matrix Composites



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2) Polymer Matrix composites (PMC's)

If polymer as a matrix material dopped with some fiber material then it is called PMC's.

We use different matrices in PMC's such as Thermo plastic Material and thermoset material and Rubber materials as a matrix,

PMC's

- Thermo-plastic
  Ex: PC, PP, PE
- Thermoset plastic
  Ex: Epoxy
- Rubber

physical & chemical properties of the matrix and reinforcement materials place vital role to get ultimate performance of the PMC's.

In PMC's reinforcement used such as Glass fiber Silica fiber, Kevlar fiber, Carbon fiber are some important fibers are used.

Glass fibers, Carbon fiber, & Kevlar fibers are used in automobile Industries & Space Industries and some domestic applications as well.

Applications,

Car bodies
Fuel tanks
Helmets
Rackets
Car bumpers — (3M)

D. Metal Matrix Composites (MMCs)

MMC's are require to resist high temp ie beyond 250°c we need these materials.

High strength, stiffness, toughness, density, wear resistance, damping & modulus are very high for this MMC's.

Matrix and fibers combinations are as mentioned below.

Matrix	Fibers
Aluminium, Magnesium, Lead, Copper	Graphite
Al, Mg, Titanium	Boron
Aluminium, Lead, Magnesium	Al_2O_3 (Alumina)
Al, Ti, Super alloys (Cobalt bases)	SiC

— (3M)

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Advantages

1. Low thermal co-eff of expns.
2. High fibre resistance
3. High wear resistance
4. High transverse stiffness, strength and modulus
5. No moisture absorption
6. High electrical & thermal conductivity
7. Better radiation resistance.

Disadvantages

1. complex fabrication processes
2. High cost of Reinforcements
3. Machining is difficult
4. Furnace is required
5. poor corrosion resistance
6. Fiber & matrix interactions at high temp degrade the MMCs

(c) Ceramic Matrix Composites (CMCs)

Advantages

1. Excellent wear & corrosion resistance
2. High strength to wear ratio
3. High strength retention at elevated temp. — (3M)
4. High chemical stability
5. Non-catastrophic failure

(2) High load carrying capa

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Disadvantages

1. Difficult in fabrication
2. Highly brittle
3. Expensive processing
4. Difference in α_f and α_m leads to Thermal stresses on cooling

Application

Heat shields

components for gas turbines, stators, vanes, blade

Brake discs,

Slide bearing

Gas ducts,

flame holder

Burners

Carbon-Carbon Matrix Composites

They resist temp above 2500°C
They overcome all the problem in CMC's
It's requires costly furnaces
High production cost. — (2M)

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Code: A0338158R0321

RGM COLLEGE OF ENGINEERING & TECHNOLOGY (AUTONOMOUS)

31st March-2021

IV B.Tech I Semester (R15) End Examinations (Regular)

MECHANICS OF COMPOSITE MATERIALS

MECH

Time: 3 Hrs **Total Marks: 70**

Note 1:Answer Question No.1 (Compulsory) and 4 from the remaining

2:All Questions Carry Equal Marks

1a Define mass volume fraction.

b Mention the applications of spray layup process?

c Mention two types of thermoplastic resins.

d Mention the advantages of Invariant Form of Stiffness and Compliance Matrices for an Angle Lamina.

e What are the functions of reinforcements in polymeric composites?

f Differentiate between a lamina and isotropic homogeneous material.

g What are semi-epirical models?

2 a) What is a composite material? Differentiate composite material from metallic alloy. (8)

b) Explain potential applications of composites in the fields of marine, electronics, aerospace and automobile. (6)

3 Explain the mechanics of materials approach to determine four elastic moduli of a composite lamina. (14)

4 a) Explain Resin Transfer Molding with a neat sketch. (10)

b) Discus Advantages, disadvantages and applications of Resin Transfer Molding. (4)

5 a) Explain clearly different types of matrix materials. (6)

b) Discuss about the following: (8)

i) Silicon carbide fiber

ii) Boron carbide fiber

6 The Engineering constants for an orthotropic material are found to be E_1= 40Gpa, E_2= 9Gpa, E_3= 9Gpa, ν_{12}= 0.26, ν_{23}= 0.21, ν_{13}= 0.21 G_{12}= 4.41Gpa, G_{23}= 3.8Gpa, G_{13}= 3.8Gpa. Find the stiffness matrix[C] and compliance matrix [S] for the above orthotropic material. (14)

7 Find the Engineering constants for a 30^0 angle ply lamina. Use the following properties.

E_1= 204 Gpa, E_2= 18.5Gpa, ν_{12}= 0.23, G_{12}= 5.59 Gpa. (14)

- xxx -

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$S_{xx} = 7.27 \times 10^{-11}\ Pa^{-1}$

$S_{yy} = 6.38 \times 10^{-11}\ Pa^{-1}$

$S_{xy} = -2.312 \times 10^{-11}\ Pa^{-1}$

$S_{ss} = 4.52 \times 10^{-10}$

$S_{xs} = -8.59 \times 10^{-11}$

$S_{ys} = 4.33 \times 10^{-11}$

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31st March 2021

Dr. M. Ashok Kumar
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IV B.Tech I Sem R15 End Exams (Regular)
Sub: MCM ; Branch: ME
Scheme of Evaluation

Time: 3 Hrs
Max. Marks: 7
Code: A0338158R0321

1(a) Defination of mass fraction

- It's also known as mass percentage or percentage by ma
- It's the ratio of mass of the constituent to that of the total mass of the composite — (1)

W_m = Mass fraction of the ~~composite~~ matrix,
W_f = Mass fraction of the fiber

$$W_m = \frac{w_m}{w_c}$$
$$W_f = \frac{w_f}{w_c}$$ — (1)

Where, w_m = ~~Wet~~ mass of the ~~composite~~ matrix.
w_f = mass of the fiber
w_c = mass of the composite

$$\boxed{W_m + W_f = 1}$$

1b

Applications of Spray lay up

- Making of custom parts
- Bath tubs,
- Boat hulls,
- Storage tanks
- Furniture components
- Swimming pools. — (1)

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1c **Thermoplastic materials**

- polycarbonate (PC)
- polystyrine (PS) } — ①
- poly-vinyl-chloride (PVC)
- Nylon (polyamides. } — ①

1d **Advantages**

- stresses and strains on pricipal axes are computed
- stiffnesses are also calculated along the axes (Moduli)
- poisson's ratios can be calculated along the given planes. } — ①
- Engg. constants can also be calculated

1e **Reinforcements in Polymer Matrix composites (PMC)**

- Natural ①
  Ex:
  Coir fiber
  Sisal fiber
  Banana fiber
  Hemp fiber
- synthetic ①
  Ex: Glass fiber
  Carbon fiber
  Kevlar fiber
  Silica fiber

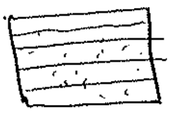
① + ① = ②

1f

Lamina ①

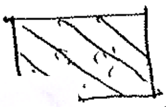
It's a layer of fiberous material arrenged in a plane with matrix material in one particular direction

Ex:



θ = 0°

θ = 90°



θ = 45°

Isotropic Homogeneous material ①

Homogeneous refers uniformity of the structure of a material, but isotropic materials are having same properties in all directions

If the properties are same in all directions in any location of the material is homogeneous

Ex: Steel

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- Developed by Halphin-Tsai

(a) E_1 = young's modulus along the longitudinal axis

$$= E_f V_f + E_m V_m$$

(b) E_2 = young's modulus along the transverse axis

$$= \left[\frac{1+\xi\eta V_f}{1-\eta V_f}\right] \times E_m$$

where

$$\eta = \left[\frac{\frac{E_f}{E_m}-1}{\frac{E_f}{E_m}+\xi}\right] \quad -(1)$$

(c) G_{12} = Inplane shear modulus

$$= \left[\frac{1+\xi\eta V_f}{1-\eta V_f}\right] \times G_m$$

where

$$\eta = \left[\frac{\frac{G_f}{G_m}-1}{\frac{G_f}{G_m}+\xi}\right] \quad -(1)$$

G_m, G_f = Inplane shear modulus of matrix and fiber resp.

E_m, E_f = Youngs modulus of matrix and fiber resp.

V_f, V_m = Volume fractions of fiber and matrix resp.

(1) + (1) = (2)

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2 a) Composite is a new material which is produced by combining two or more materials by process

(or)

Composite is material fiber is embeded in a matrix material. — (2)

- It is made up of two or more materials
- Matrix – material one
- Reinforcement – material two
- Matrix binds the other constituents
- Reinforcement improves the strength and stiffness of the material, protects from the environment (2)

Types of Composites

- Polymer matrix composite ↓ polymer is matrix material ↓ Around 250°C temp resistance
- Metal matrix composites ↓ Metal alloy is matrix material ↓ Around 700°C temp resistance
- Ceramic matrix composites ↓ Ceramic is the matrix material ↓ Around 2500°C temp resistance
- Carbon–Carbon matrix composites ↓ Carbon is matrix material ↓ Around 3000°C temp resistan

(2)

(2)

2b **Field wise applications of composites**

Marine field

- Fishing boats
- Life boats
- Anti-marine ships
- Rescue ships
- Hover crafts

- Hulls
- Decks
- Pr
- Ru
- Ya

(2)

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(3)

Electronic ...

- Switches
- optical fibers
- Led TV's
- Mother boards
- Circuit boards
- wires
- Sinks

Aerospace

Gliders
Helicopter blades
Transmission shafts
Elevators
spoilers
Rocket boosters
Nozzles
Antenna covers
Fuselage, Doors, seats
Landing gears

4/6

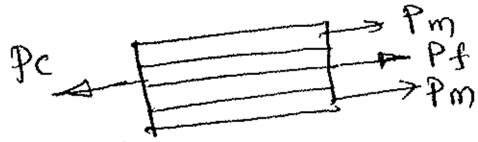
(2)

Automobile

- leaf springs
- Bumpers
- Body components
- chassis components
- Engine components
- Engine bonnet
- Mud wings
- Lamp heads
- Cabins
- Instrument panels
- window frames,

(2)

3. (a) Longitudinal modulus (E_1)



To determine this the following assumptions are made

(a) strain experienced by the composite is equal to fiber and matrix

$$\epsilon_c = \epsilon_f = \epsilon_m$$

(b) Load applied on the composite is shared by fiber and matrix

$$P_c = P_m + P_f$$

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$$\epsilon_c E_c A_c = \epsilon_f E_f A_f + \epsilon_m E_m A_m \qquad \sigma = \frac{P}{A}$$

$$P = \sigma A$$

$$\sigma = \epsilon E$$

$$E_c = E_f\left(\frac{A_f}{A_c}\right) + E_m\left(\frac{A_m}{A_c}\right) \qquad \because \epsilon_c = \epsilon_f = \epsilon_m$$

$$E_c = E_f V_f + E_m V_m \quad (a)$$

$$\because V_f = \frac{A_f}{A_c} ; \quad V_m = \frac{A_m}{A_c}$$

$$V_m + V_f = 1$$

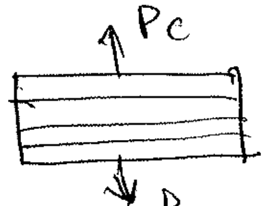
$$\therefore V_m = 1 - V_f$$

$$E_c = E_f V_f + E_m(1 - V_f)$$

$$E_1 = E_f V_f + E_m(1 - V_f) \quad \because E_c = E_1 \quad \text{—} (4)$$

(b) Transverse modulus (E_2)

Assumptions



(A) $\sigma_c = \sigma_f = \sigma_m$ — (1)

(B) $t_c = t_f + t_m$ — (2)

(C) $\delta_c = \delta_f + \delta_m$ (3)

$$\epsilon = \frac{\delta}{t} = \frac{\Delta L}{L}$$

$$\delta = \epsilon t \quad (4)$$

∴ eqn-(B) is modified as

$$\epsilon_c t_c = \epsilon_f t_f + \epsilon_m t_m$$

$$\epsilon_c = \epsilon_f\left(\frac{t_f}{t_c}\right) + \epsilon \left(\frac{t_m}{}\right)$$

$$\epsilon_c = \epsilon_f V_f + \epsilon$$

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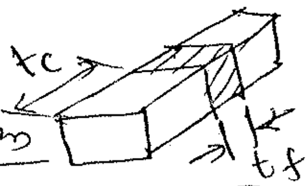
$$\frac{\sigma_c}{E_c} = \frac{\sigma_f}{E_f} V_f + \frac{\sigma_m}{E_m} V_m \qquad \because \epsilon = \frac{\sigma}{E}$$

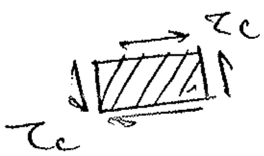
$$\frac{1}{E_c} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

$$E_c = \frac{E_f E_m}{V_f E_m + V_m E_f}$$

$$E_2 = \left[\frac{E_f E_m}{V_f E_m + V_m E_f}\right] \qquad \because E_c = E_2 \quad \text{--- (4)}$$

(c) In-plane shear modulus (G_{12})





$$G = \frac{\tau}{\gamma} \qquad \gamma = \frac{\delta}{t}$$

Assumptions

(a) $\tau_c = \tau_f = \tau_m$

$\delta_c, \delta_f, \delta_m$ are the deformations in the composites fiber & matrix resp.

$$\delta_c = \delta_f + \delta_m \quad \text{--- (1)}$$

$$\gamma_c t_c = \gamma_f t_f + \gamma_m t_m \quad \text{--- (2)}$$

$$G_{12} = \frac{\tau_c}{\gamma_c} \Rightarrow \gamma_c = \frac{\tau_c}{G_{12}} \quad \text{--- (3)}$$

$$\gamma_f = \frac{\tau_f}{G_f} \text{ (4)}, \quad \gamma_m = \frac{\tau_m}{G_m} \text{ --- (5)}$$

Sub. eqn (3), (4), (5) in eqn (2)

~~$\frac{\tau_c}{\gamma_c} t_c = $~~

$$\frac{\tau_c}{G_{12}} t_c = \frac{\tau_f}{G_f} t_f + \frac{\tau_m}{G_m} t_m$$

$$\frac{1}{\ } = \frac{1}{G_f} \frac{t_f}{t_c} +$$

$$\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}$$

where $\therefore V_f = \frac{t_f}{t_c}$

where

V_f = Vol fraction of fiber $\therefore V_m = \frac{t_m}{t_c}$

V_m = Vol. fraction of matrix, — (4)

(b) Major poisson's ratio (ν_{12})

$$\boxed{\nu_{12} = \nu_f V_f + \nu_m V_m}$$

where

ν_f = poisson's ratio of fiber

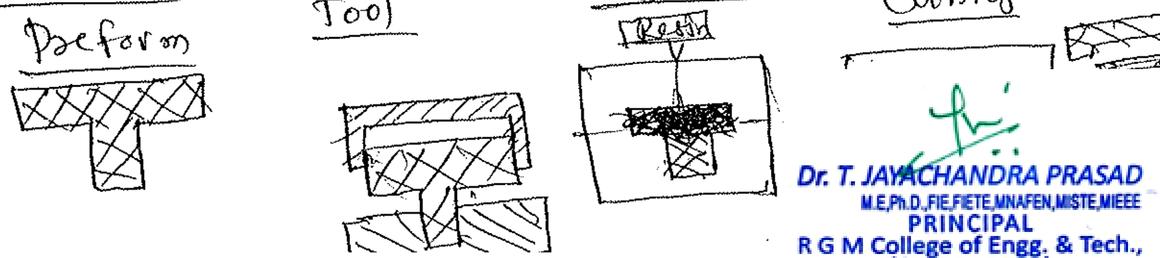
ν_m = poisson's ratio of matrix

V_m, V_f = Vol. fractions of matrix & fiber resp. — (2)

4(a) Resin transfer mould (RTM)

- RTM is an intermediate volume moulding process for producing composites
- In RTM resin is injected under pressure in mould cavity.
- This process produces parts with two finished surfaces.

Steps in RTM



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Advantages

- Low skilled labour is required
- Low tooling cost
- Low volatile emission
- Required design tailorability
- Good surface finish
- Very large complex shapes can be made
- less material wastage
- Good dimensional tolerances
- fast production, — (2)
- less emission due to closed mould

Disadvantages

- Preforms are labour intensive
- Waste may be high
- Chances of moisture entrapment
- Distortion of fiber during injection of resin due to fiber wash — (1)
- Control of resin uniformity is difficult.

Applications

- Complex structures can be produced
- Automotive body parts, big containers, bath tubs, helmets etc
- Vehicle panels (1)
- Boat hulls,
- Wind turbine blades
- Aerospace parts,

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5 (a) 4 types of matrix materials

1) polymers
2) Metals
3) ceramics
4) Carbon

Matrix
- PMC's (250°c)
  - Thermoplastics
  - Thermoset plastics
- MMC's (700°c)
- CMC's (2500°c)
- CCMC's (3000°c)

polymers

– Two different matrix materials are used in polymer matrix Composites (PMC's)

Thermosets	Thermoplastics
Ex: – Epoxy	polyethylene (PE)
– polyester	polycarbonate (PC)
– vinylester	polystyrene (PS)
	polyvinyl chloride (PVC)

(2)

Metals

Cermets, TiC, TiCN, ~~Cemen~~ Cemented carbides

(2)

Ceramics

Ceramics
Al_2O_3, SiC
Application: tool materials,

Carbon

Carbon, graphite
Application: ~~[illegible]~~ Brake pads

(2)

5 (b) **Silicon Carbide fiber (SiC)** (1)

(4M)

- High strength at elevated temp.
- High oxidation resistance
- High micro-structural stability
- High stiffness
- High tensile strength
- Low thermal expansion
- Low weight

(4)

Boron - carbide fiber

- Extreme hardness
- Difficult to sinter to high relative densities
- Good chemical resistance
- Good nuclear properties
- It's elastic modulus is close to diamond. (4)

(6) **Given Data**

$E_1 = 40\,GPa$	$G_{12} = 4.41\,GPa$	$\nu_{12} = 0.26$
$E_2 = 9\,GPa$	$G_{23} = 3.8\,GPa$	$\nu_{23} = 0.21$
$E_3 = 9\,GPa$	$G_{13} = 3.8\,GPa.$	$\nu_{13} = 0.21$

Find

(a) Compliance matrix [S] (2)

(b) Stiffness matrix [C]

Given material

orthotropic material

Sol:- Using Betti Reciprocal law

$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}$$

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$

$\nu_{21} = \frac{E}{E}$

$= \frac{9}{40}$

$$\frac{\nu_{31}}{E_3} = \frac{\nu_{13}}{E_1}$$

$$\nu_{31} = \frac{E_3}{E_1}\nu_{13} = \frac{9}{40} \times 0.21$$

$$\boxed{\nu_{31} = 0.047}$$

$$\frac{\nu_{23}}{E_2} = \frac{\nu_{32}}{E_3}$$

$$\nu_{32} = \frac{E_3}{E_2}\nu_{23} = \frac{9}{9} \times 0.21$$

$$\boxed{\nu_{32} = 0.21} \quad (4)$$

WKT

$$\{\epsilon\} = [S]\{\sigma\}$$

where $[S]$ = compliance matrix.

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{2} \\ \tau \\ \tau \end{Bmatrix}$$

$$S_{11} = \frac{1}{E_1} = \frac{1}{40\times10^9} = 2.5\times10^{-11}\ Pa^{-1}$$

$$S_{12} = -\frac{\nu_{21}}{E_2} = -\frac{0.0585}{9\times10^9} = -6.5\times10^{-12}\ Pa^{-1}$$

$$S_{12} = S_{21} = -\frac{\nu_{21}}{E_2} = 6.5\times10^{-12}\ Pa^{-1}$$

$$S_{13} = -\frac{\nu_{31}}{E_3} = -\frac{0.047}{9\times10^9} = -5.22\times10^{-12}\ Pa^{-1}$$

$$S_{13} = S_{31} = -\frac{\nu_{31}}{E_3} = -5.22\times10^{-12}\ Pa^{-1}$$

$$S_{22} = \frac{1}{E_2} = \frac{1}{9\times10^9} = 1.11\times10^{-10}\ Pa^{-1}$$

$$S_{23} = -\frac{\nu_{23}}{E_2} = -\frac{0.21}{9\times10^9} = -$$

$$S_{23} = -\nu_{23} = -9.23\times10^{-11}\ Pa^{-1}$$

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$S_{33} = \frac{1}{E_3} = \frac{1}{9\times10^9} = 1.11\times10^{-10}\ Pa^{-1}$ (13)

(47a)

$S_{44} = \frac{1}{G_{23}} = \frac{1}{3.8\times10^9} = 2.63\times10^{-10}\ Pa^{-1}$

$S_{55} = \frac{1}{G_{13}} = \frac{1}{3.8\times10^9} = 2.63\times10^{-10}\ Pa^{-1}$

$S_{66} = \frac{1}{G_{12}} = \frac{1}{4.41\times10^9} = 2.26\times10^{-10}\ Pa^{-1}$

$$[S] = \begin{bmatrix} 2.5\times10^{-11} & -6.5\times10^{-12} & -5.22\times10^{-12} & 0 & 0 & 0 \\ -6.5\times10^{-12} & 1.11\times10^{-10} & -2.33\times10^{-11} & 0 & 0 & 0 \\ -5.25\times10^{-12} & -2.33\times10^{-11} & 1.11\times10^{-10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.63\times10^{-10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.63\times10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.26\times10^{-10} \end{bmatrix}$$ (4)

WKT

Stiffness matrix is given by

$\{\sigma\} = [Q]\{\epsilon\}$ (or) $\{\sigma\} = [C]\{\epsilon\}$

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

$C_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} = \frac{40\times10^9}{(1-0.26\times0.0585)}\ Pa = 4.061\times10^{10}\ Pa$

$C_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = \frac{9\times10^9}{(1-0.26\times0.0585)}\ Pa =$

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$$C_{33} = \frac{E_3}{1-\nu_{13}\nu_{31}} = \frac{(\times 10^9)}{(1-0.21\times 0.047)} Pa = 9.089\times 10^9 Pa = 9.089 GPa$$

$C_{44} = G_{23} = 3.8\times 10^9 Pa = 3.8 GPa$

$C_{55} = G_{13} = 3.8 GPa$

$C_{66} = G_{12} = 4.41 GPa$

$$C_{12} = \frac{\nu_{12}E_{12}}{1-\nu_{12}\nu_{21}} = \frac{0.26\times 9\times 10^9}{(1-0.26\times 0.0585)} Pa = 2.37\times 10^9 Pa$$

$= 2.377 GPa$

$$C_{13} = \frac{\nu_{13}E_3}{1-\nu_{13}\nu_{31}} = \frac{0.21\times 9\times 10^9}{(1-0.21\times 0.047)} Pa = 1.90\times 10^9 Pa$$

$C_{13} = 1.90 GPa \quad \because C_{13} = C_{31}$

$$C_{21} = \frac{\nu_{21}E_1}{1-\nu_{21}\nu_{12}} = C_{12} = 2.37\times 10^9 Pa$$

$$C_{23} = \frac{\nu_{23}E_3}{1-\nu_{23}\nu_{32}} = \frac{0.21\times 9\times 10^9}{1-0.21\times 0.21} Pa = 1.977\times 10^9 Pa$$

$C_{23} = 1.97 GPa$

$= C_{23} = C_{32} \quad \because C_{ij} = C_{ji}$

$$[C] = \begin{bmatrix} 40.61 & 2.37 & 1.90 & 0 & 0 & 0 \\ 2.37 & 9.13 & 1.977 & 0 & 0 & 0 \\ 1.89 & 1.977 & 9.089 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.41 \end{bmatrix} GPa \quad (4)$$

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(7) Given Data

$\theta = 30^\circ$

$E_1 = 204\ GPa$

$E_2 = 18.5\ GPa$

$\nu_{12} = 0.23$ (2)

$G_{12} = 5.59\ GPa$

$S_{11} = \frac{1}{E_1} = \frac{1}{204\times10^9} = 4.901\times10^{-12}\ Pa^{-1}$ (15)

$S_{22} = \frac{1}{E_2} = \frac{1}{18.5\times10^9} = 5.405\times10^{-11}\ Pa^{-1}$

$S_{12} = \frac{-\nu_{12}}{E_1} = \frac{-0.23}{204\times10^9} = -1.127\times10^{-12}$

$S_{66} = \frac{1}{G_{12}} = \frac{1}{5.59\times10^9} = 1.78\times10^{-10}$

WRT

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xs} \\ S_{yx} & S_{yy} & S_{ys} \\ S_{sx} & S_{sy} & S_{ss} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

↓

Reduced transformed compliance matrix

$C = \cos\theta = \cos 30^\circ$

$S = \sin\theta = \sin 30^\circ$

where

$S_{xx} = ~~m^4 S_{11} + n^4 S_{22} + 2\times m^2n^2 S_{12} + m^2n^2 S_{66}~~$

$= C^4 S_{11} + S^4 S_{22} + 2C^2S^2 S_{12} + C^2S^2 S_{66}$

$S_{xx} = (0.866)^4\times4.901\times10^{-12} + (0.5)^4\times5.405\times10^{-11} + 2\times(0.866)^2\times(0.5)^2\times(-1.127\times10^{-12}) + (0.866)^2\times(0.5)^2\times1.788\times10^{-10}$

$S_{xx} = ~~3.922\times10^{-11}\ Pa^{-1}~~$ $7.27\times10^{-11}\ Pa^{-1}$

$S_{xy} = C^2S^2 S_{11} + C^2S^2 S_{22} + (C^4+S^4) S_{12} - C^2S^2 S_{66}$

$= (0.866)^2\times(0.5)^2\times4.901\times10^{-12} + (0.866)^2\times(0.5)^2\times5.405\times10^{-11} + ((0.866)^4+(0.5)^4)\times-1.127\times10^{-12} - (0.866)^2\times(0.5)^2\times1.788\times10^{-10}$

$S_{xy} = ~~2.317\times10^{-11}\ Pa^{-1}~~$ $= -2.317\times10^{-11}\ Pa^{-1}$

$S_{yy} = S^4 S_{11} + C^4 S_{22} + 2C^2S^2 S_{12} + C^2S^2 S_{66}$

$= (0.5)^4\times4.901\times10^{-12} + (0.866)^4\times5.405\times10^{-11} + 2\times(0.866)^2\times(0.5)^2(-1.127\times10^{-12}) + (0.866)^2\times(0.5)^2 ~~\times1.788\times10^{-10}~~$ (2)

$S_{yy} = ~~3.63\times10^{-11}\ Pa^{-1}~~$

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$$S_{ss} = 4c^2s^2 S_{11} + 4c^2s^2 S_{22} - 8c^2s^2 S_{12} + (c^2-s^2)^2 S_{66}$$

$$= 4 \times (0.866)^2 \times (0.5)^2 \times 4.901\times10^{-12} + 4 \times (0.866)^2 \times (0.5)^2 \times 5.405\times10^{-11}$$
$$- 8 \times (0.866)^2 \times (0.5)^2 \times (-1.127\times10^{-12})$$
$$+ [(0.866)^2 - (0.5)^2]^2 \times 1.788\times10^{-10}$$

$S_{ss} =$ ~~$2.24\times10^{-10}\ pa^{-1}$~~ $4.52\times10^{-10}\ pa^{-1}$

$$S_{xs} = 2c^3s S_{11} - 2cs^3 S_{22} + 2(cs^3 - c^3s) S_{12} + (cs^3 - c^3s)$$

$$= 2 \times (0.866)^3 \times 0.5 \times 4.901\times10^{-12} - 2 \times 0.866 \times (0.5)^3 \times 5.405\times10^{-11}$$
$$+ 2(0.866 \times (0.5)^3 - (0.866)^3 \times 0.5) \times (-1.127\times10^{-12})$$
$$+ (0.866 \times 0.5^3 - (0.866)^3 \times 0.5) \times 1.788\times10^{-10}$$

$S_{xs} =$ ~~$-4.67\times10^{-11}\ pa^{-1}$~~ $-8.59\times10^{-11}\ pa^{-1}$

$$S_{ys} = 2cs^3 S_{11} - 2c^3s S_{22} + 2(c^3s - cs^3) S_{12} + (c^3s - cs^3) S_{66}$$

$$= 2 \times 0.866 \times (0.5)^3 \times 4.901\times10^{-12} - 2 \times (0.866)^3 \times 0.5 \times 5.405\times10^{-11}$$
$$+ 2((0.866)^3 \times 0.5 - 0.866 \times (0.5)^3) \times 1.788\times10^{-10}$$

$S_{ys} = 4$~~$.336\times10^{-11}\ pa^{-1}$~~ (or) $4.336\times10^{-11}\ pa^{-1}$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_y} & \frac{\eta_{xs}}{G_{xy}} \\ -\frac{\nu_{yx}}{E_x} & \frac{1}{E_y} & \frac{\eta_{sy}}{G_{xy}} \\ \frac{\eta_{sx}}{E_x} & \frac{\eta_{ys}}{E_y} & \frac{1}{G_{xy}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad -(1)$$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xs} \\ S_{yx} & S_{yy} & S_{ys} \\ S_{sx} & S_{sy} & S_{ss} \end{bmatrix} \begin{Bmatrix} \sigma_x \end{Bmatrix} \quad (2)$$

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Let us equate ① & ②

eqn ① = eqn ② in terms of compliance matrix

$$\begin{bmatrix} S_{xx} & S_{xy} & S_{xs} \\ S_{yx} & S_{yy} & S_{ys} \\ S_{sx} & S_{sy} & S_{ss} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & \frac{-\nu_{yx}}{E_y} & \frac{\eta_{xs}}{G_{xy}} \\ \frac{-\nu_{xy}}{E_x} & \frac{1}{E_y} & \frac{\eta_{sy}}{G_{xy}} \\ \frac{\eta_{sx}}{E_x} & \frac{\eta_{ys}}{E_y} & \frac{1}{G_{xy}} \end{bmatrix}$$

$$S_{xx} = \frac{1}{E_x} \Rightarrow E_x = \frac{1}{S_{xx}} = \frac{1}{7.27\times10^{-11}} = 1.37\times10^{10}\ Pa$$

$$S_{yy} = \frac{1}{E_y} \Rightarrow E_y = \frac{1}{S_{yy}} = \frac{1}{6.38\times10^{-11}} = 1.56\times10^{10}\ Pa$$

$$S_{ss} = \frac{1}{G_{xy}} \Rightarrow G_{xy} = \frac{1}{S_{ss}} = \frac{1}{4.52\times10^{-10}} = 2.22\times10^{9}\ Pa$$ ②

$$S_{xy} = \frac{-\nu_{yx}}{E_y}$$

$$\nu_{yx} = -E_y \times S_{xy}$$

$$= -(1.56\times10^{10}) \times -2.317\times10^{-11}$$

$$\boxed{\nu_{yx} = 0.3614}$$ $\because \nu_{ij} = \nu_{ji}$

$$\boxed{\nu_{yx} = \nu_{xy} = 0.3614}$$

$$S_{xs} = \frac{\eta_{xs}}{G_{xy}} \Rightarrow \eta_{xs} = S_{xs} \times G_{xy}$$ ②

$$= -8.59\times10^{-11} \times 2.22\times10^{9}$$

$$\boxed{\eta_{xs} = -0.188}$$ $\boxed{\because \eta_{ij} = \eta_{ji}}$

$\eta_{xs} =$ ~

$$\boxed{\eta_{sx} = -0.188}$$

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$$S_{ys} = \frac{\eta_{sy}}{G_{xy}} \Rightarrow \eta_{sy} = G_{xy} \times S_{ys}$$

$$\eta_{sy} = 2.22 \times 10^{9} \times 4.33 \times 10^{-11}$$

$$\eta_{sy} = 0.095$$

$$\because \eta_{ij} = \eta_{ji}$$

$$\boxed{\eta_{sy} = \eta_{ys} = 0.095}$$

(2)

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THE END

7/1/4/21

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7. Given data

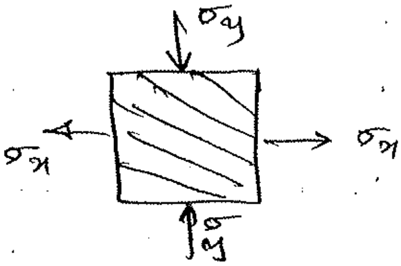
$(F_1)_t = 2280\ MPa$

$(F_2)_t = 59\ MPa$

$F_6 = 69\ MPa$

$(F_1)_c = 1450\ MPa$

$(F_2)_c = 228\ MPa$ — (1M)



As 'θ' is not given let us assume θ = 60°

$m = \cos 60 = 0.5$

$n = \sin 60 = 0.866$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [T] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

where

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.749 & 0.866 \\ 0.749 & 0.25 & -0.866 \\ -0.433 & 0.433 & -0.499 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} 0.25 & 0.749 & 0.866 \\ 0.749 & 0.25 & -0.866 \\ -0.433 & 0.433 & -0.499 \end{bmatrix} \begin{Bmatrix} F_0 \\ -F_0 \\ 0 \end{Bmatrix}$$ — (4M)

$\sigma_1 = 0.25 F_0 - 0.749 F_0 + 0 = -0.499 F_0$

$\sigma_2 = 0.749 F_0 - 0.25 F_0 + 0 = 0.499 F_0$

$\tau_{12} = -0.433 F_0 - 0.433 F_1 \quad = -0.433 F$

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Tsai - Hill Theory

$\sigma_1 = -0.499 F_0$

$\sigma_2 = 0.499 F_0$

$\tau_{12} = -0.433 F_0$

$$\left(\frac{\sigma_1}{(F_1)_t^u}\right)^2 - \left(\frac{\sigma_1 \sigma_2}{[(\sigma_1)_t^u]^2}\right) + \left(\frac{\sigma_2}{(F_2)_t^u}\right)^2 + \left(\frac{\tau_{12}}{(\tau_{12})^u}\right)^2 \geq 1$$

$$\left(\frac{-0.499 F_0}{2280}\right)^2 - \left(\frac{-0.499 \times 0.499 F_0^2}{(2280)^2}\right) + \left(\frac{0.499 F_0}{59}\right)^2 + \left(\frac{-0.433 F_0}{69}\right)^2 \geq 1$$

$1.2387 \times 10^{-4} F_0 \geq 1$ — (4M)

$$\boxed{F_0 = 8072.67 \text{ MPa}}$$

Ans:

$\sigma_x = F_0 = 8072.67$ MPa

$\sigma_y = -F_0 = -8072.67$ MPa

$\tau_{xy} = 0$

— (5M)

17/11/18

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